Effective Area Calculation for Piston-Cylinder Assembly with Large Nominal Diameter

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Abstract *– Two piston-cylinder assemblies with large nominal diameters of 50 mm and 35 mm, which are intended to establish new primary pressure standards, have been set up in the National Metrology Centre, Agency for Science Technology and Research, (NMC, A*Star), the national metrology institute of Singapore. These two assemblies have been calibrated by Geometry Metrology Standards for the properties of straightness, roundness, and diameters. Based on the calibrated data of properties, a known and proven least square method for three-dimensional geometry is applied to determine the effective areas at atmospheric pressure and temperature 20ºC for these two assemblies. The present study proposed improved models with higher-order polynomial equation and inclusion of frequency term. By using the present improved models, the residual uncertainty in modelling can be reduced to better fit the roundness and straightness of the cylinder. The determined effective areas from dimensional properties are compared with those by the cross float method based on proven pressure standards. The comparison results show good agreement, validating the traceability of the present data by using least square method. It is also verified that least square method is suitable to be used for the piston-cylinder assemblies with large nominal diameter to calculate the effective area accurately. In addition, cross-float of these two large piston-cylinders with the effective areas determined is applied to calculate the pressure. The deviation of the pressure does not exceed 6 x 10⁻⁶ over the pressure range (50 to 520) kPa in gauge mode.*

Keywords: *piston-cylinder assembly, effective area, least-squares method, cross-float method, pressure deviation.*

1. INTRODUCTION

The effective area of a piston-cylinder assembly is used as the primary pressure standard in pressure balances. The derivation of the effective area, representing the largest contribution to the uncertainty, is of great significance in pressure measurements [1]. Traditionally the effective area is determined by Dadson's theory [2] which is based on the dimensional properties of the piston and cylinder, namely, the straightness (S), roundness (R), and diameters (D) of the piston and cylinder in an assembly. The main disadvantage is the contradictions between the S, R and D measurements. An approach, based on the least squares method, was proposed by PTB [3]. This method has been applied to three gas-operated piston-cylinder assemblies with nominal effective areas of 10 cm² and 5 cm² for the range up to 2 MPa, and the final relative

standard uncertainties was reported to be smaller than 2x10⁻⁶ [3]. By linking the measurement data of S, R and D with minimum discrepancies, this approach can improve the consistency of three-dimensional data describing the geometry of the piston-cylinder assembly.

In the present study, this proven least squares method is applied to the piston-cylinder assemblies with large nominal diameters of 50 mm and 35 mm to determine the effective areas given the calibrated dimensional properties of straightness (S), roundness (R), and diameters (D). The determined effective areas are also compared with those by the cross float method based on proven pressure standards. Moreover, cross-float of these two large piston-cylinders with the effective areas determined is applied to calculate the pressure values.

2. EXPERIMENTAL SETUP

Two piston-cylinder assemblies with large nominal diameters of 50 mm and 35 mm have been set up in the National Metrology Centre, Agency for Science Technology and Research, (NMC, A*Star), the national metrology institute of Singapore. New primary pressure standards are intended to be established by using these two assemblies. Three-dimensional plots of the two piston-cylinder assemblies are shown in Figures 1 and 2. The summary of the main characteristics of these two piston-cylinder assemblies is listed in Table 1.
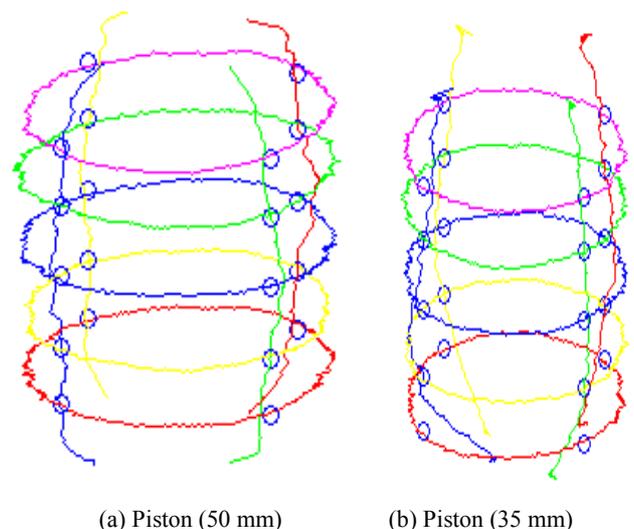

(a) Piston (50 mm)   (b) Piston (35 mm)

Figure 1: Three-dimensional plots of pistons with the diameters of (a) 50 mm, and (b) 35 mm
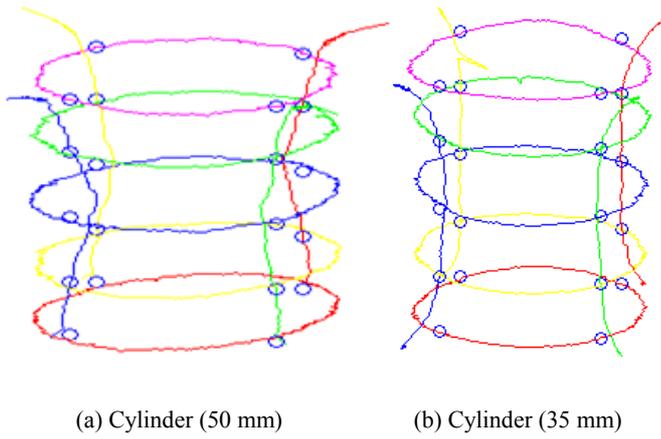
(a) Cylinder (50 mm)   (b) Cylinder (35 mm)

Figure 2: Three-dimensional plots of cylinders with the diameters of (a) 50 mm, and (b) 35 mm

Characteristics of large area piston-cylinder		
Serial Number	1618	1595
Mounting system	Simple Free Deformation	Controlled Clearance
Nominal Diameter [mm]	35	50
Nominal pressure/mass [kPa/kg]	10	5
Maximum Mass [kg]	100	100
Piston and cylinder material	Tungsten carbide	Tungsten carbide
Young Modulus [$\times 10^{11}$ N/m^2]	6	6
Poisson ratio	0.22	0.22
Thermal expansion coefficient ($\times 10^{-6}$) /°C	9	9
Equivalent piston density [kg/m^3]	10080	11988

Table 1: Characteristics of large area piston-cylinder

3. RESULTS AND DISCUSSION

3.1 CALCULATION OF THE EFFECTIVE AREA FROM DIMENSION DATA

The two assemblies have been calibrated by Geometry Metrology Standards [4] for the properties of straightness (S), roundness (R), and diameters (D). Least square method proposed by PTB [3] is applied to calculate the effective areas of the present two assemblies based on calibrated dimensional properties of S, R and D. The determined effective areas are compared with those calculated by cross-float method. The two dimensional plot of the piston and cylinder profiles for 35 mm and 50 mm are shown in Figure 3 and 4 respectively with the diameters measured shown in small cross.
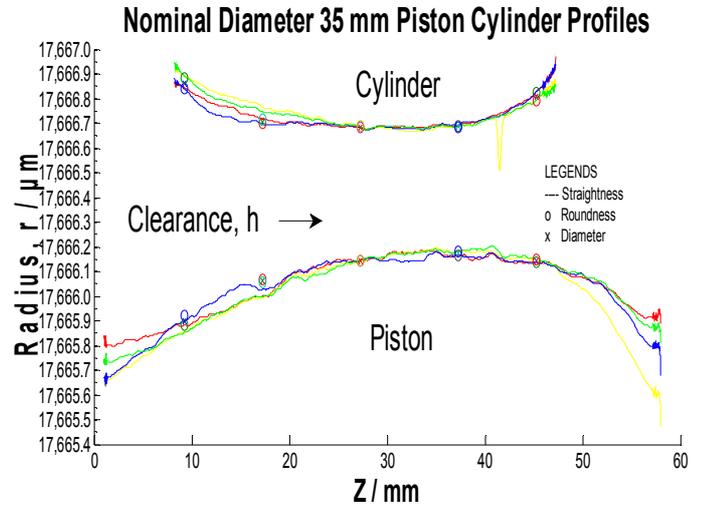

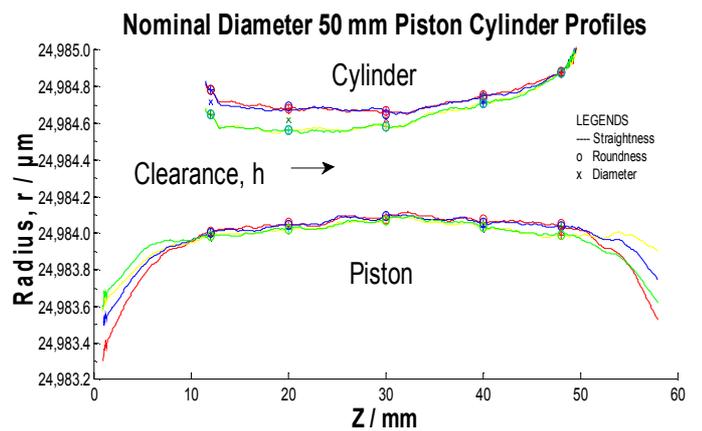Figure 3: Profile of 35 mm large area piston-cylinder

Figure 4: Profile of 50 mm large area piston-cylinder

The effective area at zero applied pressure is determined based on Dadson's proposed zero applied pressure [2] for compressible fluid, as expressed:

$$A_o = \pi r_o{}^2 \left\{ 1 + \frac{h_o}{r_o} + \frac{1}{r_o} \frac{\int_o^l \frac{(u+U)}{h^3} dx}{\int_0^l \frac{1}{h_3} dx} \right\}$$

The uncertainty of effective area at zero pressure is determined from geometrical data of diameters, straightness and roundness is based on the following equations proposed in the References [3] and [4]:

$$u(r_R) = \left\{ \left[\frac{u(D)}{2} \right]^2 + \delta^2(r_{D-R}) + \delta^2(r_{R-S}) \right\}^{0.5}$$

$$u(r_S) = \left\{ \left[\frac{u(D)}{2} \right]^2 + \delta^2(r_{D-S}) + \delta^2(r_{R-S}) \right\}^{0.5}$$

The residue results of the least squared radius between diameter, roundness and straightness from the reported data is about 50 nm based on first order polynomial for straightness and single sinusoidal for roundness. Higher order of polynomial shown in the equation below to model the actual straightness lines more accurately is applied with the results of residue reduced to 20 nm and below. Similarly for each of the roundness reported data shows approximately two sinusoidal cycles in the domain of 360 degree is more accurately represented by equation for roundness with the introduction of omega, ω. In this case a value of about ω = 2 reduced the residue from over 50 nm to 20 nm and below. The comparisons are illustrated in Figure 5 and 6 for straightness and roundness respectively.

Straightness,

$$r_{S,j}(z) = S_j(z) + a_j + b_j z + c_j z^2 + d_j z^3 + e_j z^4$$

Roundness,

$$r_{R,i}(\varphi) = R_i(\varphi) + P_i + W_i \cdot \cos \omega\varphi + V_i \cdot \sin \omega\varphi$$
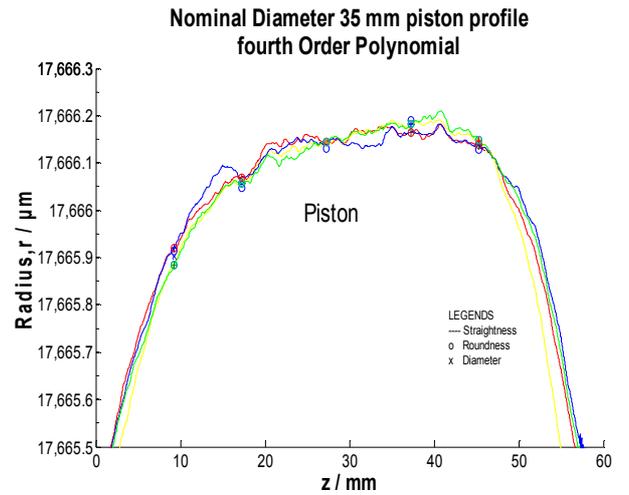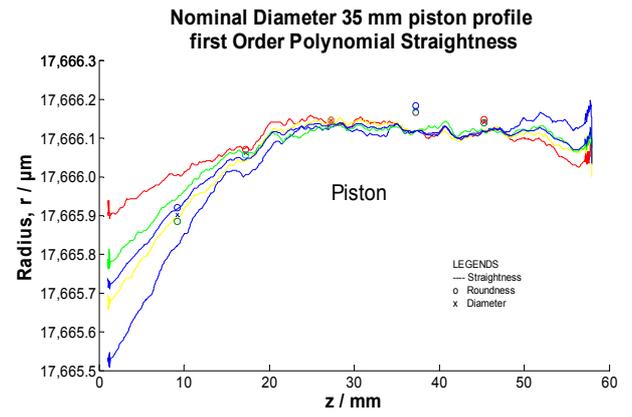

Figure 5: Profile of 35 mm large area piston straightness using first order polynomial and higher order polynomial.
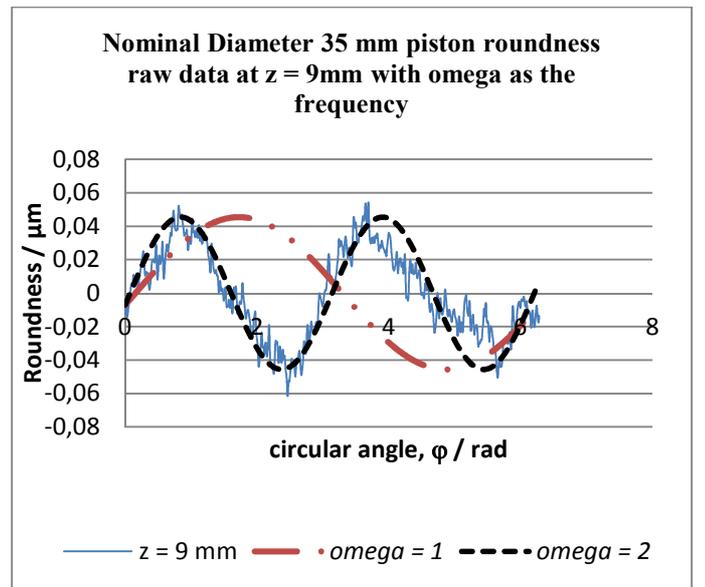

Figure 6: Roundness data at z = 9 mm with omega = 1 and omega = 2.

3.2 CALCULATION OF THE EFFECTIVE AREA FROM CROSS-FLOAT

The effective area from cross-float method traceable to PTB standard references is also performed. The piston cylinders are calibrated by Fluke-DHI with both absolute and gauge mode results. Their results are included in the comparison below.

Table 2 shows the comparison of the effective areas for the present two assemblies. Good agreement of the comparison results confirms the traceability of the present data by using least square method, and verifies that least square method is suitable to be utilized for the piston-cylinder assemblies with large nominal diameter to calculate the effective areas accurately.

Effective Area of Piston Cylinder diameters 35 mm and 50 mm in gauge mode	Fluke-DHI reports [4]	PTB reports Cross float results [5]	Present Dimensional Results based on Dadson's proposed zero applied pressure
Piston Cylinder 35 mm (Serial Number 1618)			
Piston Cylinder 35 mm Effective Area, Ao at Atmospheric Pressure and 20°C /mm²	980.4976	980.5054	980.4985
Uncertainty/x10⁻⁶	8.0	6.5	10.7
Piston Cylinder 35 mm (Serial Number 1595)			
Effective Area, Ao at Atmospheric Pressure and 20°C /mm²	1961.0411	1961.038	1961.0389
Uncertainty/x10⁻⁶	7.0	5.7	5.7

Table 2: Comparison of the Effective Area by Dimension data and cross-float

3.3 Calculated pressure

The generated pressures of the 35 mm and 50 mm large area piston cylinders are calculated from the determined effective areas. Verifications of the pressures generated in the two systems were done in 2012 to 2016. Table 3 shows the summary of 2016 generated pressure verification during cross-float between the 35 mm and 50 mm large area piston cylinders in primary pressure metrology laboratory of NMC, A-STAR. The deviation of the pressure does not exceed 6 x 10⁻⁶ over the pressure range (50 to 520) kPa in gauge mode.

Pressure [kPa gauge]			Deviation	
Nominal	PG1595 (50mm)	PG1618 (35 mm)	kPa	X 10⁻⁰⁶
50	50.00003	50.00002	0.00015	3.0
100	100.00000	99.99980	0.00007	0.7
200	200.00000	200.00000	0.00090	4.5
300	300.00010	299.99970	-0.00040	-1.3
400	400.00050	400.00030	-0.00020	-0.5
520	520.00020	520.00030	0.00010	0.2

Table 3: Verification of pressure generated in 35 mm and 50 mm large area piston-cylinders by cross-floating in NMC,A*STAR in Sep 2016.

4. CONCLUSIONS

Two piston-cylinder assemblies with large nominal diameters of 50 mm and 35 mm have been set up in the National Metrology Centre, Agency for Science Technology and Research, (NMC, A*Star), the national metrology institute of Singapore. These two assemblies have been calibrated by Geometry Metrology Standards [4] for the properties of straightness, roundness, and diameters. A known and proven least square method for three-dimensional geometry is applied to determine the effective areas at atmospheric pressure and temperature 20°C for these two assemblies given the calibrated data of dimensional properties. A high order polynomial model for the radius of the generatrix and incorporation of frequency term for the radius of circle traces were proposed to improve the known least square method in the present study. The results show that the residual can be reduced from 50 nm to 20 nm which gives a better fit for the roundness and straightness of the cylinder. The determined effective areas by least squares are compared with the reference data, and good agreement of the comparison verifies the traceability of the present data, and confirms that least square method is suitable for the calculation of the effective areas in piston-cylinder assemblies with large nominal diameters. Moreover, generated pressures in the systems is calculated by cross-float of these two large piston-cylinders based on determined effective areas. The deviation of the pressure does not exceed 6 x 10⁻⁶ over the pressure range (50 to 520) kPa in gauge mode.

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[6]   PTB reports Cross float results