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Long period oscillations of sea level data in Genoa

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***Abstract* – Long term time series of annual mean sea level data collected from 1928 to 2018 in the tidal station of Genova are analized to investigate long period oscillations.**

I. INTRODUCTION

Over the last decade, there has been a significant progress in understanding future sea level values as a result of improved satellite and in "situ" observations and numerical models [1]. As a result, there is a huge demand for improved projections of sea level scenarios, particularly at local and regional level. At the Thomson gauge of Genoa (44°25' N, 08°54' E) a permanent sea land reference mark is used to measure sea level changes [2]. A statistical investigation on the mean annual data from 1928 to 2006 has produced a linear positive trend of about 1.1 mm per year [3]In a recent investigation [4] the mean sea level from 2006 to 2016 has shown a positive rate of 10.7 cm compared to the 1937 to 1946 average value which is the standard reference mark for the Italian terrestrial topography. In the present paper we show the results of spectral analyses of long period oscillations of the annual mean sea level data of Genoa from 1928 to 2018.

II. SUBHARMONIC ANALYSIS

It is well known that when a periodic force is applied to a linear system the resulting motion is obtained by a superposition of the transient and the steady state solutions. Thus, as far as linear systems are concerned, the forced oscillation is determined once the system and the external forces are given, and are by no means affected by the initial condition with which the oscillation was started. The nonlinear systems, however, can possess a wide variety of periodic oscillations in addition to those which have the same period as the external force. It has been pointed out by Trefftz [6] that if the solution of a differential equation:

$$d^2v/dt^2 = F(v, dv/dt, t)$$

$$\text{with} \quad F(v, dv/dt, t+T) = F(v, dv/dt, t) \qquad (1)$$

is stable, it must finally lead to a periodic solution in which the least period is equal to period T of the external force, or equal to an integral multiple of T.

Corresponding to these two cases the terms "harmonic" and "subharmonic" oscillations are respectively applied. Contrary to many cases of linear differential equations, it is possible to find the general solution of equation (1) for the given initial conditions. A conventional method of solution is to assume for v(t) a Fourier series development with undetermined coefficients, and then to fix them by nonlinear relations obtained by substituting the series into the original equation (1). It should, however, be noticed that this method of solution is merely to find out the periodic states of equilibrium, which are not always sustained, but last so long as they are stable. The circumstances under which this condition obtains are determined by the stability of the periodic states of equilibrium. The study of the periodic solutions is closely related to the investigation of the stability of the periodic states of equilibrium correlated with singular points. We now deal with the subharmonic oscillations whose frequencies are a fraction 1/2, 1/3, ..., of the frequency of the external force. As a typical example of analysis of subharmonic oscillations by means of integral curves, we shall treat the subharmonic oscillation of order 1/3 which will be discussed in this note. Consider the following equation:

$$d^2v/dt^2 + k\, dv/dt + v^3 = B \cos(3t) \qquad (2)$$

where the restoring force is expressed by a cubic non-linear function of v and the argument of the external force is 3t. The solution of the equation (2) is assumed of the form:

$$v(t) = x(t) \sin(t) + y(t) \cos(t) + W \cos(3t) \qquad (3)$$

where amplitudes x(t) and y(t) are both functions of t in the transient state, but are reduced to constants in the steady state. Following Mandelstam and Papalexi [7] amplitude W may be assumed to be:

$$W = 1/(1 - 3^2)\, B = -0.125\ B \qquad (4)$$

Substituting (4) into (2) and equalling the terms containing sin(t) and cos(t) separately to zero, we obtain:

$$dx/dt = 1/2\ [\ -k\ x + A\ y + 3/4\ W\ (\ x^2 - y^2)] = X(x,y)$$
$$dy/dt = -1/2\ [\ A\ x + k\ y + 3/4\ W\ 2\ x\ y] = Y(x,y) \qquad (5)$$

where: $A = 1 - 3/4\ (x^2 + y^2) - 3/2\ W^2$. Equations (5) play a significant role in the study of the transient state and the steady state. Oscillations in the steady state are correlated with the singular points determined by:

$$X(x, y) = 0, \quad Y(x, y) = 0 \qquad (6)$$

and the transient solution can be obtained by the integral curves of:

$$dy/dx = Y(x, y) / X(x, y) \qquad (7)$$

where X(x, y) and Y(x, y) are given by equations (5). Since time t does not occur explicitly in equation (7) we can draw integral curves in the x–y plane with the aid of the isocline method. Periodic solutions are related with x(t)=constant, y(t)=constant of equations (5) and therefore with the singular points of equation (7), i.e. the points at which X(x, y) and Y(x, y) both vanish.
According to the initial conditions an oscillation can show a subharmonic character or on the contrary no subharmonic oscillation takes place if the final singular point is a stable spiral point.

III RESULTS

In particular, we have found a significant oscillation with a period of 18.6 years associated with the period of the retrograde orbital motion of the Moon's nodes through a complete cycle: the Moon's nodes are the points where the Moon's orbit intercepts the Earth celestial ecliptic [5]. It is well known that when a periodic force is applied to a linear system the forced oscillation is determined once the system and the external force are given, and is by no means affected by the initial condition with which the oscillation was started. The nonlinear systems, however, can possess a wide variety of periodic subharmonic oscillations whose frequencies are a fraction 1/2, 1/3, ..., of the frequency of the external force [6,7]. Having established four components in the harmonic analysis of the mean sea level data we computed four FIR filters centred at the angular velocities: 0.3376 rad/yr (period =18.6 years), 0.6756 rad/yr (period=9.3 years), 1.0134 rad/yr (period=6.2 years), 1.3659 rad/yr (period=4.6 years). The superposition of the four filter outputs corresponding to the first four harmonics is close to the experimental sea level data in the time interval from 1928 to 1972 (Fig1).

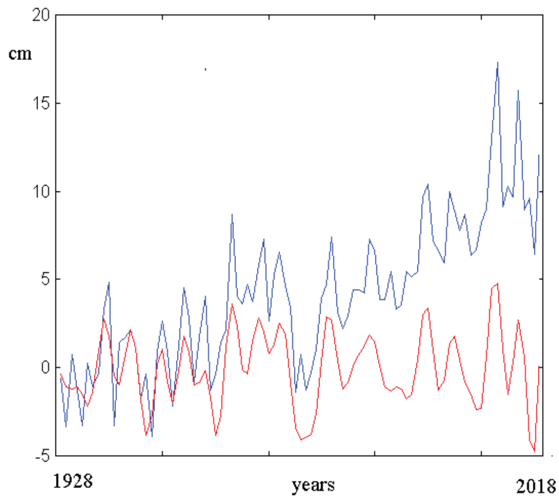

Fig.1. Mean annual sea level data in Genoa (blue) and the superposition of four harmonics (red)

At present our understanding of the mechanism responsible for the development of subharmonic waves in the mean annual sea level values in Genoa is quite limited; nevertheless we make the hypothesis that from 1973 to 2018 nonlinear effects have produced sea level oscillations whose frequencies are a fraction 1/3,1/5 of the frequency of the external force produced by the retrograde orbital motion of the Moon's nodes. The superposition of the harmonic and the subharmonic oscillations significantly reproduces the positive sea level trend of the last decades (Fig.2) and this result might play an important role to predict the sea level changes in Genoa over coming years .

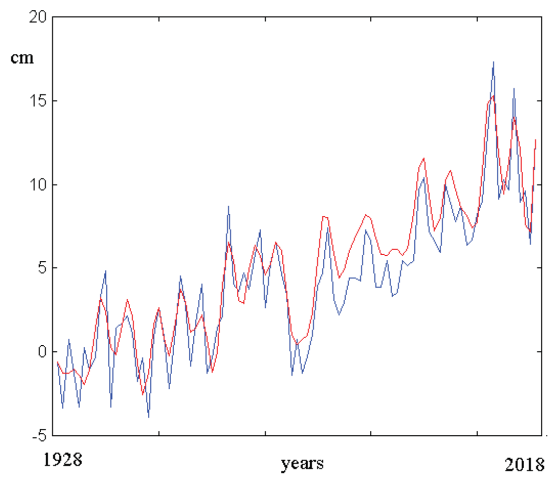

Fig.2 Mean annual sea level data in Genoa (blue) and the superposition of four harmonics and two subharmonic waves (red)

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