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Harmonics detection in frequency sparse signal

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Abstract **– The article shows the possibilities of using the compressive sensing technique to detect harmonics in a frequency sparse signal. Such features characterize signals in the ship's electrical network. A fast algorithm is presented, in which few randomly sampled projections of the input signal are used for its correct reconstruction. The ℓ_1-minimization problem in the compressive signal reconstruction is resolved by means of the convex optimization method with thresholding and the additional use of the *K*-rank-order filter in the signal's sparse domain in order to accelerate the convergence of the solution. The preliminary results of numerical simulation confirm the effectiveness of the algorithm used but also indicate its limitations.**

I. INTRODUCTION

The electric power quality (PQ) in ship system is described by the set of parameters characterizing a process of generation, distribution and utilization of electrical energy in all operation states of the ship (maneuvering, sea voyage, stay in the port) and its impact on the operation and safety of the ship as a whole. This set of parameters under consideration contains i.e. voltage and current parameters at all points of the analyzed system [1].

These parameters are mainly expressed by the coefficients of rms voltage value and its frequency deviations, coefficients of voltage asymmetry and coefficients characterizing the shape of voltage and current waveforms, it means characterizing the supply voltage distortion from a sinusoidal wave. Accordingly, to the lately updated IACS requirements [2], newly built ships must be equipped with devices for continuous monitoring of the level of harmonic distortion, while PQ factors should be measured on existing ships annually under seagoing conditions. Related measurements therefore require the collection of large amounts of digital data, which in turn leads to a huge demand for memory and significant burden in data processing.

To reduce these inconveniences, the implementation of a compressive sensing (CS) technique for data acquisition and further use of appropriate algorithms for data reconstruction may be a promising solution.

Various new techniques for identifying and estimating harmonic sources in electricity supply systems have been proposed in the literature [3, 4, 5].

In contrast to the typical approach, the CS technique provides an estimate of the signal being tested based on a small number of linear incoherent measurements [6, 7, 8]. The basic assumption in the CS approach is that most signals in real applications have a sparse representation in a certain transformation domain, which means that only a few coefficients are significant, while others are negligible or zero [6]. Many of the signals in real applications have a sparse representation in the Discrete Fourier transform (DFT) domain. Another relevant condition for using the CS technique is the incoherence between the measurement (observations) basis and the domain in which the signal has a sparse representation.

Recent research on CS technique indicates the possibility of accurate signal frequency analysis [9, 10].

This paper proposes the use of a fast reconstruction algorithm based on CS technique for detecting harmonics in the examined signal. The procedure uses random projections as measurements. The measurement matrix, generated from Bernoulli's random variables, allows the signal to be recovered with high accuracy. The ℓ_1-minimization problem in CS signal reconstruction is solved by means of a convex optimization method using a *K*-rank-order filter in the signal's sparse domain to accelerate solution convergence.

The paper is organized as follows. Section II presents the CS model in three aspects: signal sparsity, acquisition process and reconstruction condition. Section III explains the algorithmic implementation of the reconstruction procedure. Section IV shows exemples of simulation results obtained for selected multi-tone signals. Finally, the concluding remarks are formulated in Section V.

II. COMPRESSIVE SENSING MODEL

The CS theory consists of three main issues. The first is to find the sparsest representation of a signal. The second is the proper choice of measurement matrix that approximates well the original N-length signal for the least M coefficients. The last one concerns the implementation of a reconstruction algorithm that can recover the original signal from the observed M coefficients.

A. Sparsity of a signal

Let's assume that real signal $x \in R^N$ has a K-sparse representation in an orthonormal basis $\psi \in R^{NxN}$. This means that the signal can be expanded to K non-zero coefficients in basis ψ ($K<<N$). The approximation x of a signal can be expressed as follow [6]:

$$x = \sum_{i=1}^{N} a_i \cdot \psi_i = \psi \cdot a \tag{1}$$

where: $a_i \in R^N$ are the coefficients of a sparse transform domain of signal x.

In the paper, the domain of transformation is defined directly by the discrete Fourier DFT transform.

Let's consider a multicomponent signal that consists of K sinusoids and is described by:

$$x_n = \frac{1}{N} \sum_{k=1}^{K} X_k \cdot \exp(j \cdot \frac{2\pi}{N} \cdot n \cdot k) \tag{2}$$

The equation (2) shows the K-sparse representation of the signal x_n in terms of DFT, since:

$$X_k = \sum_{n=1}^{N} x_n \cdot \exp(-j \cdot \frac{2\pi}{N} \cdot n \cdot k) \tag{3}$$

where: X_k is a vector of DFT coefficients in which at most K coefficients are non-zero.

The transformation matrix ψ, created on the Fourier basis, is determined by [9]:

$$\psi_{n,k} = \frac{1}{\sqrt{N}} \cdot \exp(j \cdot \frac{2\pi}{N} \cdot n \cdot k) \tag{4}$$

B. Acquisition process

The measurement process is modelled by projections of the signal x onto vectors $\{\varphi_1, \cdots, \varphi_M\}$ forming the measurement matrix $\varphi \in R^{MxN}$. The vector of acquired samples $y \in R^M$ is defined as:

$$y = \varphi \cdot x \tag{5}$$

The transformation matrix ψ (related to the sparsity of signal) and the measurement matrix φ (used in measurement procedure) must be incoherent to ensure adequate reconstruction. The coherence measure is describe by [7]:

$$\mu(\varphi, \psi) = \sqrt{N} \cdot \max_{0 \le i, j \le N} \left| \left\langle \varphi_i, \psi_j \right\rangle \right| \tag{6}$$

The coherence takes values from the interval [7]:

$$1 \le \mu(\varphi, \psi) \le \sqrt{N} \tag{7}$$

The coherence should be as small as possible.

In the paper, the random Bernoulli matrix as the measurement matrix is used to ensure the incoherence of bases. Then, the acquisition process is described by following expression [11]:

$$y = \left\langle \varphi, x \right\rangle = \sum_{j=0}^{N-1} \varphi_{i,j} \cdot x_j \tag{8}$$

where: $\varphi_{i,j}$ - the *(i j)*th entry of the random binary matrix φ.

Taking into account the equation (1), the measurement signal y becomes [6]:

$$y = \varphi \cdot \psi \cdot a = \Theta \cdot a \tag{9}$$

where: $\Theta \in R^{MxN}$ is a reconstruction (sensing) matrix.

In the paper, the sensing matrix represents a partial random inverse Fourier transform matrix obtained by omitting rows from the transformation matrix ψ, that corresponds to unavailable samples positions:

$$\Theta = \begin{bmatrix} \psi_{11} & \cdots & \psi_{1N} \\ \vdots & \ddots & \vdots \\ \psi_{M1} & \cdots & \psi_{MN} \end{bmatrix} \tag{10}$$

C. Reconstruction condition

To ensure reconstruction of the sparse signal x from compressive measurements y, the inverse problem of formula (5) should be solved, which gives an infinite number of possible solutions. Consequently, optimization algorithms based on the ℓ_1-norm minimization is commonly applied [7]:

$$\hat{a} = \arg\min \|a\|_1 \quad subject\ to\ y = \Theta \cdot a \tag{11}$$

where: $\hat{a}$ denotes the estimate of a and $\|a\|_1$ means ℓ_1-norm of a.

According to (11), the estimation of the input signal can be made as [8]:

$$\hat{x} = \varphi \cdot (\Theta^T \cdot \Theta)^{-1} \cdot \Theta^T \cdot y \qquad (12)$$

where*: (matrix)*$^{-1}$ - the pseudoinverse matrix of *matrix*.

III. RECONSTRUCTION ALGORITHM

The ℓ1-minimization problem is solved by using of the convex optimization method with threshold t.

The reconstruction formula is defined as [11]:

$$\overline{x} = \frac{1}{1+i} \sum_{j=0}^{i-1} \frac{1}{p_j(1-p_j)} y_j \cdot \varphi_j \qquad (13)$$

where: p is the ones probability of Bernoulli distribution.

The algorithm works in a loop, and in each iteration, it checks whether equation (13) indeed converges.

The processing loop stops when the threshold t meets the given condition (Fig. 1).

The estimate of x_i at the i^{th} iteration is defined as [11]:

$$\hat{x}_i = F^{-1}\left\{Rank\left(F\{\overline{x}\}\right)\right\} \qquad (14)$$

where: *Rank* (*) denotes a *K*-rank-order filter.

The *K*-rank-order filter is applied in the Fourier domain to accelerate the convergence. The filter operates in the following way. Let's consider a input vector $X_k = F\{\overline{x}\} = [X_1, \cdots, X_N]$. First, the filter sorts input array with the values arranged in descending order. Then, it extracts the K most significant components from the input vector and assigns zeros to the remaining places. Thanks to that, the computational burden of the inverse Fourier transform, performed in the next step, is reduced.

IV. NUMERICAL SIMULATION

The simulations were carried out using a program designed based on an accessible application in the LabVIEW environment [11]. As an example, a multi-tone signal with fundamental harmonic 50 Hz has been simulated, according to the parameter sets shown in Tab. 1. The sampling frequency is equal to 10 kHz and the length of the time window is equal to 1000 samples. The time waveform and sparse representation of the tested signal in the Fourier domain is presented in Fig. 2.

When generating tones with slightly different levels of amplitude (set 1), the algorithm identifies harmonics with accuracy above 97%. The most accurate signal reconstruction (99,32%) is obtained for 210 sampled measurement signals, acquired with the ones probability p equal to 0,3 (Fig. 3).

For the multi-tone signal with more different levels of amplitude (set 2), the reconstruction based on 280 measurements and the same features of the measurement matrix does not allow the correct detection of all harmonics (Fig 4).

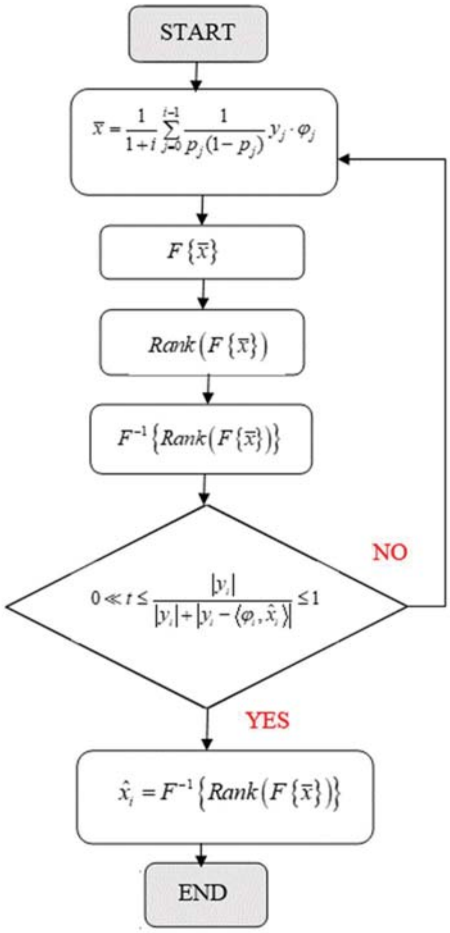

Fig. 1. Algorithmic steps of the algorithms.

The proper reconstruction requires two times more measurements, i.e. 400. The scenario with the dominant fundamental harmonic (set 3) shows that 500 sampled measurements are not sufficient to identify all components in the frequency domain (Fig. 5). The algorithm incorrectly detects the 7th harmonic. The increasing of the number of measurement results in the possibility of harmonics identification. In this case, the number of random samples reaches 800.

Table 1. The parameters of the input signals.

Harmonic order	Amplitude [V/V]		
	set 1	set 2	set 3
1	10	50	100
3	6	6	6
5	4	4	4
7	2	2	2

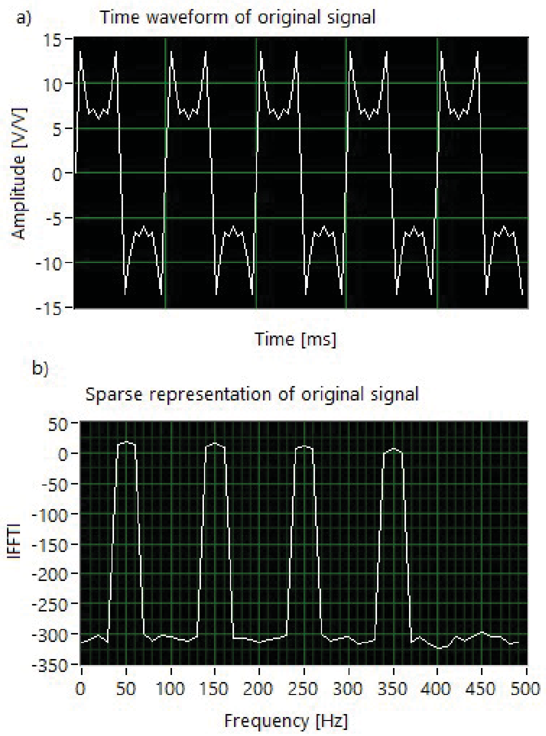

Fig. 2. The segment of time waveform (a) and DFT components (b) of input signal (set 1).

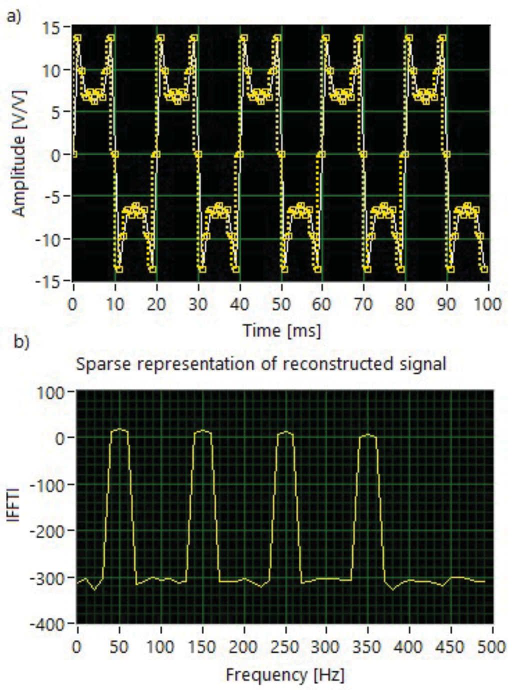

Fig. 3. The segments of time waveforms of the original (white line) and reconstructed signal (dotted yellow line) (a) and DFT components of the reconstructed signal (b). The set 1.

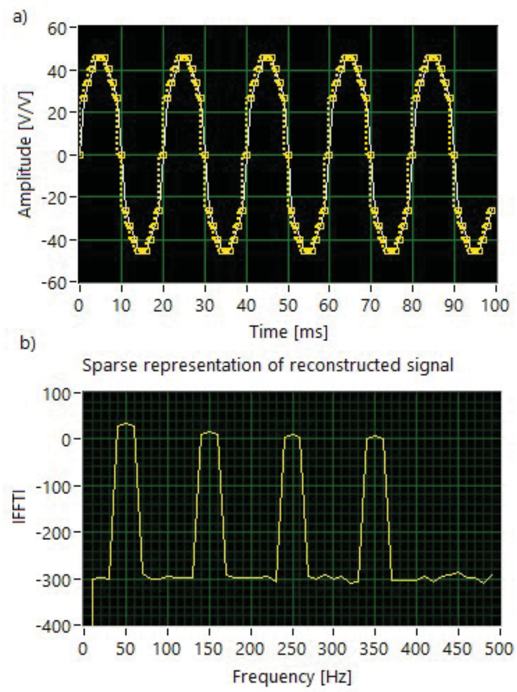

Fig. 4. The segments of time waveforms of the original (white line) and reconstructed signal (dotted yellow line) (a) and DFT components of the reconstructed signal (b). The set 2.

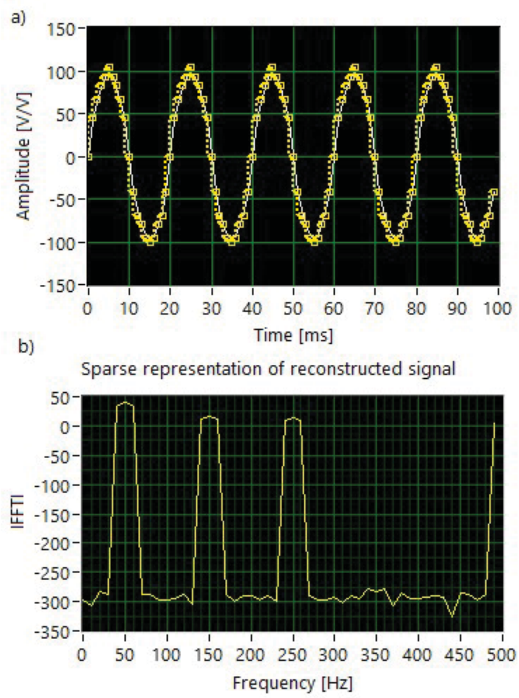

Fig. 5. The segments of time waveforms of the original (white line) and reconstructed signal (dotted yellow line) (a) and DFT components of the reconstructed signal (b). The set 3.

To study the effect of noise on the efficiency of signal reconstruction, white Gaussian noise is added at a signal to noise ratio (S/N) of 20 dB. Figure 6 shows the result of a sparse reconstruction of a multi-tone waveform with insignificant differences of amplitude levels (set 1) for 210 iterations (measurements). The presence of noise bothers to correct identification of harmonics. In the case of set 2, in which one tone is characterized by a much higher amplitude level, adding noise does not affect the detection of harmonics (Fig. 7). Noise interference distorts the precision of the spectral analysis of a multi-tone signal when the signal has a strong dominant component (set 3). Performing a spare signal reconstruction allows for more accurate detection of harmonics in the signal (Fig. 8). However, this is only possible with a very large number of iterations in the measurement algorithm.

V. FINAL REMARKS

The preliminary results of numerical simulations performed, using the fast reconstruction algorithm, show the limitations of effective reconstruction based on the CS method. The good reconstruction accuracy is in the case of multi-tone signal consisting of components whose amplitude levels do not differ significantly.

The application of the presented reconstruction algorithm to the signal in which the dominant harmonic occurs requires the pre-conditioning of the signal. It consists in the rejection of the fundamental component from the examined waveform.

The preliminary results of carried out simulations are promising therefore, the future task is to improve and develop of the presented procedures.

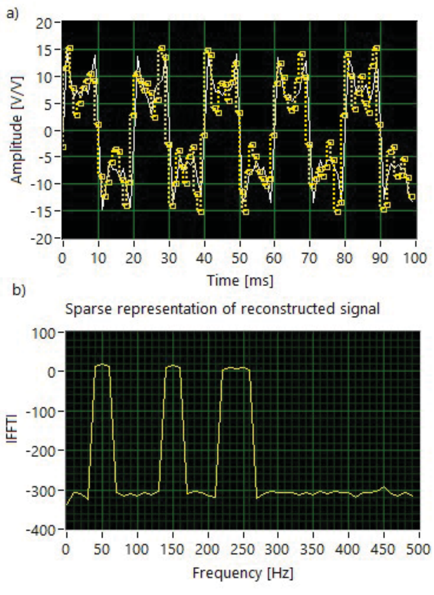

Fig. 6. The segments of time waveforms of the input (white line) and reconstructed signal (dotted yellow line) (a) and DFT components of the reconstructed signal (b). The input signal (set 1) with the additive noise.

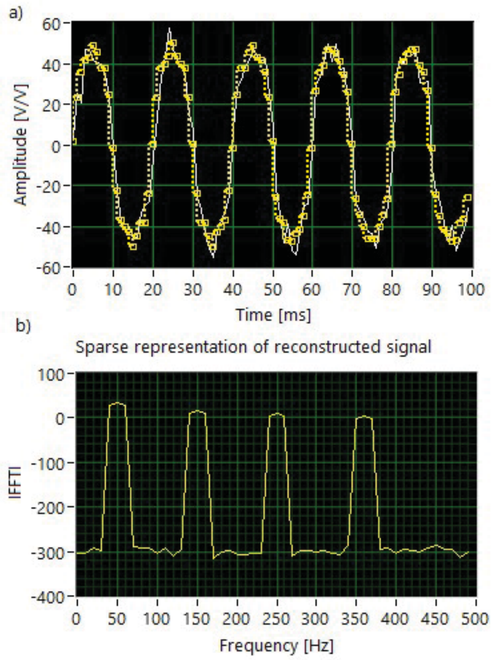

Fig. 7. The segments of time waveforms of the input (white line) and reconstructed signal (dotted yellow line) (a) and DFT components of the reconstructed signal (b). The input signal (set 2) with the additive noise. The number of measurments is equal 400.

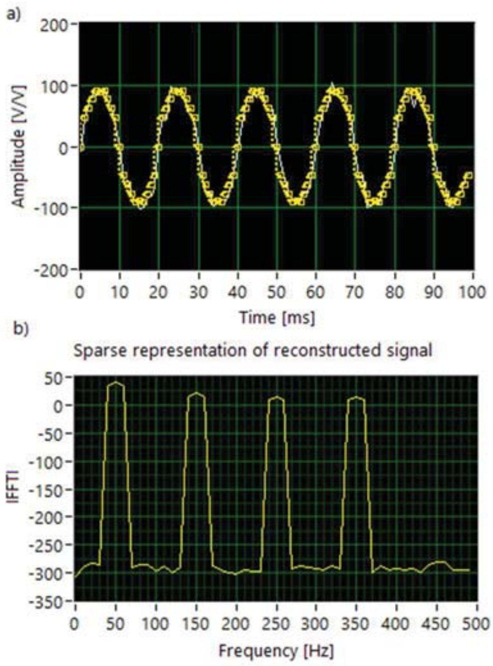

Fig. 8. The segments of time waveforms of the input (white line) and reconstructed signal (dotted yellow line) (a) and DFT components of the reconstructed signal (b). The input signal (set 3) with the additive noise. The number of measurments is equal 788.

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