

# Data processing for the accurate evaluation of sea-waves parameters

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**Abstract** – The monitoring of sea waves, either for weather forecasting or for comfort evaluation in navigation, typically includes spectrum measurement of wave elevation, either as the final result or as an intermediate step for the measurement of the relevant wave-motion parameters. Therefore, different spectrum estimation methods have been compared, by applying them to simulated wave-motion signals. Their performance is reported and guidelines for their application are provided.

## I. INTRODUCTION

Nowadays ships are equipped with a variety of sensors useful to obtain information about relevant motions and accelerations in a seaway and to increase, by means of proper active weather routing techniques, both the level of onboard comfort experienced by passengers [1] and the crew safety during the routine working operations. In this respect, the correct estimate of ship motion spectra is a basic issue to provide reliable information to the active weather routing decision system [2].

Really, before assessing the main parameters of ship motion spectra in a seaway, the basic problem of sea spectrum resampling from wave time history needs to be investigated, provided that some concerns arise with reference to: (i) the proper selection of the most reliable technique that allows efficiently resampling the wave spectrum; (ii) the minimum duration of the wave time record, required to obtain a robust estimate of the wave spectrum parameters and (iii) the amplitude of the window embodied in the short-time Fourier transform (STFT).

This basic issues reveal to be essential for the optimal fitting of wave spectra to measured sea states, as widely discussed by Mansard and Funke [3]. In fact, the variability of spectral estimate, based on conventional spectral density analysis, is appreciable and mainly depends on the quality of recorded data that in some cases can be also corrupted by noise, so as the selection of upper and lower cut-off frequencies may play a fundamental role in the wave spectrum resembling procedure. In this respect, the estimation of the three principal spectral

parameters, namely the wave peak frequency, the peak enhancement factor and the significant wave height has been performed in the past by means of different algorithms. As concerns the estimation of the wave peak frequency, different peak frequency estimators are available in literature [3]. The peak enhancement factor, instead, is generally assessed through the use of a bivariate Rayleigh probability density function [4], while the significant wave height is generally estimated by the truncated 0-order spectral moment [5].

Based on previous remarks, the main aim of current research is to compare different methods for sea spectrum resampling from wave time history that, in turn, is currently generated based on the theoretical JONSWAP spectrum. This method allows comparing the main data of the resembled spectrum with the theoretical ones, in order to efficiently compare the applied methods and investigate the incidence of the above mentioned parameters on the assessment of the main data of wave spectra.

## II. CONCISE REVIEW OF SPECTRUM ESTIMATION METHODS

Spectrum measurement is a key topic in dynamic measurement and, due to its theoretical and practical importance, several estimation methods have been proposed [6-8], since the appearance of the Fast Fourier Transform (FFT), in the late 1960s, which made possible their implementation as digital procedures and application to ocean waves have also been studied, to some extent [9-11]. Such methods may be parsed in two main groups, namely non parametrical and parametrical.

The seminal idea under the first group was the "periodogram", that is the square of the FT of the series of observations, divided by the observation time,  $T$ . The periodogram, firstly proposed by Schuster as a way for identifying hidden periodicities in a highly noisy signal, constitutes a rough estimate of the power spectral density (PSD), which may be improved by tapering the signal with smoothing windows, that allow to reduce spectral leakage. This is often called the modified periodogram. A further improvement may be obtained, by dividing the signal into

segments, calculating the modified periodogram for each of them and then taking the average of them. Averaging allows to reduce the variance of the estimate. Two main variations were developed, either considering non overlapping segments (Bartlett method), or allowing some overlap, typically from 20% to 50% (Welch method).

An alternative approach, developed since the 1970s, consists in considering parametrical models of the observed time series, such as the auto-regressive (AR) one, and obtaining estimates for the parameters involved. Once such estimates are available, the PSD of the signal may be approximated by that of the output of the model when driven by a white noise input.

Methods differ in the type of model (AR, moving average, i.e., MA, or ARMA) and in the way the parameters are estimated. For AR models, typical approaches are Burg's and the covariance method.

Lastly, a significant variation on the non-parametrical approach was proposed by Thomson, in the early 1980s [7]. The idea was to taper the data with different tapers, since the denomination of multi taper approach, each able to highly different features of the signal, also accounting, in a way, to phase information.

### III. COMPARISON OF THE PERFORMANCE OF THE CONSIDERED METHODS

The different approaches just presented have been tested on simulated signals, from a JONSWAP wave spectrum, which is defined as follows:

$$S_{\zeta}(\omega) = A_{\gamma} S_{PM}(\omega) \gamma \exp\left(-0.5\left(\frac{\omega - \omega_p}{\sigma \omega_p}\right)^2\right) \quad (1)$$

having denoted by:  $\omega$  the wave circular frequency,  $\omega_p = 2\pi/T_p$  the spectral peak frequency as a function of the wave peak period, by  $\gamma$  the peak enhancement factor and by  $\sigma$  the spectral width parameter, equal to 0.07 if  $\omega \leq \omega_p$  and 0.09 otherwise. In eq. (1)  $S_{PM}$  denotes the Pierson-Moskowitz spectrum:

$$S_{PM}(\omega) = \frac{5}{16} H_s^2 \omega_p^4 \omega^{-5} \exp\left(-\frac{5}{4} \left(\frac{\omega}{\omega_p}\right)^{-4}\right) \quad (2)$$

with significant wave height  $H_s$ , while  $A_{\gamma}$  is a normalizing factor:

$$A_{\gamma} = 1 - 0.287 \ln(\gamma) \quad (3)$$

It is noticed that the wave mean period  $T_m$  is related to the wave peak period by the following equation [12]:

$$\frac{T_m}{T_p} = a + b\gamma + c\gamma^2 + d\gamma^3 \quad (4)$$

with  $a=0.7303$ ,  $b=0.04936$ ,  $c=-0.006556$  and  $d=0.000361$ . In current analysis, the numerical simulation is performed based on the following main input data:

$H_s=3.00$  m,  $T_m=5.00$  s,  $\gamma=1.86$ . Particularly, two time-series, sampled at 2 Hz, have been generated for a total observation time  $T$  of either 3600 and 600 s, as shown in Fig. 1 with reference to the latter duration. In this respect, the wave random elevation is determined as follows:

$$\zeta(t) = \sum_i A_i \cos(\omega_i t + \varphi_i) \quad (5)$$

where  $\omega_i$  is the wave circular frequency of the  $i$ -th wave component,  $A_i = \sqrt{2S_{\zeta}(\omega_i)\Delta\omega}$  and  $\varphi_i$  is the random phase. In current analysis the wave circular interval  $\Delta\omega$  was set equal to 0.005 rad/s, provided that it allows obtaining a good resolution of the input wave spectrum in the wave frequency interval up to 5.0 rad/s.

For  $T = 3600$  s, in the case of the Welch method, a best window duration of  $T_0 = 120$  s has been identified, and the resulting PSD is presented in Fig. 2. It may be noted that a very good approximation can be obtained, although with some underestimation of the peak value.

Concerning the AR parametric methods, both Burg's algorithm and the covariance method have been tested, the former reported in Fig. 3, the latter in Fig. 4. It may be noted that the results of both methods are quite similar, and that they show a significant bias in peak localization. Lastly the result of Thomson's method are reported in Fig. 5. It may be noted that in spite of a more "noisy" appearance, this method provides an unbiased result in terms of both peak localization and width. Concerning tests on signal with  $T=600$  s, only results from non-parametrical methods are here reported, in Fig. 6, for Welch method, with  $T_0 = 80$  s, and in Fig. 7, for Thomson's method. With this limited observation time, the performance of Thomson method seems to be superior.

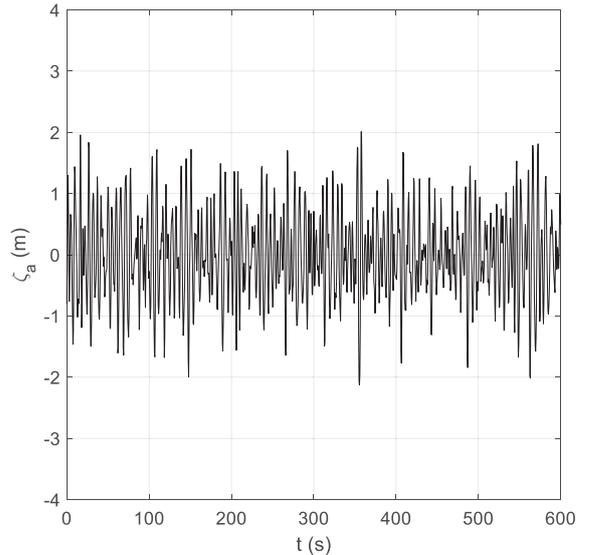


Fig 1. Simulated signal with  $T=600$  s

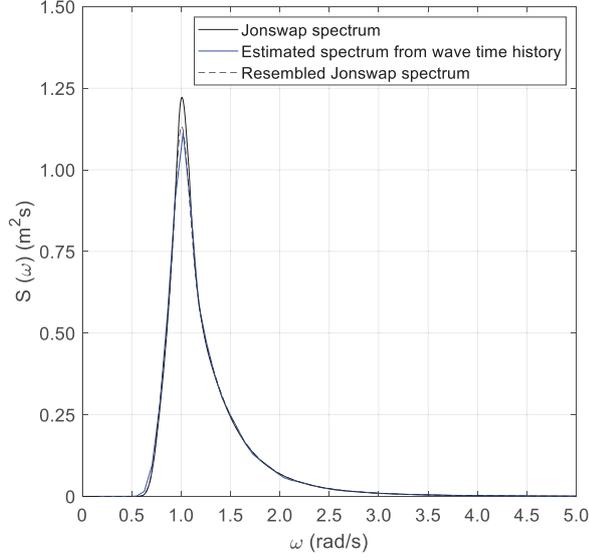


Fig 2. Estimated spectrum by the Welch method with  $T=3600$  s and  $T_0=120$  s

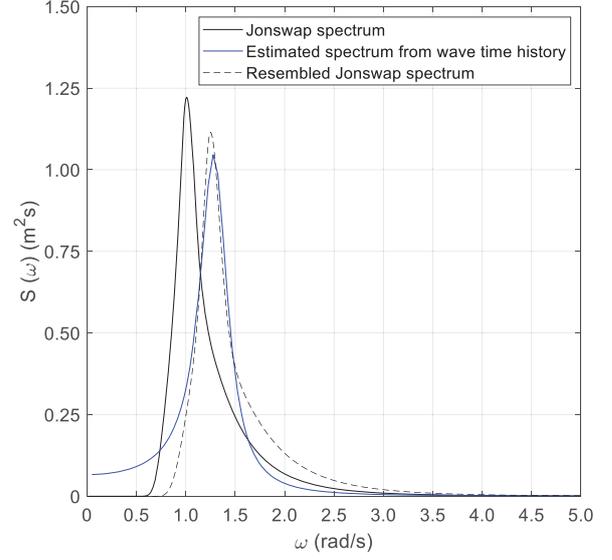


Fig 4. Estimated spectrum by the covariance method with second order model with  $T=3600$  s

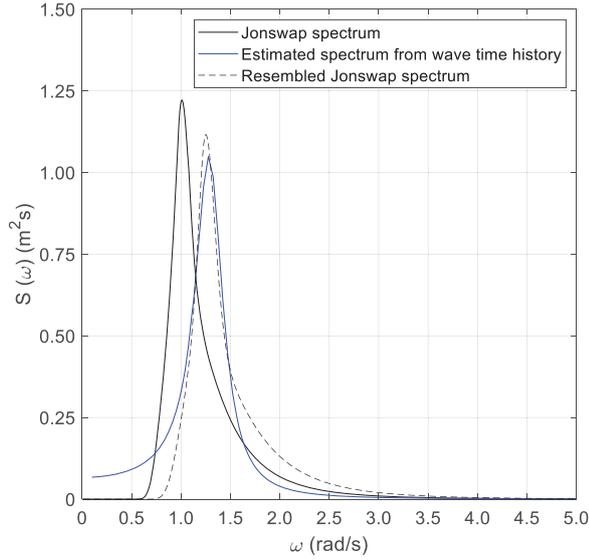


Fig 3. Estimated spectrum by the Burg method with second order model with  $T=3600$  s

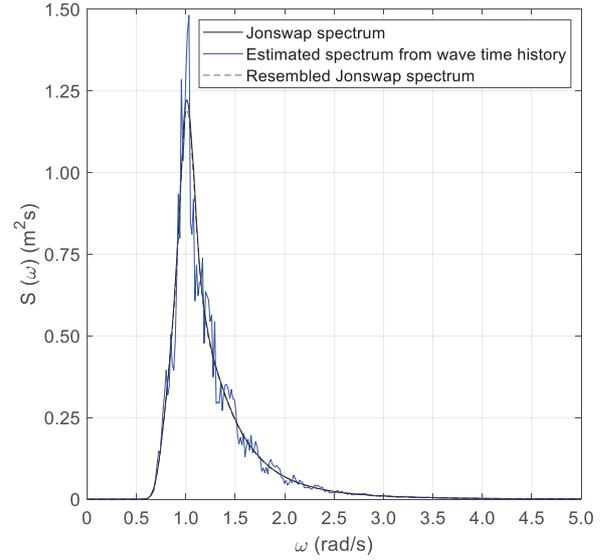


Fig 5. Estimated spectrum by the multi-taper Thomson method with  $T=3600$  s

#### IV. CALCULATION OF WAVE-MOTION PARAMETERS BASED ON SPECTRUM ESTIMATION

After assessing the estimated spectra by different techniques, the main parameters, namely the significant wave height ( $H_s$ ), the wave mean ( $T_m$ ) and peak period ( $T_p$ ), as well as the peak enhancement factor ( $\gamma$ ) can be easily determined by the following procedure. Firstly, if the random wave elevation is a Gaussian narrow-banded process, the significant wave height can be immediately determined as follows:

$$H_s = 4 \sqrt{\int_0^{\infty} S_{\zeta}(\omega) d\omega} \quad (6)$$

having denoted by  $S_{\zeta}$  the estimated spectrum amplitude as a function of the wave circular frequency  $\omega$ . Subsequently, the remaining parameters, namely the wave period and the peak enhancement factor, are easily established by curve fitting based on the least-square method. Hence, Tables 1 and 2 report a comparative analysis between the theoretical wave spectrum and the main parameters of the estimated ones by the previously investigated methods.

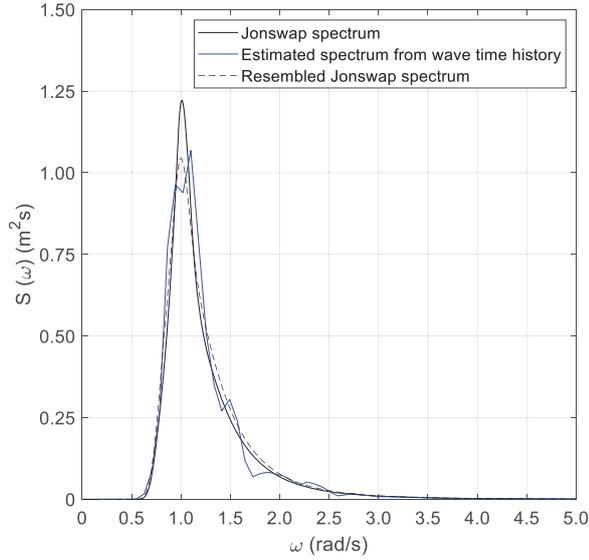


Fig 6. Estimated spectrum by the Welch method with  $T=600$  s and  $T_0=80$  s

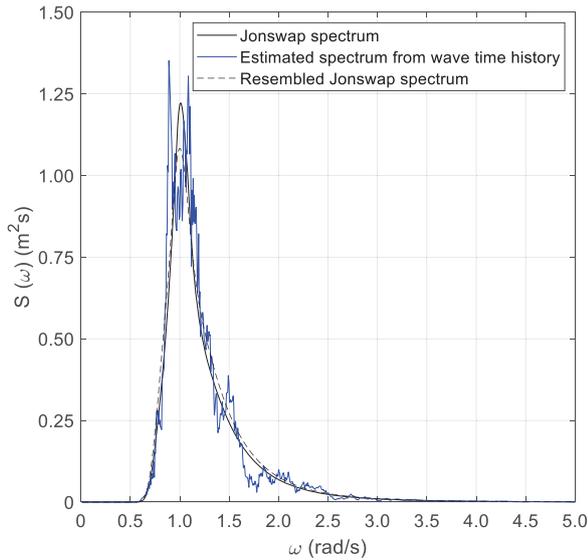


Fig 7. Estimated spectrum by the multi-taper Thomson method with  $T=600$  s

Based on current results, it seems that both Welch and Thomson methods provide a good assessment of the significant wave height and mean period of the wave spectrum, even if some differences arise with reference to the peak enhancement factor that seems to be mainly dependent on the duration of the recorded wave time history. As concerns the remaining methods, namely the Burg and Covariance ones, some appreciable differences arise with reference to the estimate mean wave period and peak enhancement factor.

TABLE I. ESTIMATED SPECTRA – 3600 s

Parameter	Input wave spectrum	Method			
		Welch	Burg	Covariance	Thomson
$H_s$ (m)	3.00	2.98	2.97	2.97	2.97
$T_m$ (s)	5.00	4.98	4.09	4.09	5.00
$\gamma$ (---)	1.86	1.68	2.35	2.32	1.84

TABLE II. ESTIMATED SPECTRA – 600 s

Parameter	Input wave spectrum	Method			
		Welch	Burg	Covariance	Thomson
$H_s$ (m)	3.00	3.07	3.03	3.04	3.08
$T_m$ (s)	5.00	4.95	4.12	4.12	4.96
$\gamma$ (---)	1.86	1.36	2.36	2.35	1.42

## V. CONCLUSIONS

The paper focused on the application of different methods to resemble the wave spectrum parameters from a recorded wave time history. In this respect, two wave time histories have been generated from a theoretical JONSWAP spectrum and subsequently analysed by different spectrum estimation methods, namely Welch, Burg, Covariance and Thomson. The analysis clearly shows that the selection of the estimation method plays a fundamental role for the reliable assessment of the main spectrum parameters, namely the significant wave height, the mean wave period and the peak enhancement factor. Based on current results, Welch and Thomson methods seems to be the most suitable techniques to resemble the input JONSWAP spectrum, while some differences arise with reference to the remaining techniques. Furthermore, the length of time history is a basic issue to correctly resemble the wave spectrum parameters. In this respect, Welch and Thomson methods provide a good estimate of all input parameters if the duration of the wave time history is equal to 1 h, while they are still unsatisfactory for the estimation of the peak enhancement factor if the time duration reduces up to 10 min.

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