MULTI-READING HEAD CYLINDRICAL GRATING ANGLE ERROR COMPENSATION BASED ON HARMONIC ANALYSIS

Lao DABAO[1], Zhou WEIHU[1], Li WANGHONG[1], Shi DONG[1]

1 Academy of Opto-electronics, Chinese Academy of Sciences,

Num.9, Dengzhuang south Road, HaiDian District, Beijing, 100094, China

LAO Da-bao: laodabao@aoe.ac.cn

ZHOU Wei-hu: zhouweihu@aoe.ac.cn

LI Wan-hong: wanhongli28@163.com

SHI Dong: shidong135@sohu.com

Abstract:

On high precision one-dimension turntable, an angle measuring system was built with the cylindrical grating and uniform multi-reading head. Based on high precision multi-mirror polyhedron with autocollimator, angle measurement error was obtained. Then, the angle measuring error was analyzed by FFT, the compensation mechanism of multi-reading head was analyzed in detail. Each order of the angle error was compensated by using the harmonic analysis method, and sources of angle error were exposed. The experimental results show that the multi-reading head cylinder grating sensor angle measuring accuracy up to the ±0.7" after the compensation, which proves the feasibility of the system and compensation method.

Keywords: Angle error compensation; cylindrical grating; Multi-reading head; harmonic analysis

1. INTRODUCTION

Precision rotary table is extensively applied in many industries, with the improvement of technology, higher accuracy and smaller size is required in the meantime. Usually, high-precision angle measurement sensor is used to establish high accuracy table, but owe to the various error factors, such as axis sway and installation error, the accuracy of sensor cannot reach its so-called index. Thus, compensation should be introduced to improve accuracy. Error Compensation can be implemented in hardware and software aspects. In the hardware aspect, multi-readhead style is adopted instead of one-readhead style to restraint some errors [1]. In the software aspect, there are many methods for compensation. RBF network [2], particle swarm optimization [3], self-calibration with EDA-method is studied in [4-5]. By these effective compensations, angle measurement error can be reduced.

In this paper, error factors of one dimension rotary table are analyzed in detail, each order of the angle measurement error can be separated by FFT. Then, harmonic analysis method is adopted to fit compensation curve.

2. ERROR MAPPING

To test and verify error analysis and compensation method, one-dimension table with cylindrical grating and four readheads is designed as figure 1.

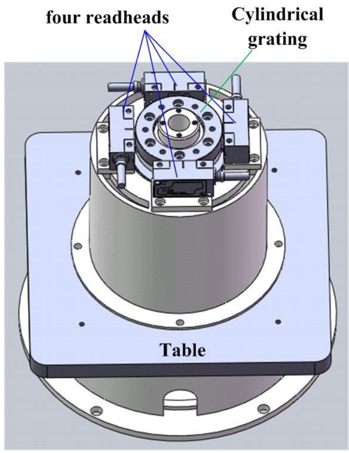


Fig.1: One dimension table

The angle measurement sensor, which is manufactured by renishaw, is consist of cylindrical grating, readheads, subdivision boxes, connector. Parameters of the sensor are as the following table 1.

Tab.1 Technical parameters of renishaw RESM

Diameter(mm)	Lines	Accuracy(″)	Resolution(″)
52	8192	±4.28	0.16

The four readheads are arranged at equal angle interval around the cylindrical grating. Error mapping of the table is finished by autocollimator and polyhedron, the installation looks like the following Fig. 2.

When we use n-mirror polyhedron to map the error, Formula 1 is used for each error at some angle position:

$$\Delta\theta_i = \theta_i - \left[360^\circ (i-1)/n + \Delta\alpha_{i,1} + (m_i - m_1)\right] \quad (1)$$

Where $\Delta\theta_i$ is the error at the angle θ_i, m_1 means the indication of autocollimator when the first mirror aim at the autocollimator. m_i means the indication of autocollimator when the i mirror aim at the autocollimator. i is a natural number from 1 to n. $\Delta\alpha_{i,1}$ is deviation between *i* mirror and first mirror.

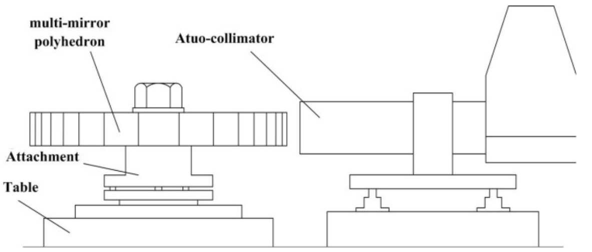


Fig.2: Error mapping installation

3. ERROR ANALYSIS

Angle measuring error factor include sensor self error, axis shifting, installation error. Sensor self error is consist of groove error, subdivision error, signal processing error, etc. Axis shifting bring about grating eccentricity and tilt, there are the source of one and two order harmonic component of the angle error. Installation error not only bring about the above component, but also cause deformation, which are the source of high order component of angle error.

3.1 Error of Sensor self

Groove error and subdivision error of sensor self produce system error and high order error.

Groove error of the grating adopted in this paper is 0.5 *μm*, as the interval between lines is 20*μm*. Suppose the diameter as D, angle measuring error δ_{gra} can be calculated by the following formula:

$$\delta_{gra} = \frac{0.5}{D/2\times1000}\times\frac{180}{\pi}\times3600 \qquad (2)$$

Subdivision error is nonlinear error caused by signal processing, it is related to installation and environment. The error is equivalent to groove error, which is about 0.2% of the interval between lines, that means 40 *nm* additional groove error is caused:

$$\delta_{SDE} = \frac{40\times10^{-3}}{D/2\times1000}\times\frac{180}{\pi}\times3600 \qquad (3)$$

Devote to the two major error, the sensor' error δ_s is:

$$\delta_s = \delta_{gra} + \delta_{SDE} \qquad (4)$$

3.2 Installation error

Installation error will cause eccentricity error, tilt error and distortion error.

A. Eccentricity

As showed in figure 3, eccentricity means the centre axis of grating and rotating axis are not the same. In the Fig. 3, *O* represents as the rotate axis. *e* is the eccentricity. *r* means radius of the grating. In the wake of rotating the grating, the track of grating centre is a circle whose centre is *O* and radius is *r*.

Assumed *O'* as the zero position, θ_p as the initial phase angle, when the grating centre move around *O* from *O'* to *O"*.

The actual rotation angle is θ, while the measuring angle by readhead is θ', thus, the angle measuring error is $\delta_e = \theta' - \theta$.

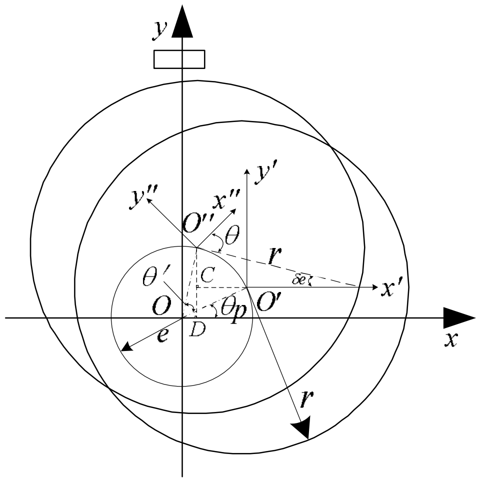


Fig.3 Angle error caused by Eccentricity

From the geometry in Fig.3, we can get $O''C = O''D - CD$, which means:

$$r\sin\delta_e = e\sin(\theta+\theta_p) - e\sin\theta_p \qquad (5)$$

Where $O''C = r\sin\delta_e$, $O''D = e\sin(\theta+\theta_p)$, $CD = e\sin\theta_p$.

We can get another equation as:

$$r\sin\delta_e = e\sin(\theta' - \delta_e + \theta_p) - e\sin\theta_p \qquad (6)$$

Since δ_e is a small angle, we can simplify formula (6):

$$\delta_e = \frac{e}{r}\left[\sin(\theta'+\theta_p) - \sin\theta_p\right] \qquad (7)$$

If initial phase angle $\theta_p = 0$, formula (7) can be simplified further:

$$\delta_e = \frac{e}{r}\sin\theta' \qquad (8)$$

From the formula (8), we can see that eccentricity caused one order angle measuring error. To restrain the error , two or four readheads are adopted to arrange around grating.

B. Tilt

Tilt of grating lead to number and width of moir fringes, which is one of the secondary order angle error source. Grating tilt is caused by installation tilt and axis shafting.

Fig.4 shows the influence on angle measurement by ring tilt. $\Delta\theta_p$ is the tilt angle between rotating axis and centre line of the grating. The actual rotation angle is θ, while the measuring angle by readhead is θ', thus, the angle measuring error is :

$$\delta_e = \theta' - \theta = -\frac{1}{4}\sin^2\Delta\theta_p\sin2\theta' \qquad (9)$$

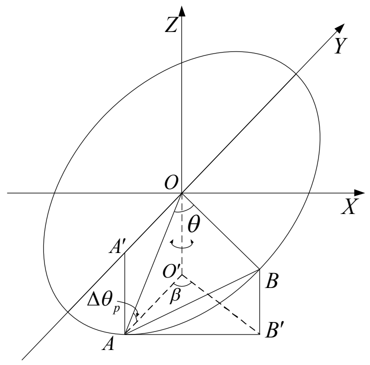


Fig.4 Influence on angle measurement by ring tilt

C. Distortion

Usually, we employ conical surface to assemble the grating, which can make the grating centre line and rotary axis alignment. But it may make the distortion of grating, like ellipse distortion, triple leaf distortion and high order distortion, as show in Fig. 5.

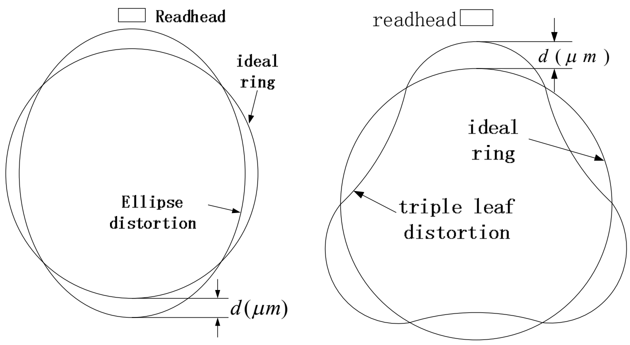


Fig.5 ellipse and triple leaf distortion

The angle error can be calculated by the following formula:

$$\delta_d = 412.5 \times \frac{d}{kD} \qquad (10)$$

Where k means the distortion order, k=2 when there is ellipse distortion, and k=3 when triple leaf distortion.

There may be four or higher order distortion in the installation course. when k is higher, d will be smaller.

3.3 Axis shifting

Axis shafting can be decomposed to the radial and axial direction shafting. It is mainly caused by tolerance of bearings, shafting of rotor, and installation error of axis.

Supposed the value of axis shafting in radial direction as w (unit:mm), angle measurement error δ_w can be expressed by the formula:

$$\delta_w = 412.5 \times \frac{w}{D} \qquad (11)$$

The error is high order component of angle measurement error. Angle error converted by Axis tilt caused by installation is similar to grating tilt analyzed above.

4. ERROR COMPENSATION METHOD

Angle measurement error can be compensated by harmonic analysis method is benefit from that amplitude and phase of some errors are stable, for example, eccentricity and tilt error, etc. But, the errors are not constant, they may be two-order or higher order component.

As a result, we can use the following harmonic expression for describing angle error.

$$\varepsilon(\theta_i) = \sum_{k=0}^{m} C_k \sin(k\theta_i + \varphi_k) \qquad (12)$$

Where $\varepsilon(\theta_i)$ means the error in the position θ_i, k is the order of error. C_k means the amplitude, and φ_k means phase.

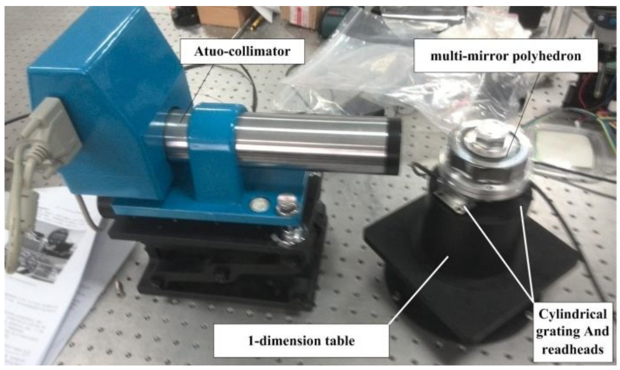
With formula (12), we can construct optimization fuctions.

$$\begin{aligned} e &= \min\left\{\sum_{i=1}^{n}[\varepsilon(\theta_i) - \Delta\theta_i]^2\right\} \\ &= \min\left\{\sum_{i=1}^{n}[\sum_{k=0}^{m} C_k \sin(k\theta_i + \varphi_k) - \Delta\theta_i]^2\right\} \end{aligned} \qquad (13)$$

If we get $\varepsilon(\theta_i)$ in the angle position θ_i, which is determined by polygon, C_k and φ_k can be resolved by LSM or other algorithm. As a result, we finish error mapping of the angle measurement system.

5. EXPERIMENT

With double-axis CCD autocollimator (CSZ-1A) and 23 mirror polygon, we get 23 angle errors in the 23 positions, the experiment set is showed in Fig. 6. By solving equations (13), the fitting curve can be got by harmonic analysis.



. Fig.5 error mapping experiment

In order to improve reliability, we obtain errors with lots of experiments in variable situation. We found repeatability error is about ±1.2 arc second. With FFT (Fast Fourier Transform), we can get the orders of system error present like that in Fig. 6.

In the same time, the amplitude and phase of 0,1,2, and 6 order is larger than order, also they are stable.

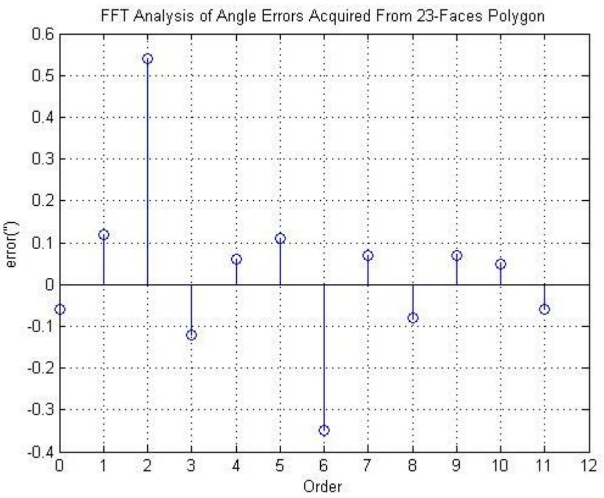


Fig. 6 angle error analyzed by FFT

Thus, the expression described by 0,1,2, and 6 order is (20) :

$$\varepsilon(\theta) = C_0 + C_1 \sin(\theta + \varphi_1) + C_2 \sin(2\theta + \varphi_2) + C_6 \sin(6\theta + \varphi_6) \tag{20}$$

To testing the error fitting curve, we use 8 mirror polygon for obtaining experiment data. The result of compensation is that the error is reduced from ±3 arc second to ±0.7 arc second. Thought the nominal error of the sensor is ±4.28 arc second, the value decrease to ±3 arc second because of multi-readhead. The result is showed intuitively in Fig. 7.

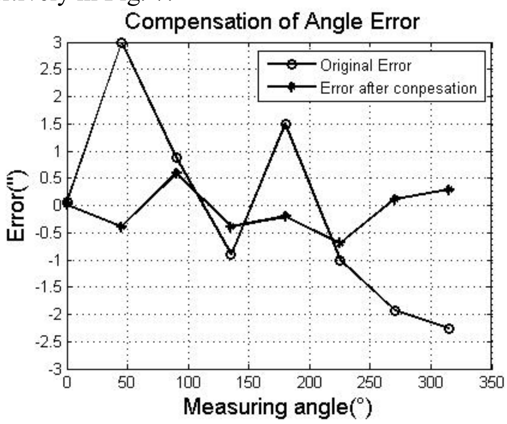


Fig.7 Result of Angle Error Compensation

6. CONCLUSION

Error factors of one dimension rotary table are analyzed in detail in this paper, and each order of the angle measurement error can be separated by FFT. Then, harmonic analysis method is adopted to fit compensation curve. From the result of experiments we can see that the compensation method is effective and feasible.

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