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# Feedforward command computation of a 3D flexible robot

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## 1 Introduction

Robotic manipulators with a lightweight structure can present some interesting features. Thanks to their reduced weight and stiffness, lightweight robots could achieve high speed tasks while being safer and more efficient than traditional rigid robots. However, when designing the controller of such systems, elastic behaviors should be accounted for in order to prevent unwanted vibrations.

In order to have a motion of the manipulator with reduced vibrations, the latter can be fed back to the controller so that proper compensation can be done [1]. By analyzing the system, appropriate feedforward inputs can also be designed in such way that the resulting motion has decreased elastic deflections. Both feedforward and feedback action can be combined to achieve robust performances [2, 3].

This work focuses on the feedforward control of 3D flexible manipulators. Based on a model of such flexible multibody systems (MBS), the inverse dynamics is solved to compute the feedforward input of the manipulator. Different methods can be used to model flexible robotic arms. Lumped mass elements models are widely used to model robotic systems [4]. Indeed, to represent the manipulator and its flexibility, this modelling technique uses a limited number of parameters and is therefore quite suitable for control purposes [5]. On the other hand, the finite element modelling approach is a general way to model MBS [6] that is able to represent distributed link flexibility. Here, the case study of a flexible joint Sawyer robot is presented. Flexibility in the joint of the robot is modelled using lumped spring and damper elements and the inverse dynamics of the system is solved using the optimization approach [7] extended to 3D problems. The computation of the feedforward input command is discussed and experimental results on the real system are presented.

## 2 Inverse dynamics formulation

The dynamics of multibody systems, such as robotic manipulators, can be described using rigid bodies and flexible bodies connected through kinematic joints, springs and dampers elements. The kinematics of such system is described using its generalized coordinates $\mathbf{q}$. In the case of a robotic manipulator, some actuators can exert some torques (or forces) $\mathbf{u}$ on the bodies to move a specific end-effector along a given trajectory. The dynamics of the system can be governed by

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{g}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{B}^T\boldsymbol{\lambda} = \mathbf{A}\mathbf{u} \tag{1}$$

$$\boldsymbol{\Phi}(\mathbf{q}) = \mathbf{0} \tag{2}$$

$$\mathbf{y}_{eff}(\mathbf{q}) - \mathbf{y}_{presc}(t) = \mathbf{0} \tag{3}$$

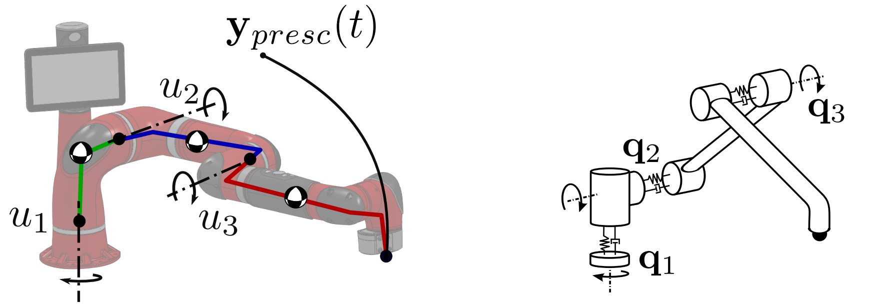
where $\mathbf{q}$ are the generalized coordinates of the system, $\mathbf{M}$ is the system symmetric mass matrix, $\mathbf{v}$ is the vector of nodal velocities, $\mathbf{g}$ is the vector of internal and complementary inertia forces, $\mathbf{B}$ is the gradient of the kinematic constraints $\boldsymbol{\Phi}$, which are used to represent the connections imposed by the kinematic joints. The matrix $\mathbf{A}$ is a boolean matrix that applies the controls $\mathbf{u}$ on the system. The $m$ dimensional vector $\boldsymbol{\lambda}$ is the Lagrange multipliers related to the $m$ kinematic constraints $\boldsymbol{\Phi}$. The last equation is called the servo constraint and fixes a part of the motion. It assures that the end-effector position $\mathbf{y}_{eff}$ follows the prescribed trajectory $\mathbf{y}_{presc}(t)$.

In this work, the inverse dynamics problem i.e., finding the control inputs that satisfy the servo constraint, is solved by formulating a constrained optimization problem minimizing an objective function $J(\mathbf{q})$

$$\min_{\mathbf{q}} \; J(\mathbf{q}) \tag{4}$$

related to the configuration of the robotic manipulator under the constraint defined by the equations of motion Eqs. (1)-(3).

## 3 Lumped mass element model




(a) Model of a 3 actuated dof simplified Sawyer robot.

(b) Flexible lumped mass element model with 3 actuated dof.

Fig. 1: Model of an elastic Sawyer robot.

To illustrate this methodology with a real case study, the feedforward action of a flexible Sawyer robot performing a trajectory tracking tasks is considered. This seven degree of freedom (dof) elastic robot is simplified to be a three actuated dof robot by locking four of its joints. The resulting model of the robot has three links and three actuated dof controlled using inputs $u_i$, with $i = 1,2,3$, as shown in Fig. 1(a). The joints of the Saywer robot are constructed using *series elastic actuators* [8]. The flexure spring inside these actuators result in a joint with intrinsic flexibility. Here, these flexible joints are described using two angles: one describing the motor related angle $q_{M,i}$ and the other one describing the link related angle $q_{L,i}$. Flexibility is modeled using a spring-damper pair that connects $q_{L,i}$ to $q_{M,i}$ [4, 5]. Each link is modeled as a rigid body and the end effector $\mathbf{y}_{eff}$ is located at the tip of the third link.

Once the model is built, the inverse dynamics is solved by formulating a constrained optimization problem, where the objective is to minimize the elastic energy $J$ inside the spring-damper pairs. Mathematically,

$$\min_{\mathbf{q}} J = \min_{\mathbf{q}} \frac{1}{2T} \sum_{i=1}^{3} \int_t k_i (q_{M,i} - q_{L,i})^2 dt \tag{5}$$

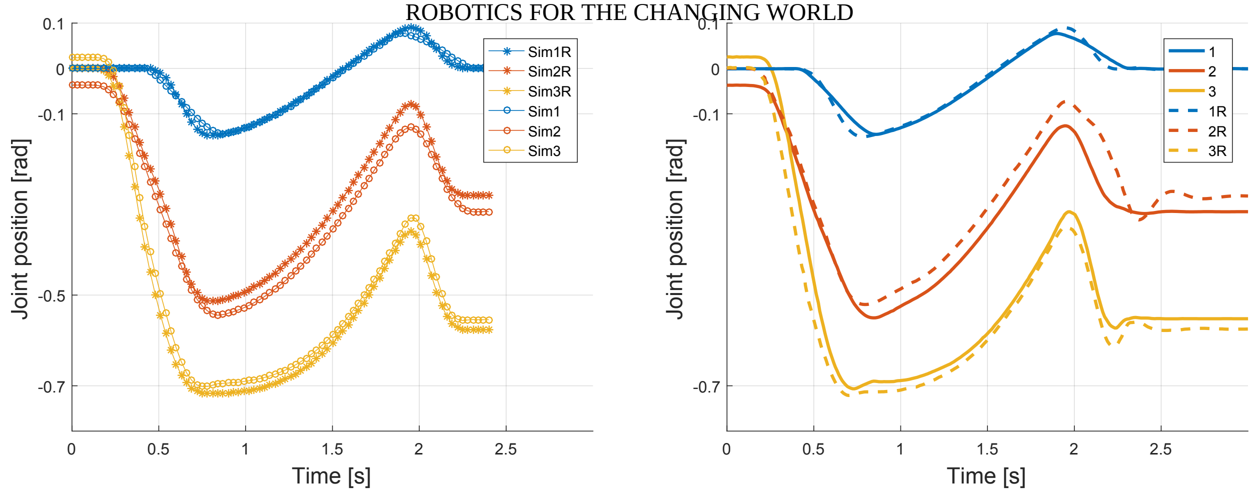
with $k_i$ is the stiffness of spring $i$ and $T$ is the total duration of the motion.

## 4 Results and discussion

The trajectory imposed at the robot end-effector is an oscillating motion in space built by combining a seventh order polynomial with a sine function. In Fig. 2(a), the computed input motor positions $q_{M,i}$ with the flexible joint assumption are represented with the bullet lines. They are compared to the inputs that would be generated for an equivalent robot with rigid joints (star line). The inputs with the rigid joint assumptions can be computed algebraically based on the trajectory at the end-effector. Besides the visible offset due to the compensation of gravity, the flexible joint assumption generates inputs that start slightly earlier than with the rigid assumption (visible for joint 1).

These inputs are sent to the real robot at a rate of 500 Hz and the resulting link positions $q_{L,i}$ measurements are shown in Fig. 2(b). With the rigid assumptions in dashed lines, one can see that some residual vibrations are present in the joints at the end of the trajectory (around 2.25 sec). One can also observe that, the second link does not manage to follow correctly the desired trajectory and a small delay can be observed at around 2 sec. When the inputs computed with the flexible assumption are used, the link angle and the input motor angle do not have significant differences: joint deflections are better compensated and almost no vibration is visible.

It is important to note that in the above results, only the default PD feedback controller on each joint is used. No additional feedback on the end-effector state was implemented here.




(a) Computed inputs $q_{M,i}$ without and with flexible joint assumption.

(b) Experimental measurements of link angle $q_{L,i}$ with flexible and rigid joint control.

Fig. 2: Computed inputs $q_{M,i}$ and measured link angle $q_{L,i}$.

# 5 Conclusion

This case study is designed to validate the optimization formulation to solve the inverse dynamics of 3D flexible robot. In this method, the inverse problem is stated as a minimization of the elastic deformation energy in the system. The optimization has to satisfy some constraints defined by the dynamics of the system and the trajectory tracking task. For this experimental example of a Sawyer robot with flexible joints, it was possible to reduce the resulting vibrations of the end-effector. More complex feedback control laws could be implemented in order to further improve the results. The end-effector acceleration could be monitored using accelerometers and additional compensations could be designed. The end-effector trajectory could also be monitored using an external camera in order to measure tracking performances.

Although not shown here, experimental tests on flexible link serial robot were also performed. In that case, flexibility is modeled using beam finite elements and the same methodology is applied to solve the inverse dynamics of the system. Those results will be presented in further publications.

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