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## Precise determination of the jacket pressure coefficient of controlled-clearance pressure balances

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### **Abstract**

We are now developing a new controlled-clearance pressure balance to upgrade the hydraulic high-pressure standard up to 1 GPa. Characterization experiments for a 500 MPa controlled-clearance piston-cylinder were performed in detail over wide pressure range, and the effective area was estimated based on the Heydemann-Welch model. The estimation based on the conventional model agrees well with the present pressure standard. However, at high pressures around 500MPa, there appears some contradictions to the assumption of the model; especially, the jacket pressure coefficient depends apparently on the jacket pressure. A possible revision on the model is briefly discussed.

*Keywords:* controlled-clearance piston-cylinder, Heydemann-Welch model

### **1. Introduction**

Controlled-clearance type piston-cylinder (CCPC) has been widely used in the national standard laboratories to establish the primary pressure standard. For CCPC, the effective area and its pressure dependence can be estimated from the experimental results using the conventional model provided by P. L. M. Heydemann and B. E. Welch [1] and its modified forms [2, 3], which are combined with the numerical methods (such as finite element analysis). The experimentally accessible factors in the models are the jacket pressure coefficient and the piston fall rate. Although these values can offer only the indirect information about the distortion of the piston-cylinder unit and the profile of the clearance, the precise determination of these values enables us to understand the characteristics of each piston-cylinder unit, and check the

consistency of the numerical results and the model itself.

We are now developing a new controlled-clearance pressure balance to upgrade the hydraulic high-pressure standard. In this paper, important features of the new pressure balance are introduced, then the characteristics for 500 MPa CCPC are shown with some discussions about the analytical method.

## 2. Features of a New Controlled-Clearance Pressure Balance

Figure 1 shows the schematics of the new controlled-clearance pressure balance and the whole setup for characterization experiments. The new pressure balance mainly consists of three parts; a weight loading unit, a pressure generation device, and a controlled-clearance piston-cylinder assembly.

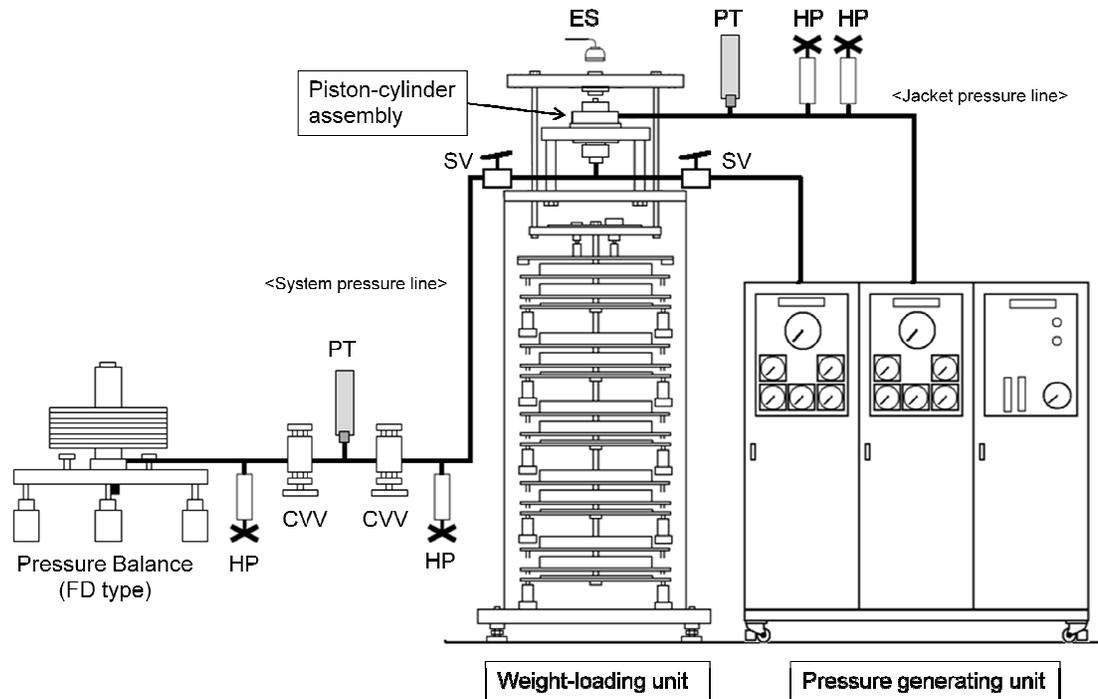


Figure 1: Schematic viewing of the new controlled-clearance pressure balance and the whole setup for characterization experiments. The devices are displayed in different scales for an easy overview. PT : pressure transducer, HP: hand pump, SV : shut-off valve, CVV : constant volume valve, ES : eddy-current sensor.

The overview of the weight-loading unit and the piston-cylinder assembly is shown in figure 2. Total mass of the weight-set is about 1100 kg, which consists of ten disk weights of 95 kg, five disk weights of 19 kg, eight cylindrical weights of 2.5 kg and ten cylindrical weights of 0.5 kg. Each weight is supported by pneumatic actuators and lifted up/down independently, which enables us to calibrate the mass of heavy weights *in situ* without moving them out of the

container. For the mass calibration, a digital mass comparator is installed on the mounting table where the piston-cylinder is usually placed. The *in situ* mass calibrations were performed twice, in 2003 and 2006. The relative difference in the mass of each weight between the two calibrations is less than 2 ppm (parts per million). Thus, it is confirmed that the mass of the weights are sufficiently stable.

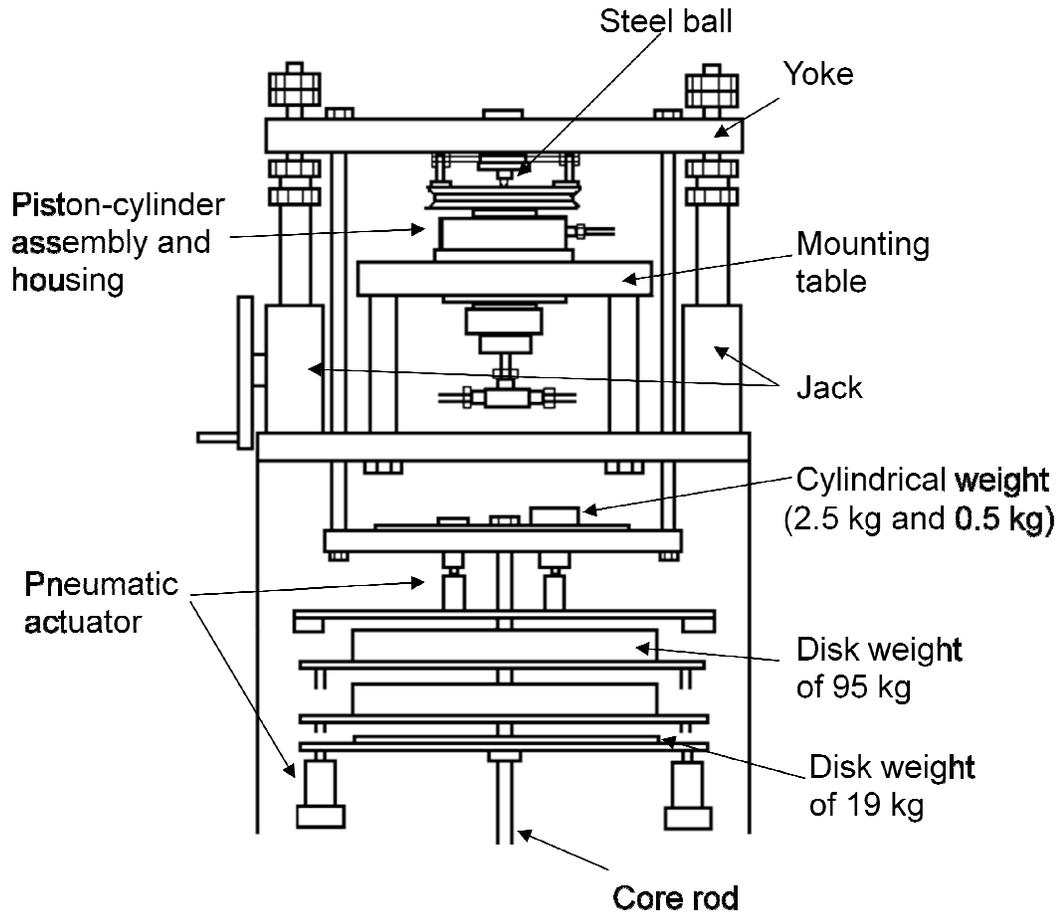


Figure 2: The overview of the weight-loading unit and the controlled-clearance piston-cylinder assembly.

The controlled-clearance piston-cylinder assembly is made from tungsten carbide and packed in a steel housing. The housing is designed to allow the jacket pressure to be applied up to about 50 % of the system pressure. There are several kinds of CCPC according to the pressure range. In this paper, results of 500 MPa CCPC are explained.

To operate CCPC, two separate pressure sources, the system pressure,  $p_s$ , and jacket pressure,  $p_j$ , are needed. Both pressures are applied by oil-operated compressors and intensifiers. Hand pumps are also used to adjust the pressure precisely. The jacket pressure is monitored by a precise pressure transducer.

For the characterization experiments, the CC pressure balance is connected to another pressure balance of the free-deformation (FD) type (see fig.1). A precise pressure transducer and constant volume valves are installed between them to perform cross-float measurements [4].

### 3. Characterization experiments for CCPC

The effective area of CCPC is evaluated based on the conventional Heydemann-Welch (H-W) model [1] as:

$$A(p_s, p_j, T) = A_0 \cdot (1 + \alpha(T - T_r)) \cdot (1 + b_p \cdot p) \cdot \{1 + d \cdot (p_z - p_j)\}, \quad (1)$$

where  $A_0$  is the effective area under the standard condition,  $\alpha$  is the sum of the thermal expansion coefficients of a piston and cylinder.  $b_p$  is the pressure distortion coefficient of a piston calculated from the elastic theory. The jacket pressure coefficient,  $d$ , and the zero clearance jacket pressure,  $p_z$ , are determined by the characterization experiments;  $d$  is determined by the cross-float measurements against the FD piston-cylinder, and  $p_z$  is determined by the change in the piston fall-rate due to the jacket pressure. These experiments were performed in steps of 50 MPa up to 500 MPa.

The procedure of the characterization experiments at each system pressure is as follows. First, the system pressure is increased to the target value without applying the jacket pressure. After waiting 15 minutes for pressure stabilization, the cross-float and piston fall-rate measurements are performed in series. After that, the jacket pressure was slowly increased in steps of  $0.05p_s$ . These two measurements are performed at each jacket pressure until the jacket pressure was increased up to  $0.5 p_s$ . During the application of  $p_j$ , it was carefully ensured that the piston was floating and rotating in the cylinder.

The jacket pressure coefficient is estimated from the change in the system pressure due to the jacket pressure, as.

$$d = \frac{1}{\Delta p_j} \frac{\Delta p_s}{p_s} \quad (2)$$

The change in the system pressure is determined by the cross-float measurements against the FD piston-cylinder. The pressure generated by CCPC without jacket pressure,  $p_s$ , is equilibrated with that by FD piston-cylinder. When the jacket pressure,  $\Delta p_j$ , is applied to CCPC, the pressure increases from  $p_s$  to  $p_s + \Delta p_s$ , which is equilibrated again with FD piston-cylinder by loading small mass on the FD piston-cylinder.

Following the cross float measurement, a piston fall-rate was measured. During the measurement, shut-off valves (shown in fig. 1 as "SV") were closed to

minimize the effect of possible leakage in the pressure line. The change in the piston position was measured by an eddy-current sensor (shown in fig. 1 as “ES”). The data are obtained every 12 seconds for about ten minutes. The fall-rate averaged for one minute is plotted against the piston position in fig. 3. The center of the piston stroke was set to be the reference height in the cross-float measurement (2.5 mm in the figure). Because  $v$  was independent of the position,  $v$  was averaged over the whole piston stroke.

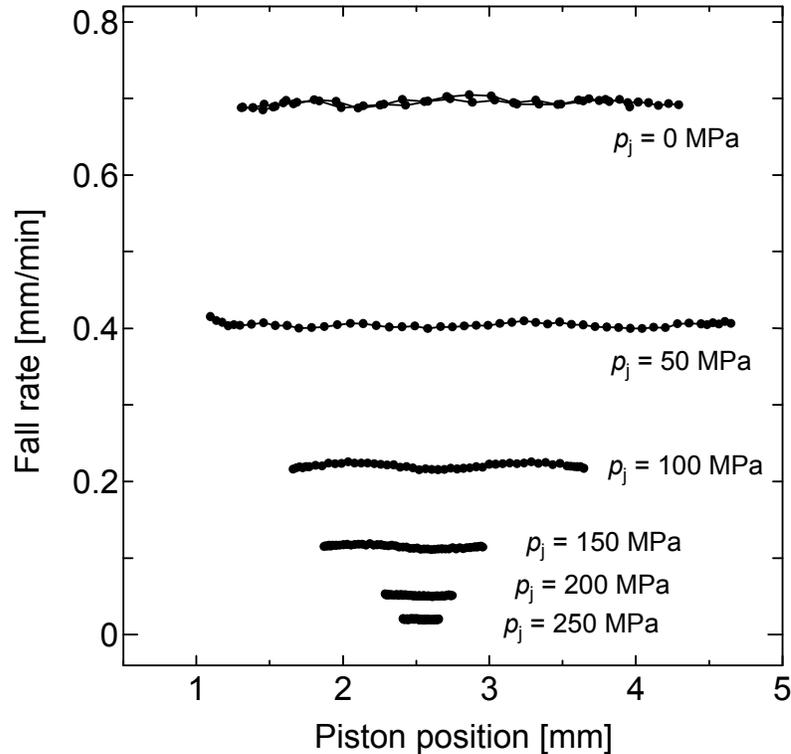


Figure 3: Examples of the piston fall-rate at several jacket pressures under  $p_s=500$  MPa.

#### 4. Results and Discussions of the characterization experiments

Figure 4 shows the jacket pressure coefficient at 250 MPa (Fig. 4(a)) and at 500 MPa (Fig. 4(b)).  $d$  was calculated using equation (2) from the difference in the system pressure between the adjacent two jacket pressure points. At 250 MPa,  $d$  looks independent of the jacket pressure. However, at 500 MPa, it appears that  $d$  decreases with increasing jacket pressure by more than 0.5 ppm/MPa. The jacket pressure dependence of  $d$  was not assumed in the original work by Heydemann and Welch [1] and also has not been reported in many experimental works especially below 300 MPa (for example see refs. [5,6]). Our 500 MPa CCPC also does not show apparent jacket pressure dependence below 300 MPa.

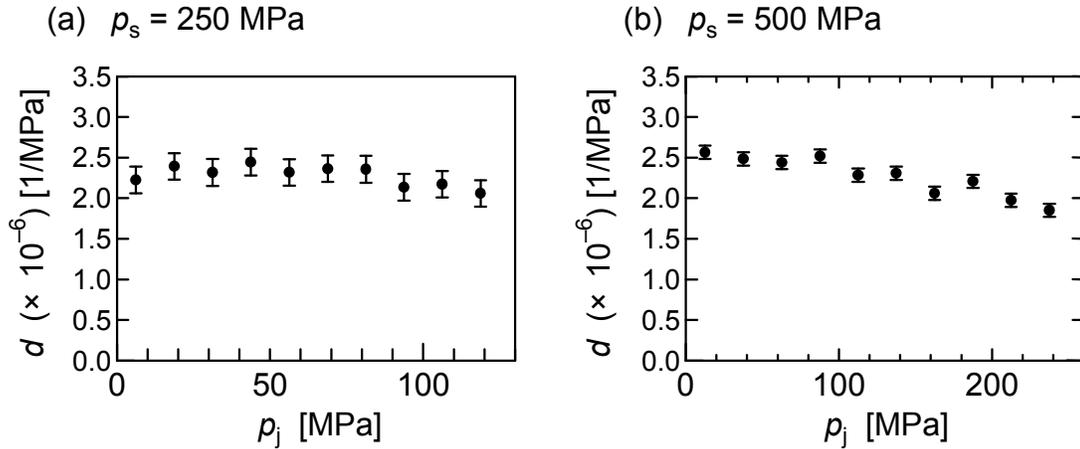


Figure 4: Jacket pressure coefficient at 250 MPa (a) and at 500 MPa (b) plotted against the jacket pressure.

In the H-W model, assuming the laminar flow, the jacket pressure and the cubic root of the piston fall-rate have a linear relationship. Figure 5 shows the plots of  $p_j$  against  $v^{1/3}$  at all system pressures. The lines on the figure are the linear fitting lines and y-intercepts of these lines become  $p_z$ . Data in fig. 5 approach very close to the y-axis; this makes the extrapolation range small and reduces the uncertainty of  $p_z$ . As shown in the figure, the data at more than 250 MPa show curvature over the wide pressure range. As a reason for the deviation from the linear relationship, Heydemann and Welch have pointed out the possibility of the temperature change due to the friction in the clearance; in this case, the deviation from the linear relationship was only occurred at very high and low jacket pressures. In our results, however, deviations were seen over the wide range of the jacket pressure. It seems that the assumption of a laminar flow through the clearance is not applied in such a wide pressure range at high pressure.

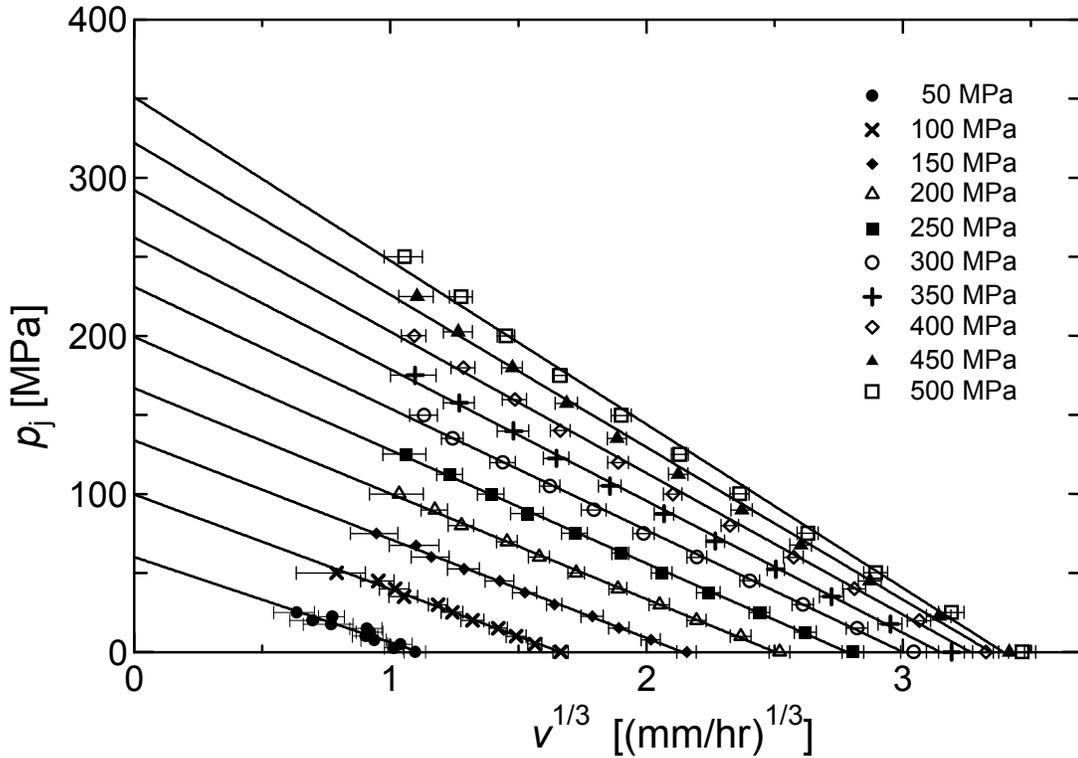


Figure 5: Plots of the jacket pressure against cubic root of the fall-rate at all system pressures up to 500 MPa.

Although the results of the characterization experiments for 500 MPa CCPC shown above are not so ideal as assumed in the H-W model,  $d$  and  $p_z$  are estimated using the conventional analysis as a first step;  $d$  is averaged over the whole jacket pressure range and  $p_z$  is determined as a y-intercept of a linear fitting in the plot of  $p_j$  versus  $v^{1/3}$ . The resultant  $d$  and  $p_z$  are shown in fig. 6. From these two parameters, changes in the effective area under three conditions were estimated using equation (1). To check the consistency between the estimation of CCPC and our present national standards, a FD piston-cylinder for 500 MPa was evaluated by cross-float measurements against this CCPC under condition of  $p_j/p_s = 0.5$ . The evaluated pressure distortion coefficient becomes  $8.22 \times 10^{-7}$  [1/MPa], while the pressure value of this FD piston-cylinder is  $8.18 (\pm 1.04) \times 10^{-7}$  [1/MPa]. Two results are, fortunately, in good agreement.

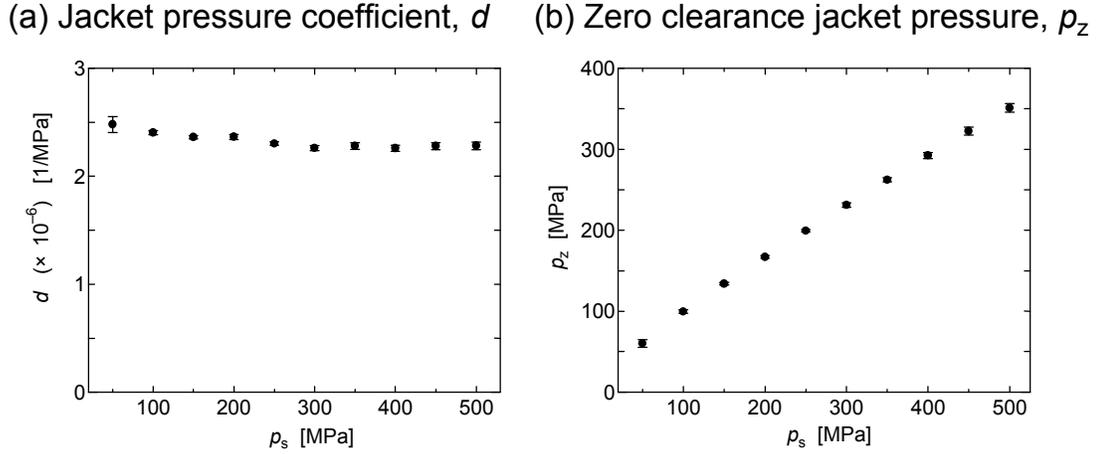


Figure 6: System pressure dependence of the jacket pressure coefficient (a), and the zero clearance jacket pressure (b). Both are estimated using the conventional method.

However, from our experimental results, there seems to be room for improvement in the H-W model at higher pressures around 500 MPa. Especially, the jacket pressure dependence of  $d$  should be taken into consideration. The fourth term in the equation (1) means the increase in the effective area due to decreasing the jacket pressure from the hypothetical  $p_z$  to  $p_j$ . When  $d$  is independent of  $p_j$ , as shown in fig. 7(a), the relative change in the effective area is expressed by the area of a colored rectangular in the figure. However, when  $d$  depends apparently on  $p_j$ , as shown in fig. 7(b), the most required is the value of  $d$  between  $p_j$  and  $p_z$ . Thus extrapolation curve of  $d(p_j)$  should be used instead of the averaged value of  $d$ . In our case at high pressures, that  $d$  decreases with increasing  $p_j$ , simple averaging of  $d$  over the whole jacket pressure range may overestimate the effective area. In general, to reflect the jacket pressure dependence of  $d$ , the term of  $1 + d \cdot (p_z - p_j)$  should be extended in a more

general form of  $1 + \int_{p_j}^{p_z} d(p_j') \cdot dp_j'$ , and precise determination of  $d(p_j)$  especially in

the high jacket pressure range is needed.

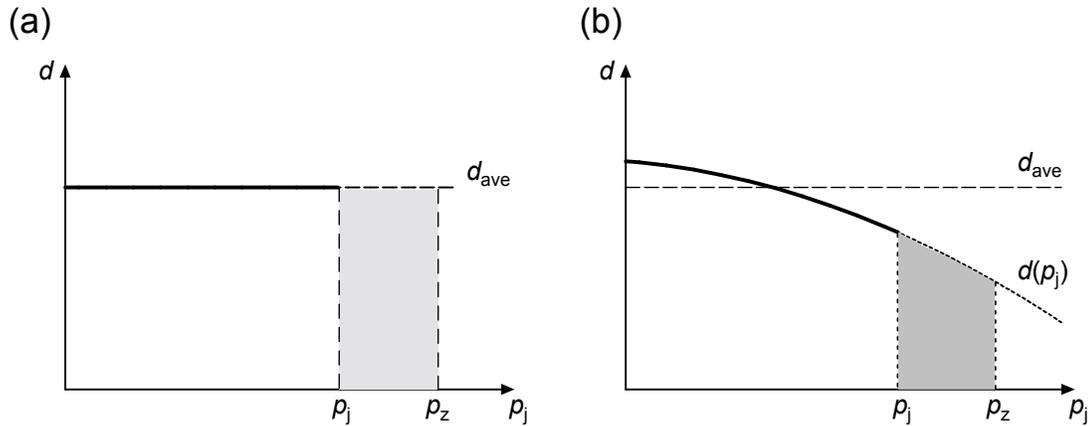


Figure 7: Estimation of the relative change in the effective area due to the jacket pressure from a plot of  $d$  against  $p_j$ . (a): in the case that  $d$  is independent of  $p_j$ , as assumed in the conventional method, (b): in the case that  $d$  depends on  $p_j$  like our results at the system pressure of around 500 MPa.

## 5. Summary

A new controlled-clearance pressure balance was introduced to upgrade the hydraulic high-pressure standard up to 1 GPa. Characterization experiments for a 500 MPa controlled-clearance piston-cylinder were performed in detail over wide pressure range to determine the parameters,  $d$  and  $p_z$ , in the Heydemann-Welch model. The estimation of the effective area based on the conventional model agreed well with the present pressure standard. However, at high pressures around 500 MPa,  $d$  depended apparently on  $p_j$  contrary to the assumptions of the model. As for the piston fall-rate, a plot of  $p_j$  against  $v^{1/3}$  showed a deviation from the linear relationship over the wide range of  $p_j$ . Based on these experimental results, a possible revision on the model was briefly discussed.

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