

Absolute sea spectrum resampling from encounter wave time history

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Abstract – The aim of the work is to describe a practical and functional method to transform a wave energy spectrum from encounter frequency domain to absolute frequency domain. An important topic in the marine engineering field is the evaluation of the directional wave spectrum from the ship's measured motions, however, when a ship advances in a seaway any spectrum estimation is directly in to the encounter frequency domain. A transformation from the encounter domain to the absolute domain, corresponding to observations from a fixed point in the inertial system, is necessary. Several cases have been examined, for different angles between the route and the prevailing sea state, to validate the presented technique showing accurate results for all the tested cases.

I. INTRODUCTION

The present paper represents a preliminary part of a wider research program whose purpose is the evaluation of the exciting sea spectrum from the measured time history of ship motions. To achieve this objective, a fundamental step is the elaboration of a method to obtain the sea spectrum in absolute frequency domain from the encounter frequency domain valid for all prevailing sea state direction and keeping unchanged the wave energy spectral density. During the measurements, the ship advanced speed and the angle between the ship heading and the wave propagation direction do not change and the condition of deep water is always assumed. For beam and head sea the spectrum transformation, from encounter to absolute domain or vice versa, can be uniquely carried out while in following sea conditions the problem has not unique solution. The problem is mathematically governed by the Doppler Shift which associates to any one single encounter frequency three absolute frequencies when two conditions occur: the ship follows the wave and the encounter frequency is less than a limit value.

In literature there are a lot of papers and naval architecture textbooks about the theoretical implications of the Doppler Shift for a ship sailing in waves. These include some milestone as: Beck et al. [1], Bhattacharyya [2], Jensen [3], Journée and Massie [4], Lewandowski [5]

and Lyod [6]. Nonetheless, all this references are very detailed and complete for the theoretical construction of the spectrum in encounter frequency domain starting from the spectrum in absolute frequency domain but much less exhaustive about the reverse transformation.

However, two transformation techniques, from encounter frequency domain to the absolute frequency domain are presented in Nielsen [7,8]. The second transformation technique allows, using a function formulated through the wave spectral moments, an optimisation of a set of characteristic wave parameters. The method was tested with numerical simulations and also with full-scale experimental data.

In the present work a specific transformation algorithm is given, based on the papers of Nielsen, to achieve the sea spectrum, exciting the ship, in absolute frequency domain starting from the same spectrum in encountered frequency.

II. PROBLEM FORMULATION

The transformation of wave energy spectra from one frequency domain to the other implies a fundamental assumption: the total energy must be conserved, as expressed in equation (1).

$$S_0(\omega_0)d\omega_0 = S_e(\omega_e)d\omega_e \quad (1)$$

where ω_0 and ω_e are respectively the absolute and the encounter frequency, while S_0 and S_e are the spectra ordinate in the absolute and encounter domain.

In deep water the relation between ω_0 and ω_e is expressed by equation (2).

$$\omega_e = \omega_0 - \omega_0^2 \psi \quad (2)$$

$$\psi = \frac{U}{g} \cos \mu \quad (3)$$

where U is the advanced ship speed and μ is the heading relative to the waves.

In current analysis, the ideal spectrum used is the JONSWAP spectrum. For a fixed U and μ the encounter

frequencies are evaluated and the encounter spectrum is assessed, distinguishing three cases. Subsequently from the encounter spectrum the wave time history elevation is generated. The encounter sea spectrum is resembled from the time history of sea surface elevation and, finally, the absolute spectrum is assessed.

The first case is when $\mu \in]90^\circ, 180^\circ]$ in this case ψ is <0 , the equation (2) is a bi-univocal relation. For each absolute frequency there is only one encounter frequency and vice-versa, in particular are valid the following relations:

$$\omega_0 = \frac{1 - \sqrt{1 - 4\psi\omega_e}}{2\psi} \quad (4)$$

$$S_e(\omega_e) = \frac{S_0(\omega_0)}{\sqrt{1 - 4\psi\omega_e}} \quad (5)$$

The second case is when $\mu = 90^\circ$ in this case $\omega_0 = \omega_e$ and, consequently, $S(\omega_0) = S(\omega_e)$.

The third case is when $\mu \in [0^\circ, 90^\circ[$ in this case ψ is >0 , the equation (2) is not a bi-univocal relation as showed in Figure 1. For each encounter frequency there are three possible absolute frequencies.

If the encounter frequency is minor than $1/4\psi$, the absolute frequencies are expressed, respectively in the three region I, II and III, by (6).

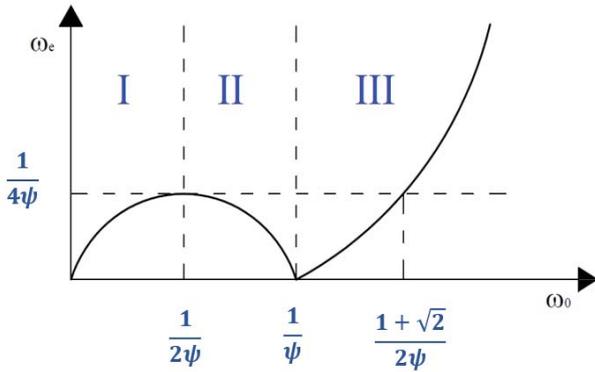


Fig. 1. Relationship between encounter wave frequencies and absolute wave frequencies

$$\begin{cases} \omega_{01} = \frac{1 - \sqrt{1 - 4\psi\omega_e}}{2\psi} \\ \omega_{02} = \frac{1 + \sqrt{1 - 4\psi\omega_e}}{2\psi} \\ \omega_{03} = \frac{1 + \sqrt{1 + 4\psi\omega_e}}{2\psi} \end{cases} \quad (6)$$

The spectrum in encounter frequency domain is given by the sum of the three contributions:

$$S_e(\omega_e) = S_0(\omega_{01}) \left| \frac{d\omega_{01}}{d\omega_e} \right| + S_0(\omega_{02}) \left| \frac{d\omega_{02}}{d\omega_e} \right| + S_0(\omega_{03}) \left| \frac{d\omega_{03}}{d\omega_e} \right| \quad (7)$$

$$S_e(\omega_e) = \frac{S_0(\omega_{01}) + S_0(\omega_{02})}{\sqrt{1 - 4\psi\omega_e}} + \frac{S_0(\omega_{03})}{\sqrt{1 + 4\psi\omega_e}} \quad (8)$$

When the encounter frequency is equal or major than $1/4\psi$, the absolute spectrum ordinates are calculated using equation (10).

$$\omega_{03} = \frac{1 + \sqrt{1 + 4\psi\omega_e}}{2\psi} \quad (9)$$

$$S_e(\omega_e) = \frac{S_0(\omega_{03})}{\sqrt{1 + 4\psi\omega_e}} \quad (10)$$

The first step is the definition of the input parameters of the JONSWAP spectrum. For an assigned velocity and heading angle, relative to the prevailing sea state, the encounter spectrum is assessed. Subsequently, starting from the frequency domain distribution of the wave components, the time-domain simulation of the irregular sea surface elevation is obtained. From the wave time history, the frequency-domain spectrum has been resembled. In Pennino et al [9] the resembled spectrum is obtained by the Fast Fourier transform, analysing the incidence of Fourier transform parameters on sea spectrum resampling and how the shape of the obtained spectrum was strictly dependent on the chosen parameters. In this paper the resembled spectrum is obtained used the periodogram which returns power spectral density (PSD) estimation of the input signal using a rectangular window.

For the transition from encounter spectrum to absolute spectrum it is possible to proceed in a specular manner, considering three different cases.

Basically, as it is clearly shown in Figure 2, when ω_e is minor than $1/4\psi$ it is not immediate to associate the energy contribution associated with a range of frequencies ω_e to three different energy rates in the field of absolute frequencies.

When $\mu \in]90^\circ, 180^\circ]$ the equation (2) is bi-univocal and the absolute spectrum is obtained by equation (12).

$$\omega_e = \omega_0 - \omega_0^2 \psi \quad (11)$$

$$S_0(\omega_0) = S_e(\omega_e)(1 - 2\psi\omega_0) \quad (12)$$

When $\mu = 90^\circ$, as a result, ψ is equal to zero and

$$S(\omega_0)=S(\omega_e).$$

When $\mu \in [0^\circ, 90^\circ]$ the equation (2) is not a bi-univocal relation and two different conditions, a and b, are distinguished:

- a) $\omega_0 \geq \frac{1+\sqrt{2}}{2\psi}$
- b) $\omega_0 < \frac{1+\sqrt{2}}{2\psi}$

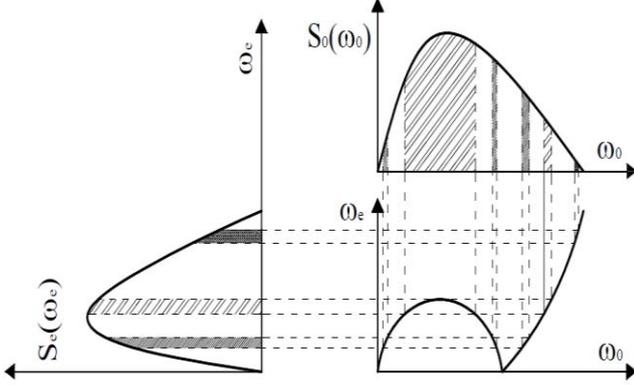


Fig. 2. Transformation of wave spectrum

When the condition a) is verified the equation (2) is an invertible function and the following relations, to obtain the spectrum in absolute frequencies domain, are applicable:

$$\omega_e = -\omega_0 + \omega_0^2 \psi \quad (13)$$

$$S_0(\omega_0) = S_e(\omega_e)(-1 + 2\omega_0 \psi) \quad (14)$$

While if the condition b) is verified the equation (2) is not an invertible function and the successive procedure has to be followed. First of all, the transformation must be divided into two parts depending on which condition of equations (15) is verified.

$$\begin{cases} \omega_0 < \frac{1}{\psi} \rightarrow \omega_e = \omega_0 - \omega_0^2 \psi \\ \frac{1}{\psi} \leq \omega_0 < \frac{1+\sqrt{2}}{2\psi} \rightarrow \omega_e = -\omega_0 + \omega_0^2 \psi \end{cases} \quad (15)$$

For each value of ω_e there will be three values of the absolute frequency as in (6). Considering that the area subtended from the spectrum is always the same, in absolute or encounter frequencies, the n th order spectral moment in the encounter domain can be assessed from the encounter spectrum, consequently, it is possible to evaluate the sea state parameters, such as significant wave height using equation (16).

$$H_S^* = 4 \sqrt{\int_0^\infty S_e(\omega_e) d\omega_e} \quad (16)$$

A theoretical and parameterised wave spectrum is introduced. The input parameters for the parameterised spectrum will, typically, be significant wave height and a characteristic wave period. It is not possible to obtain the period, from the encounter-wave spectrum, as done for the significant wave height. The peak enhancement factor, γ^* , and the peak period, T_P^* , are set arbitrarily. On first approximation it is possible to proceed considering γ^* equal to 1 as in Bretschneider spectrum and T_P^* evaluated by equation (17).

$$T_P^* = 1.4049 \cdot 2\pi \sqrt{\frac{\int_0^\infty S_e(\omega_e) d\omega_e}{\int_0^\infty \omega_e^2 S_e(\omega_e) d\omega_e}} \quad (17)$$

In this way it is possible to estimate the parameterised wave spectra: $S_1(\omega_1, H_S, T_P^*, \gamma^*)$; $S_2(\omega_2, H_S, T_P^*, \gamma^*)$ and $S_3(\omega_3, H_S, T_P^*, \gamma^*)$. According to which condition between A, B or C is verified the spectrum in absolute frequencies will be evaluated, respectively, by the first, the second or the third equations of (18).

$$A. \quad \omega_0 \leq \frac{1}{2\psi}$$

$$B. \quad \frac{1}{2\psi} < \omega_0 \leq \frac{1}{\psi}$$

$$C. \quad \frac{1}{\psi} < \omega_0 \leq \frac{1+\sqrt{2}}{2\psi}$$

$$\begin{cases} S_0(\omega_0) = \frac{S_{01}}{S_{01} + S_{02} + S_{03}} S_e(\omega_e)(1 - 2\omega_0 \psi) \\ S_0(\omega_0) = \frac{S_{02}}{S_{01} + S_{02} + S_{03}} S_e(\omega_e)(-1 + 2\omega_0 \psi) \\ S_0(\omega_0) = \frac{S_{03}}{S_{01} + S_{02} + S_{03}} S_e(\omega_e)(-1 + 2\omega_0 \psi) \end{cases} \quad (18)$$

III. APPLICATION TO SIMULATED DATA

The presented method has been applied for different values of H_S , T_P and γ , that define the JONSWAP spectrum, and for several advanced speed and μ angles showing always good results. In the following figures are reported: JONSWAP spectrum, encounter wave spectrum, resembled encounter wave spectrum, resembled JONSWAP spectrum for $H_S = 3\text{m}$, $T_P = 12\text{s}$, $\gamma = 2$, ship velocity equal to 15 kn and several μ values.

In Table 1 the values of significant height, peak enhancement factor and peak period evaluated for all the absolute spectra, rebuilt from the encounter spectra for different encounter angles, are reported. The results show

values very close to the starting ones, the differences are greater only in the case of encounter angle equal to zero, especially for the assessed peak enhancement factor. Indeed, the maximum difference has been found between the γ value for the starting spectrum and the reconstructed one for a zero encounter angle, all the other differences are much lower showing the reasonableness of the presented method.

Table 1. Parameters of rebuilt spectrum for different encountered angle direction

Input data for Jonswap Spectrum	H_S	T_p	γ
	3	12	2
$\mu = 0^\circ$	2,99	12,87	2,39
$\mu = 30^\circ$	2,92	12,18	2,14
$\mu = 45^\circ$	2,98	12,05	1,91
$\mu = 60^\circ$	3,01	12,04	2,06
$\mu = 90^\circ$	2,99	11,99	1,88
$\mu = 120^\circ$	2,99	12,00	2,10
$\mu = 150^\circ$	2,98	12,00	1,92
$\mu = 180^\circ$	2,98	12,06	1,96

IV. CONCLUSIONS

This work is a part of a project whose main aim is the assessment of the absolute sea spectrum, exciting the ship, starting from the ship motions. In case the observations are made on an advancing ship is necessary to transform the estimated wave spectrum from encounter frequency domain to absolute frequency domain. There is a lack, in the literature, of works describing a practical method to carry out this transformation. When the ship advances in beam or head sea there are no difficulties to apply the transformation from encounter to absolute frequency domain, while, when the ship moves on following sea the solution for the transformation is not unique. The article has proposed an algorithm, to make this transformation, based on the introduction of parameterised wave spectrum for computing scaling ratios of wave spectral densities. The algorithm has been tested for different numerical wave time history showing that it could be successfully used.

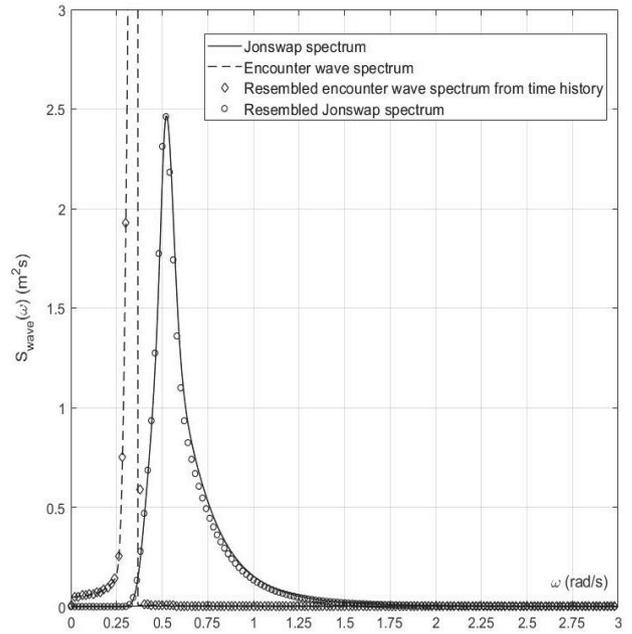


Fig. 3. Resembled spectrum for encountered angle direction equal to 30°

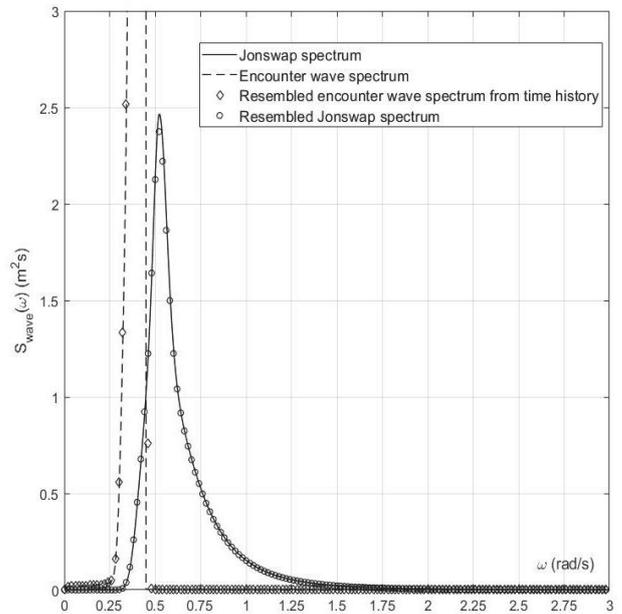


Fig. 4. Resembled spectrum for encountered angle direction equal to 45°

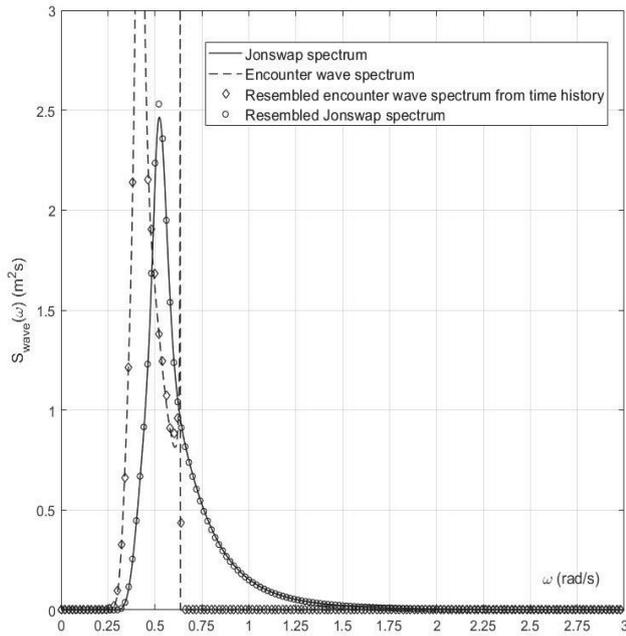


Fig. 5. Resembled spectrum for encountered angle direction equal to 60°

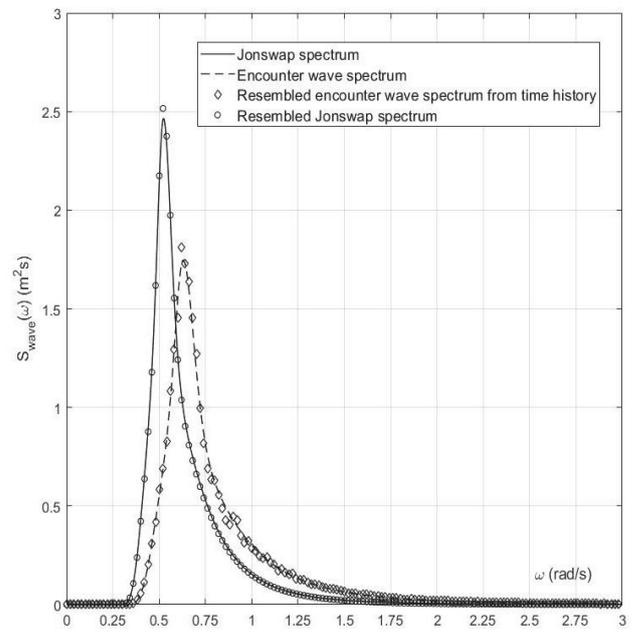


Fig. 7. Resembled spectrum for encountered angle direction equal to 120°

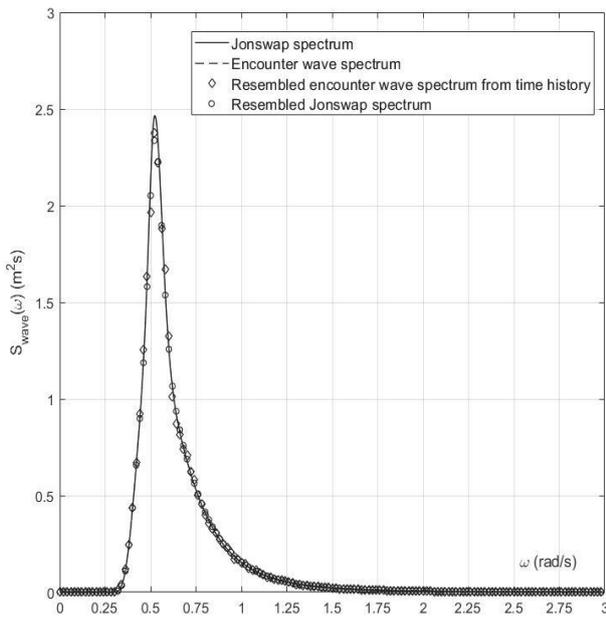


Fig. 6. Resembled spectrum for encountered angle direction equal to 90°

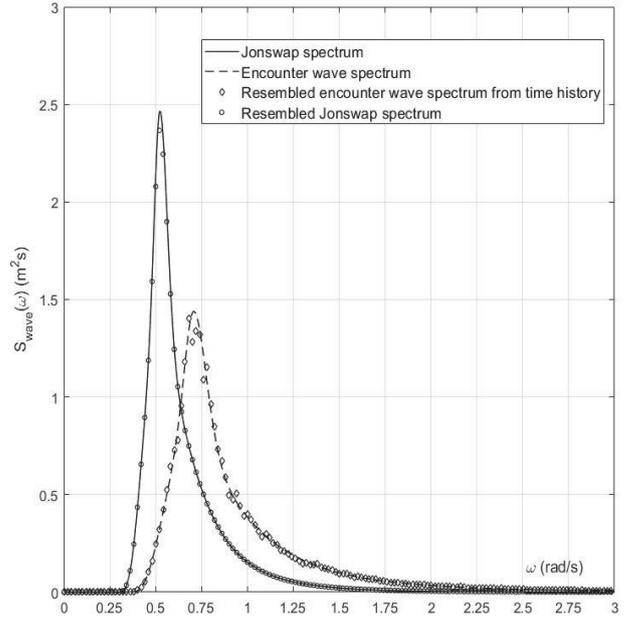


Fig. 8. Resembled spectrum for encountered angle direction equal to 150°

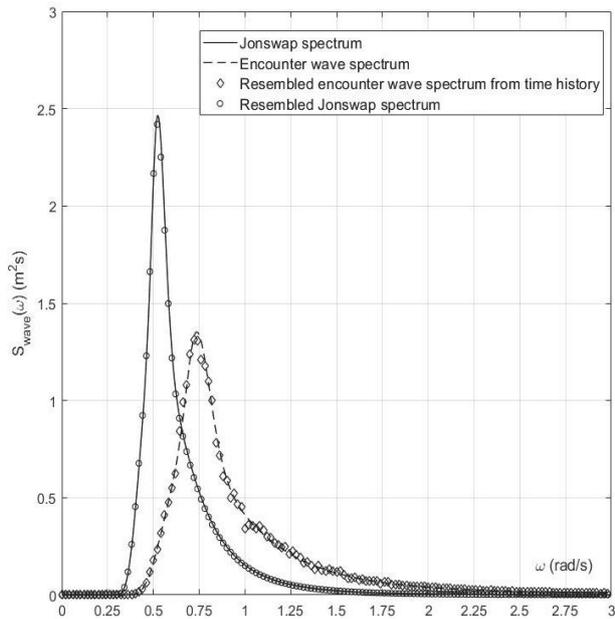


Fig. 9. Resembled spectrum for encountered angle direction equal to 180°

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