Analytical Model of Magnetostrictive/Piezoelectric Laminate Composite

Khalil Moussi*, Slim Naifar*†, Sonia Bradai*† and Olfa Kanoun†
*National Engineering School of Sfax, University of Sfax, Sfax, Tunisia
†Chair for Measurement and Sensor Technology, Technische Universität Chemnitz Chemnitz, GERMANY
Email: khalilallahmoussi@gmail.com

***Abstract*—In the past few decades, extensive researches have been conducted on magnetoelectric (ME) effect especially in composite materials. Based on ME effect, vibration energy is converted into electric energy. Using Terfenol-D and PZT materials magnetic field is transformed into electric power. However, magnetic, mechanical and electric effects appear when the two composites are coupled. Moreover, under dynamic conditions, additional effects alter the transducer performance. For these purposes, investigations were carried out on magnetostrictive and piezoelectric materials. This paper addresses the analytical modeling of magnetostrictive/piezoelectric laminate composites. First, the linear model and the equivalent circuits are presented for both magnetostrictive and piezoelectric layers. Second, the equivalent circuit of the ME laminate composites is obtained. Finally, we simulate the analytical model to determine the optimal thickness giving high electric power output.**

***Index Terms*—Analytical modeling, magnetostriction, piezoelectricity, linear modeling, equivalent circuit.**

I. Introduction

Magnetostriction is the change in shape of certain ferromagnetic materials under the influence of an external magnetic field. The cause of magnetostriction change in length is the result of the rotation of small magnetic domains. This rotation and re-orientation causes internal strains in the material structure and providing proportional, positive and repeatable expansion in microseconds. The strains in the structure lead to the stretching (in the case of positive magnetostriction) of the material in the direction of the magnetic field [1]. The ME effect obtained in composites is more than a hundred times that of single-phase ME materials [2]. Thus, the designed transducer is composed of three layers as follows; a thin piezoelectric layer bonded between two magnetostrictive layers, under time varying magnetic field, the magnetostrictive layer is excited and then, undergoes a change in shape. The piezoelectric layer is pressed by the two magnetostrictive materials, and produces a corresponding electric voltage. Depending on the direction of the magnetic field, the magnetostrictive layer could be magnetized along its length direction, i.e, longitudinal mode (L mode), or along its thickness direction, i.e, transversal mode (T mode). Similarly, for the piezoelectric layer, depending on the polarization direction there are a longitudinal mode (L mode) and a transversal mode (T mode). Currently, there has been much research into the modeling of magnetostrictive piezoelectric magnetostrictive (MPM) laminate composites [3]. Some attempts of modeling concentrated on the L-T mode [4], [5] and other focused on the L-L mode [6]. Other attempts investigate the secondary effects affecting the MPM performance such as pre-stress, bias magnetic field, hysteresis, demagnetization and magnetic losses [5], [7], [8]. Taking into account all these investigations, our objective is to establish for each configuration L-L, L-T and T-T mode, an analytical model for the MPM laminated composites, the optimal mode will be selected, and then we predict the optimal geometric dimensions in order to get higher electric power output.

II. Magnetostrictive Layer Modeling and Equivalent Circuit

The linear mathematical model proposed by G.Engdahl [9] is used to characterize the magnetostrictive laminated material. A characteristic property of magnetostrictive materials is that a mechanical strain will occur if they are subjected to a magnetic field in addition to strains originated from pure applied stresses. Moreover, their magnetization changes due to changes in applied mechanical stresses in addition to the changes caused by changes of the applied magnetic field. These dependencies can be described by mathematical functions if one assumes that the material shows reversible properties [9].

$$\varepsilon = \varepsilon(H, \sigma) \tag{1}$$

$$B = B(H, \sigma) \tag{2}$$

Where ε is the mechanical strain, σ is the mechanical stress, H is the applied magnetic field and B is the magnetic flux density. For small variations in $d\sigma$ and dH the linearized constitutive equations can be formulated as:

$$\varepsilon_i = S_{ij}^H \sigma_j + d_{ki} H_k, i, j = 1..6 \tag{3}$$

$$B_m = d_{mj} \sigma_j + \mu_{mk}^{\sigma} H_k, m, k = 1, 2, 3 \tag{4}$$

Where d is the magnetostrictive constant, μ^{σ} is the magnetic permeabilities at constant σ, and S^H is the elastic compliances at constant H.

From the general equations (3) and (4), we consider only one direction depending on the magnetic field direction, which is mainly along the length of the layer, or parallel to the thickness direction. It is of interest to consider the modeling and equivalent circuit of one mode in some detail and then to present results for the other one.

A. Longitudinal mode

Consider a layer of magnetostrictive material with its length along the x_3 direction and the major faces normal to the x_2 direction as shown in Fig. 1. The magnetic field is longitudinal $H_1 = H_2 = 0$ and $\sigma_1 = \sigma_2 = 0$.

$$\varepsilon_3 = S_{33}^H \sigma_3 + d_{33} H_3 \quad (5)$$

$$B_3 = d_{33}\sigma_3 + \mu_{33}^{\sigma} H_3 \quad (6)$$

First, we use Newton's second law

$$\frac{\partial \sigma}{\partial x} = \rho \frac{\partial^2 u}{\partial^2 t} \quad (7)$$

Where ρ is the mass density and u is the particle displacement. Substituting the stress by:

$$\sigma = \frac{1}{s^B}\varepsilon - \lambda B \quad (8)$$

And using the definition of strain

$$\varepsilon = \frac{\partial u}{\partial x} \quad (9)$$

Leads to

$$\frac{1}{\varepsilon^B}\frac{\partial^2 u}{\partial^2 x} - \lambda \frac{\partial B}{\partial x} = \rho \frac{\partial^2 u}{\partial^2 t} \quad (10)$$

Since we assumed only a longitudinal component of B, and in case of harmonic oscillation, i.e, $u = u_x \exp{(j\omega t)}$, we have

$$\frac{\partial^2 u}{\partial^2 x} = K u_x \quad (11)$$

Where the wave number K is defined as: $K = \omega\sqrt{\rho \varepsilon^B} = \frac{\omega}{c^B}$. And c^B is the sound velocity obtained at constant induction. The soultion of Eq.11 can be written as:

$$u(x) = A\cos(Kx) + B\sin(Kx) \quad (12)$$

The boundary conditions can be given in terms of the face velocities U_1 and U_2 are as follows:

$$U_1 = j\omega u_x(0) \quad (13)$$

$$U_2 = j\omega u_x(l) \quad (14)$$

Which gives the displacement

$$u(x) = \frac{U_1}{j\omega}\cos(Kx) + \frac{U_2 - U_1 cos(Kl)}{j\omega sin(Kl)} sin(Kx) \quad (15)$$

The force can be deduced from the relation $F = -\sigma A_l$ where $A_l = w \times t$ is the cross section. Then, the forces at the faces 0 and l can be given as follows:

$$F_1 = \frac{j\rho c^B A_l}{sin(Kl)} U_2 - jU_1 \rho c^B A_l \cot(Kl) + \frac{d}{\mu^{\sigma}\varepsilon^B} A_l B \quad (16)$$

$$F_2 = -\frac{j\rho c^B A_l}{sin(Kl)} U_1 + jU_2 \rho c^B A_l \cot(Kl) + \frac{d}{\mu^{\sigma}\varepsilon^B} A_l B \quad (17)$$

Eq. 16 and Eq. 17 can be written as:

$$F_1 = (U_1 - U_2){Z^l}_{2m} + U_1 {Z^l}_{1m} + \lambda A_l B \quad (18)$$

$$F_2 = (U_1 - U_2) Z_{2m}^l - U_2 {Z^l}_{1m} + \lambda A_l B \quad (19)$$

Where ${Z^l}_m = j\rho c^B A_l \tan\frac{Kl}{2}$ and ${Z^l}_{2m} = \frac{-j\rho c^B A}{sin(Kl)}$

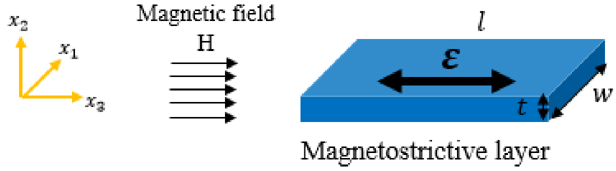

Fig. 1. Longitudinal magnetic field parallel to magnetostrictive layer strain.

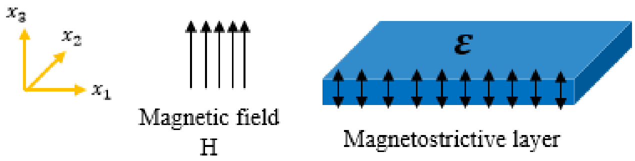

Fig. 2. Magnetostrictive layer under magnetic field along thickness direction.

B. Transversal mode

Consider a layer of magnetostrictive material with its length along the x_1 direction and the major faces normal to the x_3 direction as shown in Fig. 2. The magnetic field is transversal $H_1 = H_2 = 0$ and $\sigma_1 = \sigma_2 = 0$. Similarly, to the longitudinal mode, the forces at the faces 0 and t can be given as follows:

$$F_1 = (U_1 - U_2){Z^t}_{2m} + U_1 {Z^t}_{1m} + \frac{d_{33}}{\varepsilon_{33}^H \mu_{33}^{\varepsilon}} A_t B \quad (20)$$

$$F_2 = (U_1 - U_2){Z^t}_{2m} - U_2 Z_{1m}^t + \frac{d_{33}}{\varepsilon_{33}^H \mu_{33}^{\varepsilon}} A_t B \quad (21)$$

Here $A_t = w \times l$ is the cross section,
${Z^l}_{1m} = j\rho c^B A_t \tan\frac{Kt}{2}$ and ${Z^t}_{2m} = \frac{-j\rho c^B A_t}{sin(Kt)}$

Figure 3 and 4 present the equivalent circuit of the magnetostrictive layer in an impedance representation comprising the mechanical impedances for the longitudinal and transversal mode.

A thin piezoelectric layer of thickness t and length l and polarized along its length direction is shown in figure 5. we assume that the permittivity of the layer is sufficiently greater than that of surroundings to prevent fringing electric fields, electric flux lines will also be parallel to its length, and therefore the electric field components can be given as $D_1 = D_2 = 0$ and $\frac{\partial D_3}{\partial x_3} = 0$. In case of stress along x_3

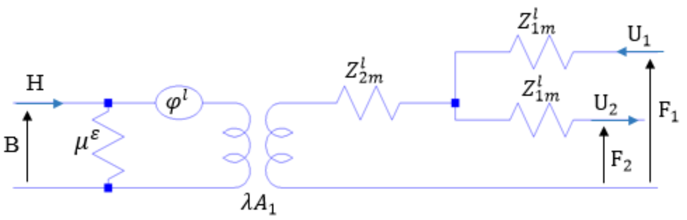

Fig. 3. Equivalent circuit of magnetostrictive layer under longitudinal excitation.

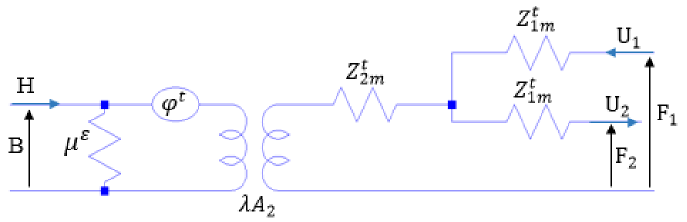

Fig. 4. Equivalent circuit of magnetostrictive layer under transversal excitation.

direction. The standard form of the piezoelectric constitutive equations can be given as

$$\varepsilon_3 = s^D_{33}\sigma_3 + g_{33}D_3 \tag{22}$$

$$E_3 = -g_{33}\sigma_3 + \beta^{\sigma}_{33}D_3 \tag{23}$$

III. Piezoelectric Layer Modeling and Equivalent Circuit

In general, poled piezoceramics can be either transversely or longitudinally polarized materials. Normally, the plane of isotropy is defined as the 12-plane. The piezoelectric material therefore exhibits symmetry about the 3-axis, which is the poling axis of the material.

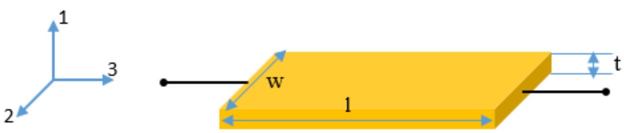

Fig. 5. Piezoelectric layer under longitudinal mode excitation.

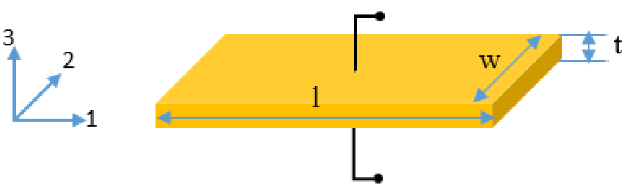

Fig. 6. Piezoelectric layer under transversal mode excitation.

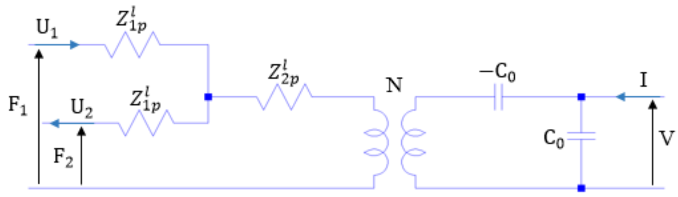

Fig. 7. Equivalent circuit of piezoelectric layer (end-electroded) under longitudinal excitation [10].

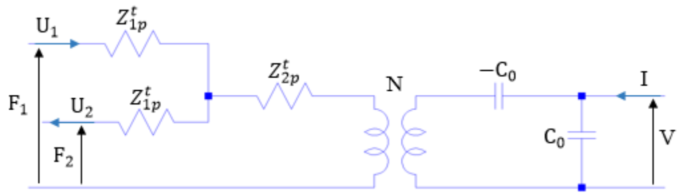

Fig. 8. Equivalent circuit of piezoelectric layer (thickness-electroded) under transversal excitation [10].

Where ε, σ, D and E represent receptively the strain, the stress, the electric field and the electric displacement of the piezoceramic layer. The appropriate wave equation can be written as

$$\rho\frac{\partial^2 u_3}{\partial t^2} = \frac{1}{s^D_{33}}\frac{\partial^2 u_3}{\partial x_3^2} \tag{24}$$

The solution to the wave equation gives particle displacement of the form

$$u_3 = [A\cos\frac{\omega x_3}{v^D_l} + B\sin\frac{\omega x_3}{v^D_l}]\exp(j\omega t) \tag{25}$$

Using boundary conditions in $x_3 = 0, l$

$$u_3|_{x_3=0} = U_1, u_3|_{x_3=l} = U_2 \tag{26}$$

Giving

$$A = \frac{1}{j\omega}U_1\exp-(j\omega t) \tag{27}$$

$$B = \frac{1}{j\omega}[\frac{U_2}{\sin\frac{\omega l}{v^D_l}} - \frac{U_1}{\tan\frac{\omega l}{v^D_l}}]\exp-(j\omega t) \tag{28}$$

The force exerted at each face of the transducer can then be expressed in terms of $I_3 = j\omega wtD_3$, $Z_{0^D} = wt(\frac{\rho}{s^D_{33}})^{0.5}$, $U_1$and U_2 as

$$F_1 = \frac{jZ^D_0}{\sin\frac{\omega l}{v^D_l}}U_2 - \frac{jZ^D_0}{\tan\frac{\omega l}{v^D_l}}U_1 + \frac{g_{33}}{j\omega s^D_{33}}I_{33} \tag{29}$$

$$F_2 = \frac{jZ^D_0}{\tan\frac{\omega l}{v^D_l}}U_2 - \frac{jZ^D_0}{\sin\frac{\omega l}{v^D_l}}U_1 + \frac{g_{33}}{j\omega s^D_{33}}I_{33} \tag{30}$$

Eq. 29 and Eq. 30 can be written as:

$$F_1 = (U_1 - U_2)Z^l_{2p} + U_1 Z^l_{1p} + \frac{g_{33}}{j\omega s^D_{33}}I_{33} \tag{31}$$

$$F_2 = (U_1 - U_2)Z^l_{2p} - U_2 Z^l_{1p} + \frac{g_{33}}{j\omega s^D_{33}}I_{33} \tag{32}$$

	Density (Kg/m^3)	Elastic constants ($10^{-12}m^2/N$)	Piezomagnetic constants	Piezoelectric constant ($10^{-3}Vm^2/N$)	Elastic stiffness ($10^{10}N/m^2$)	Coupling factor
$PZT-5A^a$	7600	$s_{33}^E = 16.7$ $s_{11}^E = 14.8$	$d_{33,p} = 580pC/N$	$g_{33} = 24.8$	$c_D^{33} = 14.7$	$k_t = 0.5$ $k_{15} = 0.685$
$Terfenol-D^b$	9230	$s_{33}^H = 40$	$d_{33,m} = 1.210^{-8}m/A$			

^{a}Cited from Sunnytec Company, Suzhou, China.
^{b}Cited from Reference 9.

TABLE I
PHYSICAL ARAMETERS OF TERFENOL-D AND PZT MATERIALS USED IN THE SIMULATION

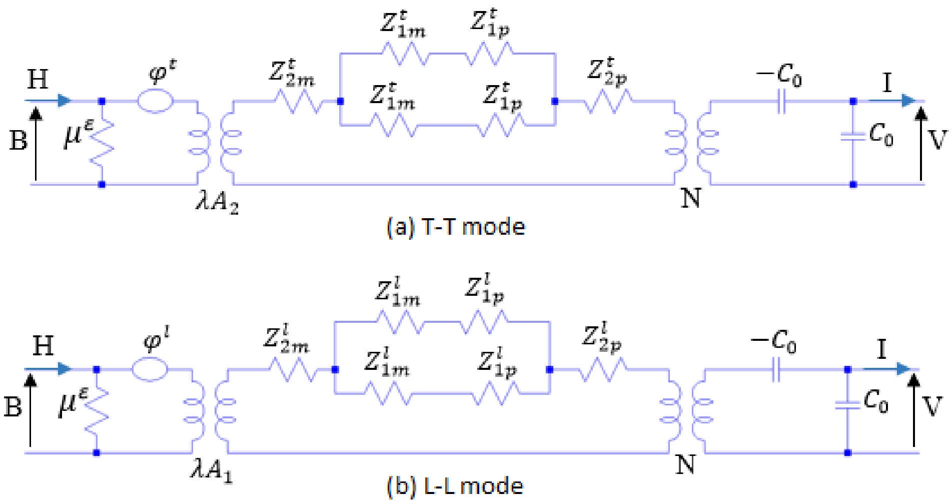

Fig. 9. Complete equivalent circuit of magnetostrictive/ piezoelectric/ magnetostrictive laminated materials

Where $Z_{1p}^l = jZ_0 \tan(\frac{\omega l}{2v^D})$ and $Z_{2p}^l = -\frac{jZ_0}{\sin(\frac{\omega l}{2v^D})}$

The voltage V across the layer is obtained in fuction of $C_0 = \frac{wt}{l\beta_{33}^\sigma}(1-k_{33}^2)$ as follows

$$V = \int_0^l E_3 dx_3 = \frac{g_{33}}{j\omega s_{33}^D}U_1 - \frac{g_{33}}{j\omega s_{33}^D}U_2 + \frac{I_3}{j\omega C_0} \tag{33}$$

An ideal electromechanical transformer can now be introduced with a turns ratio N defined as the ratio (force out)/ (voltage in).

$$N = \frac{force}{voltage} = \frac{wt}{l}(\frac{\varepsilon_{33}^\sigma}{s_{33}^E})^{0.5}k \tag{34}$$

The complete equivalent circuit for the length expander layer with end electrodes is given in figure 7.

A. Thickness mode with electric field parallel to thickness

A thin piezoelectric layer of thickness t and length l and polarized along its thickness direction is shown in figure 6. The equivalent circuit for the end-electroded and tickness-electroded piezoelectric plates are given ,respectively, in figure 7 and figure 8.

For the thickness-electroded mode the coefficients Z_{1p}^t, $Z_{2p}^t, Z_0, C_0, v_t^D$ and N are given as follows

$$Z_{1p}^t = jZ_0 \tan(\frac{\omega t}{v_t^D}),\ Z_{2p}^t = -\frac{jZ_0}{\sin(\frac{\omega t}{v_t^D})}$$

$$Z_0 = wt\rho v_t^D,\ C_0 = \frac{wl}{t\beta_{33}^\varepsilon}$$

$$v_t^D = (\frac{c_{33}^D}{\rho})^{0.5} \text{ and } N = \frac{wl}{t}(\frac{c_{33}^D}{\beta_{33}^\varepsilon})^{0.5}$$

IV. COMPLETE EQUIVALENT CIRCUIT AND SIMULATION

A. Complete equivalent circuit

Based on the previous analysis a complete equivalent circuit of the three-laminated layer is shown in figure 9 for the L-L and T-T modes. Note that V is the induced voltage from the piezoelectric layer, B is the magnetic induction exciting the two magnetostrictive layers. The first transformer represents the magnetostrictive layers which transform the magnetic excitation into a forces applied on the two faces of the piezoelectric layer. The second transformer represents the piezoelectric material that transform the forces into a corresponding electric voltage. From the complete equivalent circuit figure 9(a), the voltage coefficient is derived as follows:

$$\alpha_V = |\frac{dV}{dH}| = \frac{Ntd_{33}}{C_0} + \frac{\phi^t \lambda A_t Z_{eq} + \lambda A_t \mu^\sigma}{N} \tag{35}$$

Where $Z_{eq} = \frac{(Z_{1p}^t + Z_{1m}^t)}{2} + Z_{2p}^t + Z_{2m}^t$

B. Simulation

According to the numerical values given in table 1, the theoretical electric power output is calculated for the T-T, L-L and L-T modes using MATLAB and the results are plotted in figure 10. The dimensions of both terfenol-D and piezoelectric layers is $12mm \times 6mm \times 1mm$. As shown in Fig. 10, the L-L mode gives the highest power output compering to the other modes. This result is consistent with previous investigations [11], [12] and [13]. Alternatively, for this new analytical model, the value of the electric power output was significantly increased for magnetic flux density above 0.15 T. For a magnetic induction B=0.1T, the electric power is calculated for various thickness ratio n (n is equal to magnetostrictive thickness by piezoelectric layer thickness). The result in Fig. 11 shows that the electric power decreases for high values of the thickness ratio, i.e. the magnetostrictive layer must be thinner than the piezoelectric layer.

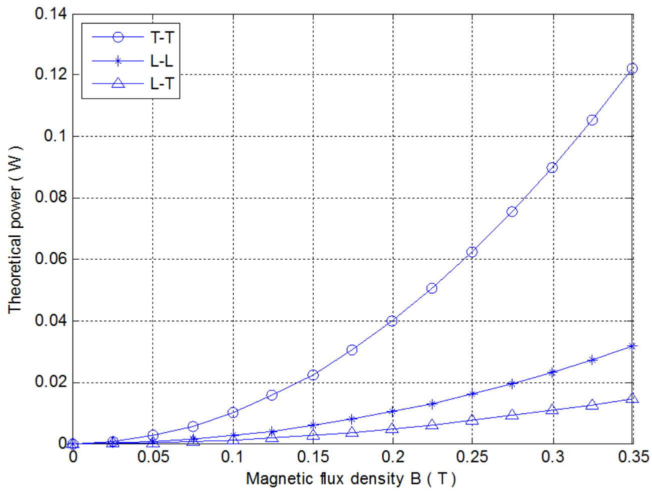

Fig. 10. Theoretical power output for T-T, L-L and L-T modes

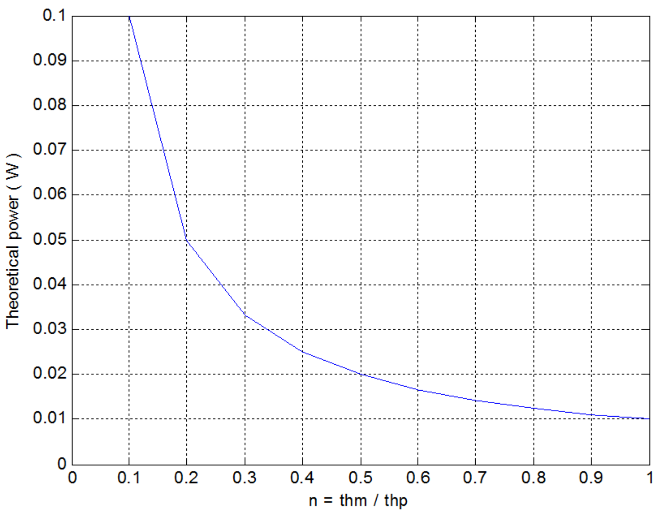

Fig. 11. Theoretical thickness ratio versus electric power output

V. Conclusion

The ME equations were derived and the corresponding equivalent circuits were presented for the magnetostrictive and piezoelectric layers. Leading to the complete equivalent circuit of the layers coupled in different configuration such as T-T and L-L mode. Simulations were done to predict the mode and the optimal thickness ratio giving higher electric power output, on one hand, the T-T mode shows significantly higher values relative to the other mode, on the other, the magnetostrictive layer must be thinner compering to the piezoelectric layer.

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