DYNAMIC CALIBRATION OF PRESSURE SENSORS WITH A SHOCK TUBE

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Abstract – Dynamic calibration of pressure sensors is becoming more important. Shock tubes can generate a pressure step which could provide the basis for dynamic calibration of pressure sensors. The amplitude of this pressure step generated upon reflection of the wave from the end face of the tube can be determined accurately from ideal gas theory using readily measured parameters, such as shock wave velocity, static temperatures and pressures which can be traceable to the national Standard. With the tests of dynamic calibration using shock tube, we could get the time response of the sensor to be calibrated. Then we take this response as the output and pressure step which is calculated from ideal gas theory as the ideal input. The transfer function is then calculated point by point and is defined as the ratio of the Fourier transform of the output to the Fourier transform of the input. We use this method to calibrate pressure sensors in CIMM shock tube and obtain remarkble results.

Keywords: dynamic calibration, pressure sensor, shock tube, transfer function

1. INTRODUCTION

At present, dynamic calibration of pressure sensors is becoming more important. Shock tubes can generate a pressure step which could provide the basis for dynamic calibration of pressure sensors. Changcheng Institute of Metrology and Measurement (CIMM) is engaged on research of dynamic pressure calibration for over thirty years. In this paper, the method of dynamic calibration of pressure sensors with shock tube of CIMM is introduced.

2. SHOCK TUBE THOREY

The amplitude of pressure step generated upon reflection of the wave from the end face of the tube can be determined accurately from ideal gas theory using readily measured parameters[1,3], such as shock wave velocity, static temperatures and pressures which can be traceable to the National Standard.

2.1. Theory

The magnitude of the pressure step Δp can be calculated by (1),

$$\Delta p = \frac{7}{3}(M^2-1)\left[(4M^2+2)/(M^2+5)\right]p_1 \qquad (1)$$

Where p_1 is the initial gas pressure, M is the Mach number which is calculated by (2),

$$M = \frac{V_s}{c_1} \qquad (2)$$

Where V_s is the velocity of the shock wave and c_1 is the speed of sound. V_s is calculated by (3),

$$V_s = \frac{d}{t} \qquad (3)$$

In which d is the distance of two side-wall pressure transducers and t is the time of the shock wave takes to travel between the transducers.

The speed of sound c_1 can be given by (4),

$$c_1 = c_0 \cdot \sqrt{\frac{T}{T_0}} \qquad (4)$$

Where c_0 is the speed of sound at $T = 273.15K$, $P = 100kPa$, and contains 0.03 molar CO_2 and no water. It's value can be given by (5),

$$c_0 = (331.45 \pm 0.05)\,m/s \qquad (5)$$

Where T is the absolute temperature, T_0 is the absolute temperature of 0℃, $T_0 = 273.15K$.

2.2. Uncertainty Budget

When the components are not related to each other, uncertainty u can be calculated by (6),

$$u^2 = a_1^2u_1^2 + a_2^2u_2^2 + \cdots + a_i^2u_i^2 \qquad (6)$$

Where a_1 □ a_i are the sensitivity coefficient, u_1 □ u_i are the standard uncertainty of every component.

I. Value of u_{c_1} can be estimated from (7),

$$u_{c_1}^2 = a_1^2u_{c_0}^2 + a_2^2u_T^2 + a_3^2u_{T_0}^2 \qquad (7)$$

Where the sensitivity coefficients are,

$$a_1 = \frac{\partial c_1}{\partial c_0} = \sqrt{\frac{T}{T_0}} \qquad (8)$$

$$a_2 = \frac{\partial c_1}{\partial T} = \frac{1}{2}c_0\frac{1}{\sqrt{T_0T}} \qquad (9)$$

$$a_3 = \frac{\partial c_1}{\partial T_0} = -\frac{1}{2} c_0 \frac{\sqrt{T}}{\sqrt{T_0^3}} \tag{10}$$

II. Value of u_{V_s} can be calculated from,

$$u_{V_s}^2 = a_4^2 u_d^2 + a_5^2 u_t^2 \tag{11}$$

Where the sensitivity coefficients are,

$$a_4 = \frac{\partial V_s}{\partial d} = \frac{1}{t} \tag{12}$$

$$a_5 = \frac{\partial V_s}{\partial t} = \frac{-d}{(t)^2} \tag{13}$$

III. Value of u_M can be calculated from,

$$u_M^2 = a_6^2 u_{V_s}^2 + a_7^2 u_{c_1}^2 \tag{14}$$

Where the sensitivity coefficients are,

$$a_6 = \frac{\partial M}{\partial V_s} = \frac{1}{c_1} \tag{15}$$

$$a_7 = \frac{\partial M}{\partial c_1} = -\frac{V_s}{c_1^2} \tag{16}$$

IV. Uncertainty of the pressure step $u_{\Delta p}$ is calculated by

$$u_{\Delta p}^2 = a_8^2 u_{p_1}^2 + a_9^2 u_M^2 \tag{17}$$

Where the sensitivity coefficients are,

$$a_8 = \frac{\partial(\Delta p)}{\partial p_1} = \frac{14(2M^4 - M^2 - 1)}{3M^2 + 15} \tag{18}$$

$$a_9 = \frac{\partial(\Delta p)}{\partial M} = 14 p_1 \cdot \frac{(8M^3 - 2M)(3M^2 + 15) - 6M(2M^4 - M^2 - 1)}{(3M^2 + 15)^2} \tag{19}$$

An uncertainty budget for the step pressure generated by shock tube of CIMM is given in Table 1.

Table 1. Uncertainty budget for the shock tube of CIMM

Parameter	Symbol	Value	Standard uncertainty	Unit	Sensitivity coefficient	$a_i^2 u_i^2$
Adiabatic index	c_0	331.45	0.05	m/s	1.05E+00	2.73E-03
Temperature	T	293.15	0.115	K	5.80E-01	4.49E-03
Temperature	T_0	273.15	0	K	-6.34E-01	0.00E+00
Speed of sound	c_1	**346.46**	**0.085**	m/s		**7.22E-03**
Distance	d	400.02	0.577	mm	1.09E+03	3.99E-01
Time interval	Δt	913.60	1.00	μs	-4.79E+05	2.30E-01
Shock speed	V_s	**437.85**	**0.79**	m/s		**6.29E-01**
Shock speed	V_s	437.85	0.79	m/s	2.89E-03	5.24E-06
Speed of sound	c_1	346.46	0.085	m/s	-3.65E-03	9.61E-08
Mach number	M	**1.2638**	**0.0023**			**5.34E-06**
Driven pressure	p_1	300000	600	Pa	1.77E-06	1.13E-06
Mach number	M	1.2638	0.0023		2.69E+00	3.85E-05
Pressure step	Δp	**0.5315**	**0.0063**	MPa		**3.97E-05**
			1.18	**%**		

From Table 1, the uncertainty of pressure step generated by shock tube of CIMM is obtained. Then we take this pressure step as the ideal input for calculating the transfer function .

3. TRANSFER FUNCTION

A sensor used for dynamic pressure cannot be characterized by its sensitivity alone. The objective of dynamic calibration is the determination of the transfer function (gain and phase as functions of frequency) which describes the behavior of the sensor in both the static and dynamic cases[2].

The transfer function is calculated point by point and is defined as the ratio of the Fourier transform of the output to the Fourier transform of the input. For shock tube, the transfer function of calibrated transducer can be calculated as following steps.

Step 1, signal extension

$$\begin{cases} DATA(n) = data(n) & n \le N \\ DATA(n) = data(N) - data(n-N) & N < n \le 2N \end{cases} \tag{20}$$

Where *data(n)* is the original signal, *N* is the length of the signal and *DATA(n)* is the signal after extension.

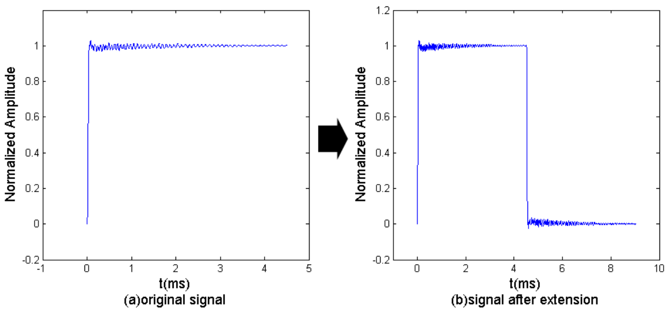


Fig. 1. Signal extension

Step 2, square wave generation

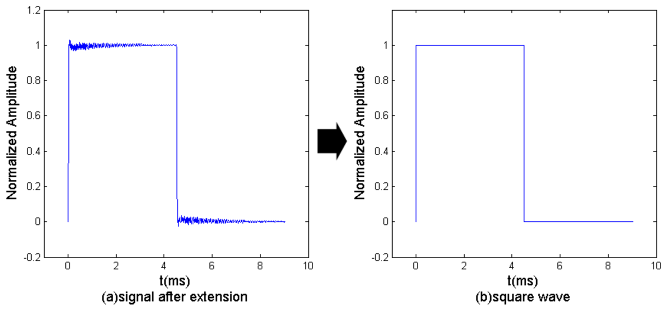


Fig.2. Square wave

Step 3, DFT

The formula of DFT is usually used as Eq.(4). There is a drawback that frequency resolution is a constant fs/N when N is selected.

$$X(k)=DFT[x(n)]_N=\sum_{n=0}^{N-1}x(n)e^{-j\frac{2\pi}{N}kn},k=0,1,\cdots,N-1$$
(21)

In order to get more variable resolutions of frequency, the Eq.(4) can be shown as

$$X(k)=\sum_{n=0}^{N-1}x(n)e^{-j\frac{2\pi}{N}kn}=\sum_{n=0}^{N-1}x(n)e^{-j\frac{2\pi}{fs}F(k)n},k=0,1,\ldots,N-1$$
(22)

Where $F(k)=\frac{k}{N}\cdot fs$, $F(k)$ can be unlimited sequence of frequency.

Step 4, calculating transfer function

$$H(f)=\frac{Y(f)}{X(f)} \tag{23}$$

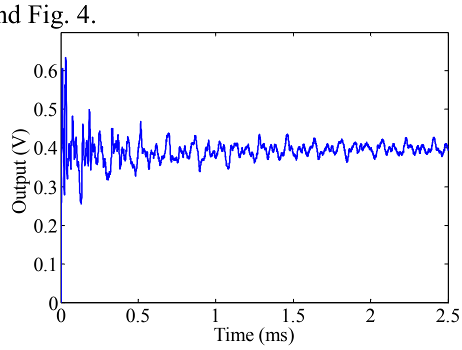
where $Y(f)$ is the DFT results of output of calibrated sensor and $X(f)$ is the DFT results of ideal input which can be calculated from ideal gas theory.

4. EXPERIMENTAL RESULTS

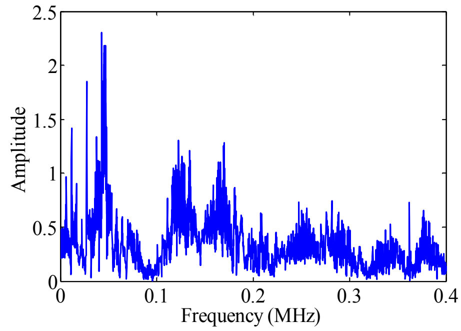
Two pressure sensors in the side wall of the driven part are used to derive the velocity. The velocity can be calculated from the known 400mm separation between the sensors and the measured time interval between the two detections. Then the Mach number can be obtained. Another pressure sensor which is calibrated statically in the driven part is used to measure the initial gas pressure.

The sampling rate of National Instrument 6133 card is 2MHz and the sampler is triggered on the rising edge of the output of the pressure transducer in the end wall of the shock tube.

The calibrated pressure sensor is Endevco 8510B-2000. Six experimental results are obtained using same thickness aluminum diaphragm. The analysis data are taken for a time of 2.5 ms. The results of experiment 1 and 2 are shown in Fig. 3 and Fig. 4.

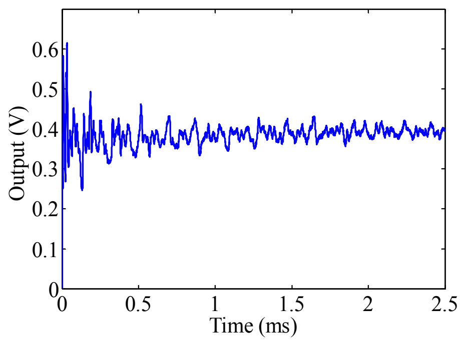


(a) time response

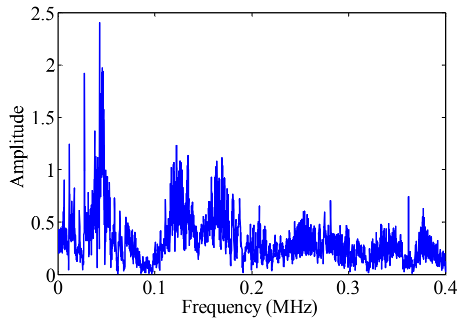


(b) Amplitude of transfer function

Fig.3. Results of experiment 1



(a) time response



(b) Amplitude of transfer function

Fig.4. Results of experiment 2

Results of mean and standard deviation of 6 experiments are shown in Fig. 5.

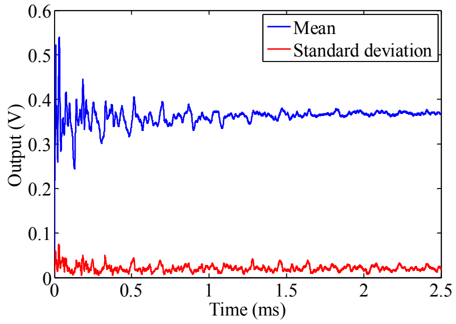


Fig. 5. Mean and standard deviation of 6 experiments

Results of relative standard deviation are shown in Fig. 6.

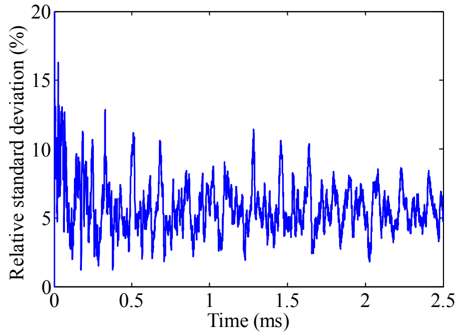


Fig.6. Relative standard deviation

5. CONCLUSIONS

A method for dynamic calibration of pressure sensors with a shock tube is introduced in this paper. With the tests of dynamic calibration using shock tube, the time response of calibrated sensor can be obtained. Then the transfer function can be calculated point by point and is defined as the ratio of the Fourier transform of the output to the Fourier transform of the input. Finally, six experimental results using this method in CIMM shock tube are shown.

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REFERENCES

[1] S. Downes, A. Knott and I. Robinson, "Towards a shock tube method for the dynamic calibration of pressure sensors", Phil. Trans. R. Soc. A, 372: 20130299, July 2014.

[2] J. P. Damion, "Means of Dynamic Calibration for Pressure Transducers", Metrologia, 30:743-746, 1994.

[3] S. Downes, A. Knott and I. Robinson, "Uncertainty estimation of shock tube pressure steps", XXI IMEKO World Congress, 336, Prague, Czech Republic, August, 2015.