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Data processing for the accurate evaluation of combined wind sea and swell spectra

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Abstract **– The reliable monitoring of sea state parameters is a key factor for weather forecasting, as well as to ensure the safety of ship and navigation. In current analysis different spectrum estimation techniques are applied to random wave signals, generated from a theoretical wave spectrum obtained by combining both wind sea and swell components, with different significant wave heights and peak periods. Hence, the proposed spectrum resembling methods are applied in order to compare the relevant performances, with reference to both short and long-time durations of the considered wave signal.**

I. INTRODUCTION

The assessment of wave spectra from the analysis of random wave elevations has been a widely investigated topic since the works of Mansard and Funke [1-2] and Battjes and val Vledder [3], since it is a key factor to detect the sea state conditions and ensure the safety of ship and navigation [4-6]. Really, the assessment of the wave spectrum parameters, namely the significant wave height, the wave peak period and peak enhancement factor, reveals to be a quite challenging issue, provided that some key factors, such as the selection of the proper spectrum estimation technique, the minimum duration of the wave time signal and the trade-off between spectral resolution and variance of the spectral estimator, represent key factors of the entire data processing procedure. Recently, some advances have been gathered by Rossi et al. [7-8] that compared different spectrum estimation techniques, applied to single peak wave spectra, concluding that the Welch and Thomson methods are the most promising techniques, combined with the Nonlinear Least Square Method (NLSM) for the assessment of sea state parameters, that reveal to be reliable even for short time signals.

Hence, the proposed procedure is now extended to double peak wave spectra, obtained by the superposition of wind sea and swell components, with different significant wave heights and peak periods. A comparative analysis between different spectrum estimation techniques is performed, in order to detect the best method combined with both short and long-time durations.

II. INPUT WAVE SPECTRUM AND RANDOM WAVE GENERATION

Combined wind sea and swell are described by a double peak wave spectrum according to the following equation [9]:

$$S(\omega) = S_{wind}(\omega) + S_{swell}(\omega) \quad (1)$$

where the wind sea and swell components are assumed to be uncorrelated and to follow the JONSWAP spectrum S_J that, in turn, is determined as follows:

$$S_J(\omega) = A_\gamma S_{PM}(\omega)\gamma^{exp\left(-0.5\left(\frac{\omega-\omega_p}{\sigma\omega_p}\right)\right)} \quad (2)$$

In eq. (2) ω is the wave circular frequency, $\omega_p = 2\pi/T_p$ is the spectral peak frequency depending on the wave peak period T_p, γ is the peak enhancement factor and σ denotes the spectral width parameter, equal to 0.07 if $\omega \leq \omega_p$ and 0.09 otherwise.

In eq. (2) A_γ is a normalizing factor, depending on the peak enhancement factor:

$$A_\gamma = 1 - 0.287ln(\gamma) \quad (3)$$

while S_{PM} denotes the Pierson-Moskowitz spectrum:

$$S_{PM}(\omega) = \frac{5}{16}H_s^2\omega_p^4\omega^{-5}exp\left(-\frac{5}{4}\left(\frac{\omega}{\omega_p}\right)^{-4}\right) \quad (4)$$

that, in turn, depends on the significant wave height H_s and the wave peak frequency ω_p. In absence of additional data, the wave peak period T_p is related to the wave mean period T_m by the following equation which is valid for γ ranging from 1 up to 7:

$$\frac{T_m}{T_p} = a + b\gamma + c\gamma^2 + d\gamma^3 \quad (4)$$

with a=0.7303, b=0.04936, c=-0.006556 and d=0.000361 [9]. After assessing the combined wind sea and swell spectrum, the random wave elevation is determined by the following equation (5), based on the superposition of N wave components, each one with circular frequency ω_i and random phase φ_i:

$$\varsigma(t) = \sum_{i=1}^{N} \sqrt{2S(\omega_i)\Delta\omega_i} cos(\omega_i t + \varphi_i) \quad (5)$$

where $\Delta\omega_i$ denotes the circular frequency interval between two subsequent wave components.

III. SPECTRUM ESTIMATION

Spectrum estimation is one of the most effective and amply used approaches for extracting the useful information from a time-series record, in the experimental investigation of a dynamic phenomenon [10]. Due to its complexity, a considerable literature has been developed along the years on the theoretical aspects of this estimation process [11], and efforts have also been put in addressing its application in specific investigation areas. In the case of sea wave monitoring, based on previous experience [7-8], in this study, two methods have been mainly considered, namely Welch's (averaged modified periodogram) and Thomson's (multi-taper method).

In Welch' method the acquired data record of duration T in segments of duration T_0, with partial overlap, typically from 20% to 50%. Each segment is pre-treated by tapering with a smooth window, to reduce spectral leakage and the periodogram (the square of the Discrete Fourier Transform) is calculated. The final estimate is obtained by averaging over segments. For a given overall duration T, the quality of the result depends upon the choice of the taper (the shape of the time window) and the duration T_0 of the segments. The combination of the window and of T_0 determines the effective bandwidth of the analyser, according to $\Delta f_e = \alpha_w T_0^{-1}$. It should be noticed that the greater Δf_e is, the lower spectral resolution is. On the other hand the variance of the estimate is proportional to the ratio T_0/T. Therefore, a trade-off is required between the conflicting needs of having a good time resolution and a small variance.

The basic idea of Thomson's method is to taper the overall data sequence with different tapers, each able to highlight different features of the signal, also accounting, in a way, to phase information. A typical choice for the taper is the discrete prolate spheroidal sequences, which have optimal properties for preventing spectral leakage. Here the main analysis parameter is usually denoted by W, which constitutes the (semi)bandwidth of the analyser. Again a trade-off is required between spectral resolution, which requires a small W, and variance, which decreases when W increases. Yet for optimal choices of analysis parameters, the results from the two estimators may be different, since the two approaches are quite different.

IV. WAVE SPECTRUM FITTING

The sea state parameters, namely the significant wave height H_s, the wave peak period T_p and the peak enhancement factor γ, are determined by the Nonlinear Least Square Method (NLSM), embodied by Rossi et al. [7-8] and purposely modified to fit double peak wave spectra, obtained by combined wind sea and swell components. The fitted wave spectrum is assessed by a two-step procedure, as detailed in the following:

(i) The peak frequencies, corresponding to the swell and wind wave components, are preliminarily detected, as they correspond to the relative maxima of the smoothed estimated spectrum.

(ii) The remaining spectral parameters, namely the significant wave height and the peak enhancement factor of the two components, are obtained by the NLSM, based on the iterative trust-region-reflective algorithm and the interior-reflective Newton method [12]. Particularly, it allows detecting the unknown parameters by iteratively solving a large set of linear equations by the method of the preconditioned conjugate gradients.

The parameters of the bimodal spectrum can be accurately detected if the peak frequencies of the wind sea and swell components are far enough to assure that the two spectral components are separated for practical purposes.

V. NUMERICAL APPLICATION

The numerical procedure, outlined in Sections IV and V, is applied to random wave elevation time histories obtained by the bimodal spectrum having the main data outlined in Table 1 and depicted in Figure 1. The time series have been obtained using eq. (5) using a circular frequency interval $\Delta\omega_i$ equal to 0.001 rad/s. Besides, two different time durations, are analysed in order to investigate the incidence of the sample length on the reliability of the resembled wave spectra.

TABLE I. INPUT BIMODAL SPECTRUM

Parameter	Method	
	Wind wave	Swell
H_s (m)	3.00	2.00
T_m (s)	12.00	20.00
T_p (s)	15.51	22.77
γ (---)	1.00	7.00

Particularly, two random wave records, having a duration of 600 and 3600 s respectively, have been generated at 10 Hz sampling frequency. The two random wave signals are analysed by the Multi-Taper Thomson (MT) and Welch (W) methods, outlined in Section III. Hence, Figures 2.1 and 2.2 provide the estimated spectrum by the MT method, with reference to the short (600s) and long (3600s) time durations, respectively. Similarly, Figures 3.1 and 3.2

provide the wave spectra estimated by the Welch method. Based on current results, the estimated spectra, based on the short time duration, are slightly poor, while the quality of the resembled spectra, combined with the long-time duration, is very good.

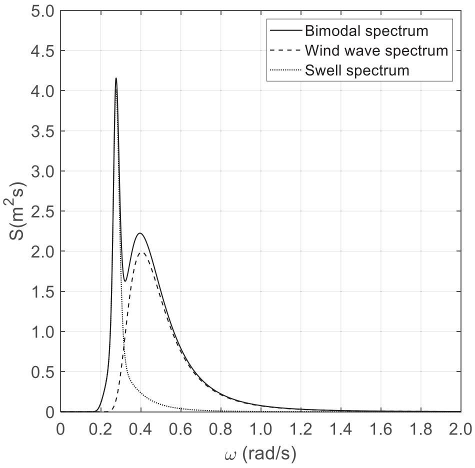

Fig 1. Bimodal wave spectrum

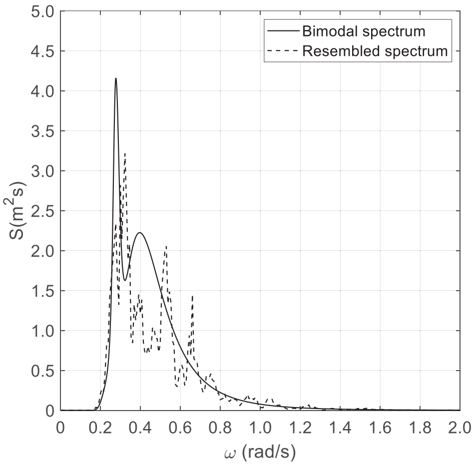

Fig 2.1 Resembled spectrum - MT method – 600 s

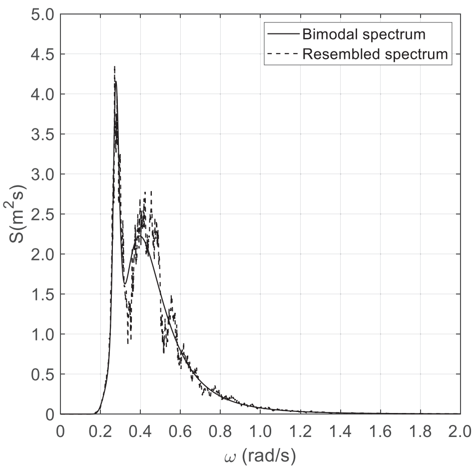

Fig 2.2 Resembled spectrum - MT method – 3600 s

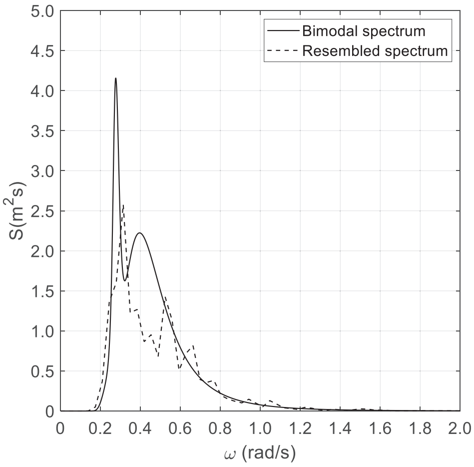

Fig 3.1 Resembled spectrum - W method – 600 s

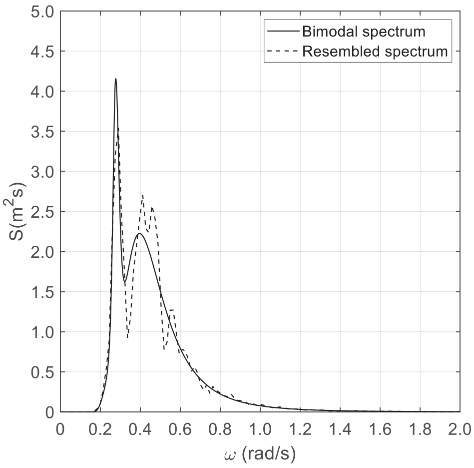

Fig 3.2 Resembled spectrum - W method – 3600 s

The main parameters of the resembled spectra, based on 1-hour wave history, are listed in Tables II and III, while the fitted spectra are plotted in Figures 4.1 and 4.2 in order to carry out a comparative analysis with the reference bimodal spectrum.

TABLE II. FITTED SPECTRUM – MT METHOD – 3600 s

Parameter	Fitted spectrum		% Difference	
	Wind wave	Swell	Wind wave	Swell
H_s (m)	3.01	2.04	0.33	2.00
T_p (s)	15.31	22.52	-1.29	-1.10
γ (---)	1.01	5.35	1.00	-23.57

TABLE III. FITTED SPECTRUM – W METHOD – 3600 s

Parameter	Fitted spectrum		% Difference	
	Wind wave	Swell	Wind wave	Swell
H_s (m)	2.94	2.17	-2.00	8.50
T_p (s)	14.71	21.68	-5.16	-4.79
γ (---)	1.00	4.00	0.00	-42.86

Based on current results, both methods allow efficiently resembling the main parameters of the bimodal spectrum, apart from the peak enhancement factor of the swell component which is in both cases underestimated, probably due to the low range of the swell spectrum. Besides, by the percentage differences between the fitted and reference parameters of the bimodal spectrum, the MT method seems to be slightly superior if compared with the W one.

VI. CONCLUSIONS

The paper focused on the application of different spectrum estimation methods to resemble the main parameters of a bimodal wave spectrum obtained by the superposition of wind wave and swell components. Two random wave time histories, with 10-min and 1-hour duration, have been generated from a theoretical bimodal spectrum and subsequently analysed by the Multitaper Thomson and Welch methods. Based on current results, it is gathered that the time duration plays a fundamental role in terms of reliability of the resembling procedure. In this respect, the short time duration, corresponding to a 10-min wave time history, is not sufficient to estimate the input wave spectrum with a sufficient accuracy. On the contrary both spectrum estimation methods provide a reliable estimate of the input bimodal spectrum, combined with the 1-hour wave record.

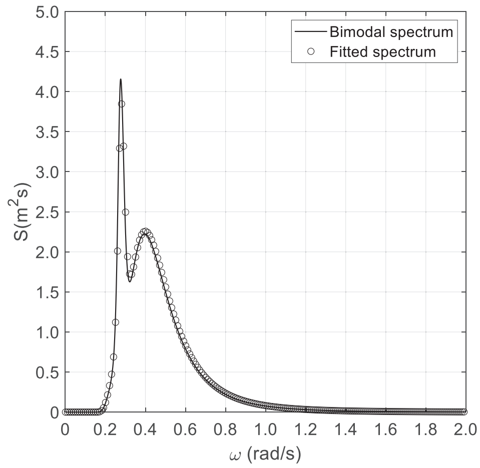

Fig 4.1 Fitted MT spectrum – 3600 s

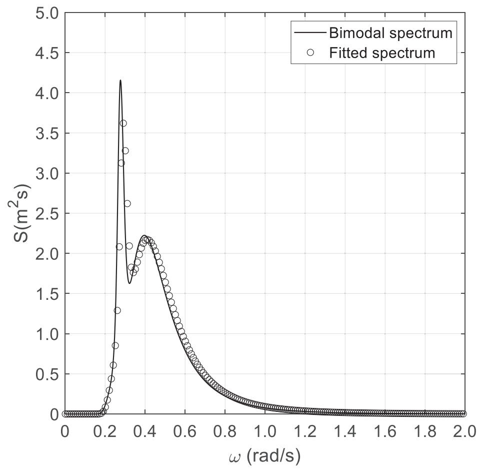

Fig 4.2 Fitted W spectrum – 3600 s

Besides, the two-step fitting procedure, outlined in Section V and based on the Nonlinear Least Square Method, seems to provide a reliable assessment of the main spectrum parameters, namely the significant wave height, the wave peak period and the peak enhancement factor. In this respect, the Multitaper Thomson method seems to be the most promising technique to estimate and resemble bimodal wave spectra. Nevertheless, this outcome needs to be further investigated by means of parametric study devoted to analyse and combine different wind wave and swell spectra.

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