

FREQUENCY STABILIZED AND HOMODYNE LASER DIODE

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Abstract

A displacement measurement with an uncertainty of less than sub-nanometer is required in many industrial fields. A laser interferometer has become a powerful tool in nanometrology, because the wavelength of the light source can be calibrated by meter definition. In this paper, we propose the use of sinusoidal phase (frequency) modulation on a laser diode (LD) to achieve both the frequency stabilization of the LD and a displacement measurement by a homodyne interferometer.

Keywords: Frequency stabilization, Laser diode, Iodine, Homodyne interferometer.

1. INTRODUCTION

A displacement measurement with an uncertainty of less than sub-nanometer is crucial to many fields of modern technologies such as nanotechnology, precision engineering, optical metrology, semiconductor manufacturing, etc. Laser interferometers are widely utilized for the displacement measurement at nanoscale because of its inherent accuracy. Moreover, this method can keep the traceability to the definition of the meter, if the wavelength is calibrated. Recently, an LD is selected as a light source because of its high power, small size and high efficiency. Moreover, the LD offers direct frequency modulation through an injection current modulation [1] and large frequency tunability that are widely utilized in laser spectroscopy. However, the long term drift of the laser frequency, typically at a rate of several megahertz per minute, must be improved before it can be applied for high accurate displacement measurements [2]. Fortunately, molecular iodine has a broad spectrum of absorption lines from the green to the near infrared (500-900nm). Its hyperfine structure components provide a spectrum of reference frequencies for frequency stabilization of lasers. The frequency stabilization of the laser diode to iodine hyperfine structure components has been extensively investigated and reported [3] ~ [6]. To detect the absorption signal buried in the noise, the frequency of the laser is modulated across the hyperfine lines.

A sinusoidal phase (frequency) modulation technique [7] ~ [9] is effective for the displacement measurement by an interferometer. Recently, Madden *et al.* have reported the availability of the sinusoidal phase (frequency) modulation on an LD to a Michelson interferometer [10]. They have used the Lissajous diagram [11] to obtain the optical path movement (equivalence to displacement) using 2-nd and 3-rd harmonic modulated signals. To draw the exact Lissajous diagram with the sinusoidal phase (frequency) modulation, it is needed to know the value of the modulation index (or modulation depth). In the sinusoidal frequency modulation

for the Michelson interferometer, the value of the modulation index varies with the modulated frequency bandwidth and the optical path difference between two arms of the interferometer. Therefore, Madden *et al.* almost fixed both the modulation bandwidth and the optical path difference, and determined the modulation index. To increase a measurable displacement of the Michelson interferometer with the sinusoidal frequency modulation on the LD, and to use the Lissajous diagram simultaneously, it is required to know the dynamic change of the modulation index due to the optical path change. Some papers [12] ~ [14] have argued that the displacement measurement of an interferometer is possible from the estimation of the dynamic modulation index change by $J_1 \sim J_4$, modified $J_1 \sim J_4$, and $J_1 \sim J_3$ methods, where J_i means i -th order Bessel function. However, since these methods have not used the Lissajous diagram, the accuracy of the displacement measurement is not enough.

In this paper, first, we propose the combination method to stabilize the LD frequency to iodine absorption line, and to measure the displacement for a Michelson interferometer using the sinusoidal frequency modulation on the LD. Secondly, we propose the enhancement of the measurable displacement range while keeping the measurement accuracy by combining the estimation of the dynamic modulation index change and the Lissajous diagram. Lastly, we report preliminary experiments and discussion.

2. THEORY

Figure 1 shows a frequency stabilized LD and a homodyne interferometer. This system is divided into two parts, the frequency stabilized LD source and the homodyne interferometer. In the frequency stabilized LD, the frequency of laser source is modulated by sinusoidal current signal and passed through a Faraday rotator (FR). A beam splitter (BS) is employed to divide the output laser into two beams, one beam passes through the iodine cell and the other enters to a Michelson interferometer. The polarized beam splitter 1 (PBS1), the quarter wave plate 1 (QWP1), and the mirror 1 (M1) are utilised to direct the absorption signal to photodetector 1 (PD1). The third harmonic signal is detected using a lock-in amplifier (LIA). In order to lock the laser frequency, the output signal from the LIA is fed back to current driver via the PID controller. Then, the LD that its frequency is lock to the hyperfine of iodine is used for the homodyne interferometer.

The output beam from BS1 is fed into a Michelson homodyne interferometer. One PBS and two QWPs are employed to direct two orthogonal polarization signals on PD 2. A polarizer is used to combine these signals to make interference signal. Normally, in the homodyne interferometer, the intensity of the interference fringes are

stable in space because they are independent time. In our research, the intensity of the interference signal has a temporal dependence because of the frequency modulation. To obtain the Lissajous diagram, we apply the sinusoidal phase (frequency) modulation technique [7].

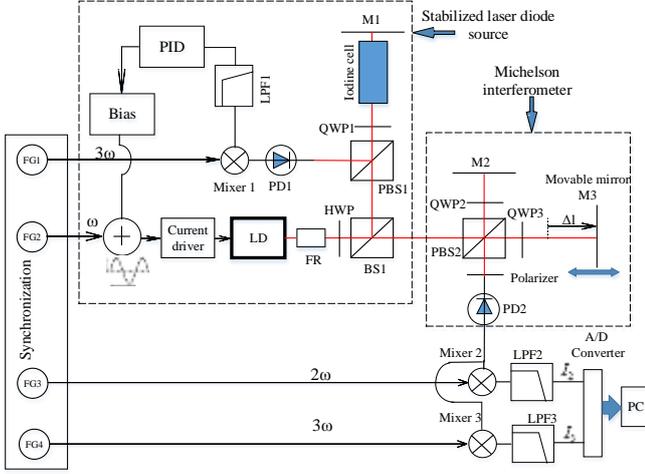


Figure 1. Schematic diagram of the frequency stabilized laser diode homodyne interferometer. LD: laser diode, FR: Faraday rotator, HWP: half wave plate, QWP: quarter wave plate, M: mirror, PD: photodetector, FG: function generator, LPF: low pass filter, PC: computer.

In general, the frequency modulation using sinusoidal signal of output laser is given as

$$f(t) = f_0 + \Delta f \sin 2\pi f_m t \quad (1),$$

where Δf is modulated frequency width, f_m is modulation frequency, f_0 is carrier frequency. The phase component can be derived by equation below

$$\phi(t) = \int_0^t 2\pi f(t) dt = 2\pi f_0 t - \frac{\Delta f}{f_m} \cos 2\pi f_m t + \phi_0 \quad (2),$$

where ϕ_0 is the initial phase of laser source. For homodyne interferometer, the phases of the reference and measurement arms can be written as

$$\phi_r = 2\pi f_0 t - \frac{\Delta f}{f_m} \cos 2\pi f_m t + \phi_0 \quad (3),$$

$$\phi_m = 2\pi f_0 (t - \tau) - \frac{\Delta f}{f_m} \cos 2\pi f_m (t - \tau) + \phi_0 \quad (4),$$

where ϕ_r, ϕ_m are the phases of the reference and measurement arms, respectively, τ is a delay time between them. The different phase between the reference and the measurement arms is given

$$\phi_m - \phi_r = -2\pi f_0 \tau - \frac{2\Delta f}{f_m} \sin \left[2\pi f_m \left(t - \frac{\tau}{2} \right) \right] \times \sin \left[2\pi f_m \cdot \left(\frac{\tau}{2} \right) \right] \quad (5).$$

Because the delay time τ is negligibly small so that equation (5) can be simplified by using trigonometric approximations [15]

$$\sin \left[2\pi f_m \cdot \left(\frac{\tau}{2} \right) \right] \approx 2\pi f_m \cdot \left(\frac{\tau}{2} \right)$$

and $\sin \left[2\pi f_m \left(t - \frac{\tau}{2} \right) \right] \approx \sin 2\pi f_m t$. Equation (5) can be

rewritten as

$$\phi_m - \phi_r = -2\pi f_0 \tau - 2\pi \Delta f \tau \sin 2\pi f_m t \quad (6).$$

The intensity of resulting beam can be derived by equation below

$$\begin{aligned} I &= E_{0r}^2 + E_{0m}^2 + 2E_{0r}E_{0m} \cos(\phi_m - \phi_r) \\ &= E_{0r}^2 + E_{0m}^2 + 2E_{0r}E_{0m} \cos(2\pi f_0 \tau + 2\pi \Delta f \tau \sin 2\pi f_m t) \\ &= E_{0r}^2 + E_{0m}^2 + 2E_{0r}E_{0m} \cos(m \sin 2\pi f_m t + \phi) \end{aligned} \quad (7),$$

$$\text{where } m = 2\pi \Delta f \tau = \frac{4\pi \Delta f n \Delta l}{c} \quad (8),$$

$$\phi = 2\pi f_0 \tau = 4\pi \frac{n \Delta l}{\lambda_0} \quad (9),$$

where n is the refractive index, Δl is the path difference, λ_0 is the wavelength of laser source. In equation (7), m implies the modulation index.

Using Bessel function, equation (7) is expanded as

$$\begin{aligned} I &= E_{0r}^2 + E_{0m}^2 + 2E_{0r}E_{0m} J_0(m) \cdot \cos \phi \\ &+ 2E_{0r}E_{0m} \left\{ -2J_1(m) \sin 2\pi f_m t \cdot \sin \phi \right. \\ &+ 2J_2(m) \cos 4\pi f_m t \cdot \cos \phi \\ &\left. - 2J_3(m) \sin 6\pi f_m t \cdot \sin \phi + \dots \right\} \end{aligned} \quad (10).$$

Using the lock-in amplifier we can extract the intensity of two orthogonal signals, 2nd and 3rd harmonic terms, from equation (8) as below [10]

$$I_2 = 2E_{0r}E_{0m} J_2(m) \cos(\phi) \quad (11),$$

$$I_3 = -2E_{0r}E_{0m} J_3(m) \sin(\phi) \quad (12),$$

From equation (9) and equation (10) we can obtain the Lissajous diagram. Then the Lissajous diagram can be utilized for tracking phase shift. The phase shift can be calculated as

$$\phi = \arctan \left(-\frac{J_2(m) I_3}{J_3(m) I_2} \right) \quad (13).$$

The relationship between phase shift and the displacement is given as

$$\Delta l = \frac{\lambda_0}{4\pi n} \phi \quad (14),$$

where λ_0 is central wavelength of laser source.

From the equations (13) and (14), we can determine the displacement using $I_2, I_3, J_2(m)$, and $J_3(m)$. To draw the exact Lissajous diagram, we must know the value of the modulation index m . Equation (8) shows m varies with the displacement Δl dynamically. However, from the power spectrum of the interferometer intensity shown in equation (10), we can estimate the modulation index m .

3. PRELIMINARY EXPERIMENT AND DISCUSSION

To confirm our measurement method for homodyne interferometer, we built an experimental system shown in figure 2. The laser source, with the wavelength of 635 nm and the frequency modulation of 33 kHz, is employed. The

initially unbalanced length between two arms was set as 1 meter. The moving mirror was driven by a piezo electric actuator (PZT). We applied a triangular signal with an amplitude of 1 μm and a frequency of 1 Hz to the PZT. An avalanche photodetector was used to detect the interference signal. The measurement time was 5 seconds and cut off frequency of the low pass filter of the LIA was 300 Hz. The measurement conditions in our experiment is shown in table 1.

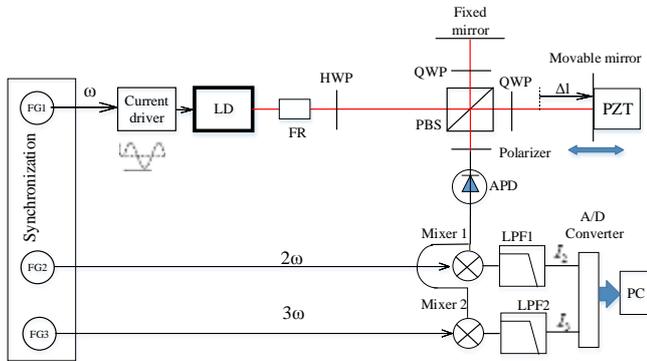


Figure 2. Experimental setup of homodyne interferometer.

Table 1. The measurement conditions application for homodyne interferometer

Laser diode source	HL6344G
LD wavelength	635 [nm]
Modulation index	4.64 [rad]
Modulation frequency bandwidth	111 [MHz]
Modulation frequency for LD	33 [kHz]
Modulation depth of PZT stage	1 [μm]
Modulation frequency of PZT stage	1 [Hz]
Sampling time	5 [s]
Cut off frequency of low pass filter of LIA	300 [Hz]

The Lissajous diagram of displacement measurement is shown in figure 3. The Lissajous figure is deformed due to the variation of the intensity of the laser source, the misalignment of the optics and the difference between $J_2(m)$ and $J_3(m)$. From figure 3, we can estimate the power spectrum of the interferometer intensity, also estimate the modulation index (lateral amplitude/vertical amplitude is proportional to $J_2(m)/J_3(m)$). In table 1, we also show the estimated modulation frequency bandwidth. Therefore, we can obtain the normalized Lissajous diagram by using the following equations

$$I_2' = \frac{I_2 J_3(m)}{\sqrt{[I_2 J_3(m)]^2 + [I_3 J_2(m)]^2}} = \cos(\phi), \quad (20)$$

$$I_3' = \frac{I_3 J_2(m)}{\sqrt{[I_2 J_3(m)]^2 + [I_3 J_2(m)]^2}} = \sin(\phi). \quad (21)$$

The normalized Lissajous diagram, is shown in figure 4. The radius and center of the normalized Lissajous diagram are 1 and the origin, respectively.

Finally, the displacement result is shown in figure 5. This experiment confirm that we can get the Lissajous diagram

from second and third harmonics of the interference signal. Thus a frequency modulation LD is a suitable light source for homodyne interferometer.

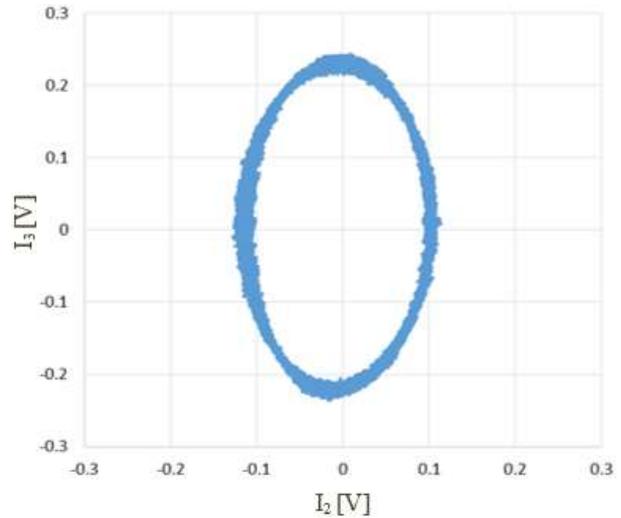


Figure 3. Lissajous figure of second and third harmonics.

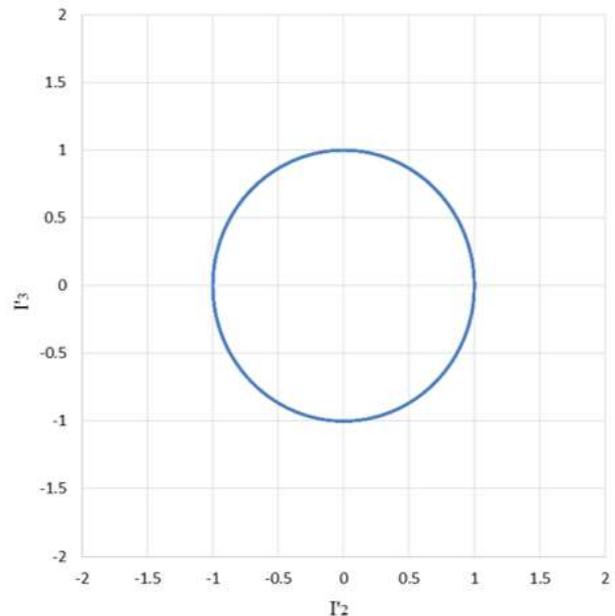


Figure 4. Normalized Lissajous figure of second and third harmonics.

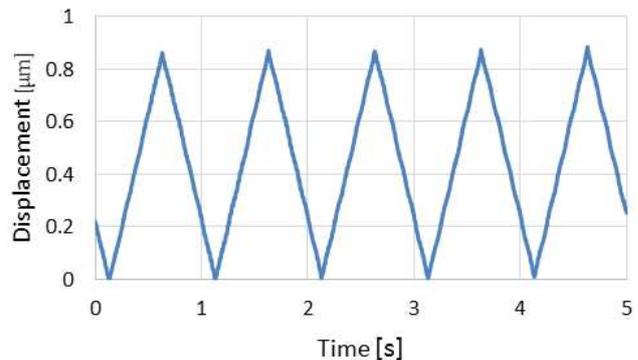


Figure 5. The displacement measurement result.

4. CONCLUSION

In this paper, we have proposed a displacement measurement method using an LD and a homodyne interferometer. Our method is based on the use sinusoidal phase (frequency) modulation on the LD to achieve both the frequency stabilization of the LD and the displacement measurement. The experimental results prove that our method is suitable for the high accuracy displacement measurement.

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