

## THE PROGRESS OF THE PRIMARY CALIBRATION SYSTEM BY SINE APPROXIMATION METHOD IN NIMT

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**Abstract:** This paper presents the design and implementation progress of the vibration calibration system by using the Sine Approximation Method at the National Institute of Metrology Thailand (NIMT). This system was set up in accordance with ISO16063-11, The Sine Approximation.

The experiments were conducted on a prototype design and constructed at our calibration laboratory. The obtained results were subsequently validated against existing systems. More specifically, the magnitude and phase sensitivity and related statistics were validated with the SPEKTRA Calibration System. In this paper, the implementation is described in details together with the comparisons of resultant magnitude and phase sensitivity values reported and discussed.

**Keywords:** Metrology, Interferometry, Vibration, Accelerometer

### 1. INTRODUCTION

The Vibration Laboratory of the National Institute of Metrology Thailand is now responsible for establishing the primary calibration system as a vibration reference protocol within mid frequency range of 50 Hz – 5 kHz. In our settings, the ISO 16063-11 standard, comprising of the description of three calibration methods based on laser interferometry, was adopted. According to this standard, there are currently 2 conventional methods that may be applied for sensitivity magnitude calibration, i.e., Fringe Counting Method (FCM) that can be used in the frequency range of 50 Hz – 800 Hz, and Minimum Point Method (MPM) that is intended for the frequency range of 800 Hz – 5 kHz.

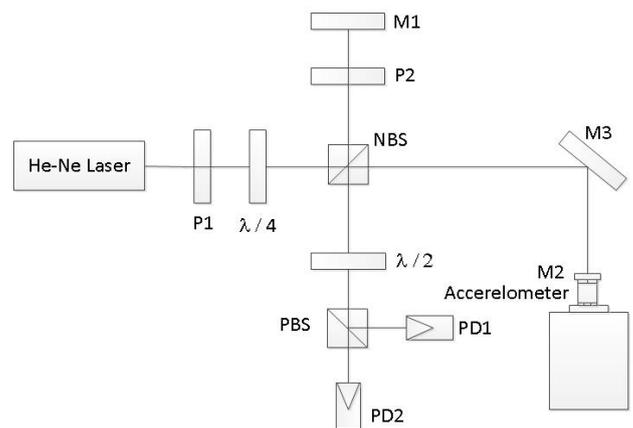
Not until recently, when a variety of related applications, the measurement and the extension of the frequency range had never been so crucial. Accordingly, not only magnitude but also phase of complex sensitivity needs to be paid considerably more attention. The third, Sine Approximation Method (SAM) has thus been established to evaluate this complex sensitivity with the phase differences, covering the frequency range of 50 Hz–1 kHz.

In this paper, the progress of the calibration system based on the Sine Approximation Method at NIMT is

presented and the corresponding preliminary experimental results are reported and discussed.

### 2. MEASURING INTERFEROMETER

The configuration diagram of our experimental set up for Sine Approximation Method is shown in Fig 1.



**Figure1.** The diagram of the SAM implemented at NIMT

In our setting, the optical parts were a modified version of Michelson interferometer with quadrature output signals. They consist of the polarizers (P), a quarter wavelength retarder (QR,  $\lambda/4$ ), a non-polarizing beam splitter, a half wave plate ( $\lambda/2$ ), a polarizing beam splitter (PBS) and mirrors.

A stabilized He-Ne laser with a wavelength of 632.8 nm is used as a light source and its output beam is linearly polarized by polarizer (P1). The quarter wavelength retarder ( $\lambda/4$ ) then converts the incident laser beam to a circularly polarized one. This beam is later split into 2 orthogonal ones with 50 – 50 ratio by a non-polarizing beam splitter (NBS). As also depicted in this configuration, the reference interferometer arm consists of reference mirror (M1) and polarizing plate (P2). After passing through the NBS, the beam is reflected by the reference mirror (M1) and become a linearly polarized at the polarization plate (P2). At the measuring arm of interferometer, the beam directed to the folding mirror (M3) and then the reflector (M2) on the UUT,

mounted on the top of shaker, which is respectively driven in sinusoidal motion.

The measuring beam, which is reflected by the reflection mirror (M3) interfere with the reference beam. This interfered beam separated into each polarization component by means of the polarizing beam splitter (PBS). These quadrature outputs were subsequently detected by the two photo detectors (PD1 and PD2) and recorded simultaneously together with the accelerometer output, by the recorder.

The quadrature outputs of the photo detectors correspond to the desired sinusoidal excitation of accelerometer given by the signal generator and power amplifier. When the shaker moves, these photo detector output signals, *i.e.*,  $V_1$  and  $V_2$ , are a phase modulation of the displacement of accelerometer. Output voltages of the photo detector signals are expressed as;

$$V_1(t) = \hat{V}_1 \cos[\varphi_0 + \hat{\varphi}_M \cos(\omega t + \varphi_s)] \quad (1)$$

$$V_2(t) = \hat{V}_2 \sin[\varphi_0 - \hat{\varphi}_M \cos(\omega t + \varphi_s)] \quad (2)$$

where

$t$  is time,  $\hat{V}_1$  and  $\hat{V}_2$  are the voltage amplitude of photo detector outputs,

$\varphi_0$  is the initial phase angle of vibration signal, depends on the difference of interferometer,

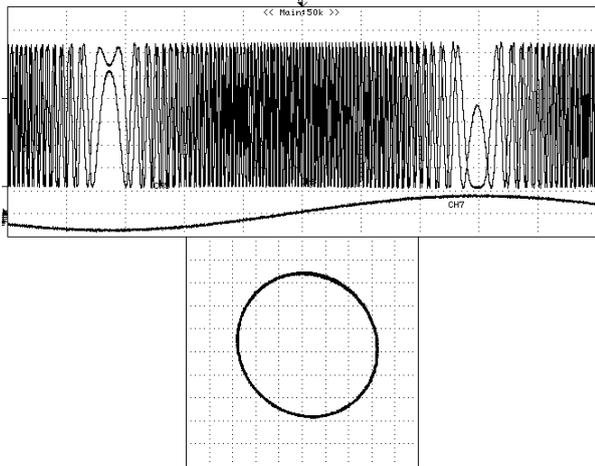
$\hat{\varphi}_M$  is the amplitude of phase modulation of accelerometer

$\omega$  is the angular frequency of vibration, where

$\omega = 2\pi f$  and  $f$  is the excited frequency

$\varphi_s$  is the vibration zero phase angle

Fig 2 depicts an example of these quadrature signals at 160 Hz and corresponding X-Y scattering plot. It is worth noting that, the modulating phase caused slight skew to what should have been orthogonal sinusoidal signals, while its fuzzy edge are resulted from various uncertainties, such as noise harmonics and modulating drifts, *etc.*



**Figure 2.** An example of photo detector output signals

Assuming for the time being that both quadrature output signals of the photo detectors are identical,  $\hat{V}_1 = \hat{V}_2$  and there is no phase shift between the two interferometric signals,

phase modulation term,  $\hat{\varphi}_M$  conveys the total interferometric phase. Therefore, by applying arctangent demodulation, the total interferometric can be calculated by the formula:

$$\varphi_M(t) = \tan^{-1} \frac{V_2(t)}{V_1(t)} + m\pi \quad (3)$$

where  $m$  is a positive integer number, chosen to avoid the discontinuity of modulation phase value for  $\varphi_M(t)$ . This process is called phase unwrapping.

The values of time varying modulating phase obtained from the sinusoidal data would then be used in subsequent acceleration calculation for accelerometer under test.

In order to obtain sufficiently accurate phases, they are thus approximated by solving a set of linear combinations between sine and cosine functions. This scheme is usually called Sine Approximation Method. By expanding the above equation,  $\varphi_M(t)$  becomes

$$\varphi_M(t) = b_1 \cos \omega t - b_2 \sin \omega t + \varphi_0 \quad (4)$$

where

$$b_1 = \hat{\varphi}_M \cos \varphi_s, \quad b_2 = \hat{\varphi}_M \sin \varphi_s$$

The linear combinations of periodic sequence in (4) over a specific period can then be expressed in a matrix form:

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon} \quad (5)$$

where

$\mathbf{Y}$  is an  $(n \times 1)$  vector of measure  $\varphi_M(t)$

$\mathbf{X}$  is an  $(n \times 3)$  matrix of known form

$\boldsymbol{\beta}$  is a  $(3 \times 1)$  vector of parameters that are expectations of  $b_0, b_1$  and  $b_2$

$\boldsymbol{\epsilon}$  is an  $(n \times 1)$  vector of errors

The least square estimated of  $\boldsymbol{\beta}$  are the values  $\mathbf{b}$  given by

$$\mathbf{b} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{Y} \quad (6)$$

where

$\mathbf{b}$  is a  $(3 \times 1)$  matrix of  $b_0, b_1$  and  $b_2$

$\mathbf{X}^T$  is the transportation of matrix  $\mathbf{X}$

$(\mathbf{X}^T \mathbf{X})^{-1}$  is the inversion of the matrix  $(\mathbf{X}^T \mathbf{X})$

This solution of  $\mathbf{b}$  from (6) minimizes the sum of squared errors, irrespective of any distribution properties of the errors. The value of  $\varphi_M(t)$  can be calculated by solving the linear system by regression analysis for each value  $t$ , where phase and amplitude can be computed. Specifically, the modulation phase amplitude ( $\hat{\varphi}_M$ ) and the displacement initial phase ( $\hat{\varphi}_s$ ) can be evaluated by using

$$\hat{\varphi}_M = \sqrt{b_1^2 + b_2^2} \quad (7)$$

$$\hat{\varphi}_s = \tan^{-1} \left[ \frac{b_2}{b_1} \right] \quad (8)$$

The amplitude of acceleration ( $\hat{a}$ ) and the initial phase of acceleration ( $\varphi_a$ ) are calculated from the modulation phase amplitude ( $\hat{\varphi}_M$ ) and initial phase angle of displacement ( $\hat{\varphi}_s$ ) using the relations

$$\hat{a} = \pi \lambda f^2 \hat{\varphi}_M \quad (9)$$

$$\varphi_a = \varphi_s + \pi \quad (10)$$

The absolute displacement amplitude of the pick-up can be calculate from (9) and expressed by

$$\hat{d} = \frac{\hat{\varphi}_M \lambda}{8\pi} \quad (11)$$

Similarly, to measure the phase lag between vibrating part and accelerometer output voltage, the sine approximation method is also applied to the corresponding output signal of accelerometer,  $U(t)$ , which can be written as follow:

$$U(t) = b_{1u} \cos \omega t - b_{2u} \sin \omega t + \varphi_u \quad (12)$$

where

$$b_{1u} = \hat{U} \cos \varphi_u, \quad b_{2u} = \hat{U} \sin \varphi_u$$

The symbol  $\hat{U}$  is the estimate amplitude of the accelerometer output and the initial phase angle,  $\varphi_u$  are thus estimated by

$$\hat{U} = \sqrt{b_{1u}^2 + b_{2u}^2} \quad (13)$$

$$\varphi_u = \tan^{-1} \left[ \frac{b_{2u}}{b_{1u}} \right] \quad (14)$$

The magnitude ( $S_a$ ) and phase shift ( $\Delta\varphi$ ) of complex sensitivity are accordingly obtained by

$$S_a = \frac{\hat{u}}{\hat{a}} \quad (15)$$

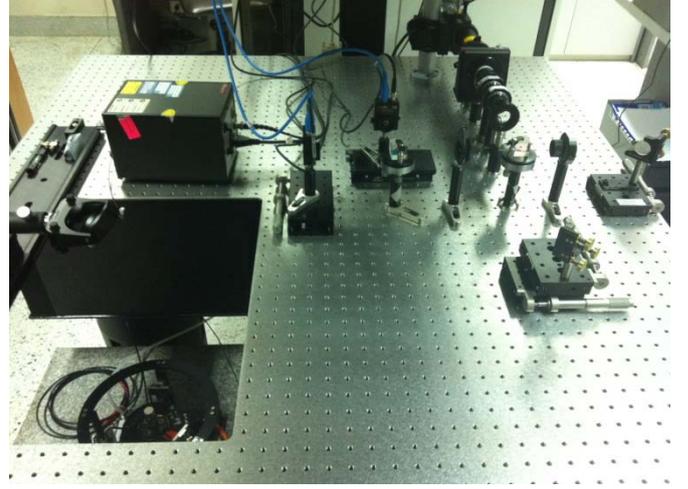
$$\Delta\varphi = \varphi_s - \varphi_u \quad (16)$$

### 3. MEASUREMENT AND RESULTS

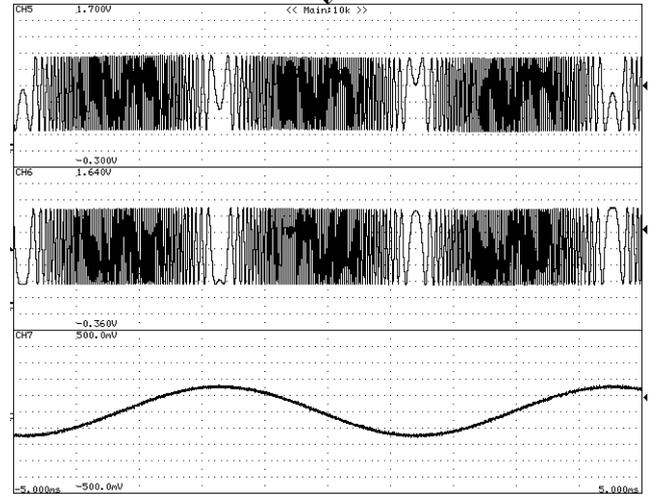
The realization of the schematic of measurement set up shown in Fig. 1 and used in our experiment is illustrated in Fig. 3, where the interferometer system was mounted on the optical breadboard of an air spring isolation table. The exciter, SPEKTRA Model SE-09, was mounted on granite rigid body. The shaker was excited in horizontal direction. The accelerometer, back to back type, was used as UUT to evaluate phase modulation result.

A 4-channel signal recorder capable of simultaneous sampling with a maximum rate of 10 MS/s and 12 bit resolution was employed for data collection and sampling. In our experiment, the quadrature output signals from photo detectors and the accelerometer output were sampled synchronously and recorded in the scope recorder. An example of the typical output signals are shown in Fig.4.

It order to properly evaluate the adverse factors due to the misalignment of interferometer signals, the difference gain of photo detectors and also the non-linearity effects, the quadrature signals were corrected by using a least squares method [2] with the numerical treatment as presented in [3].

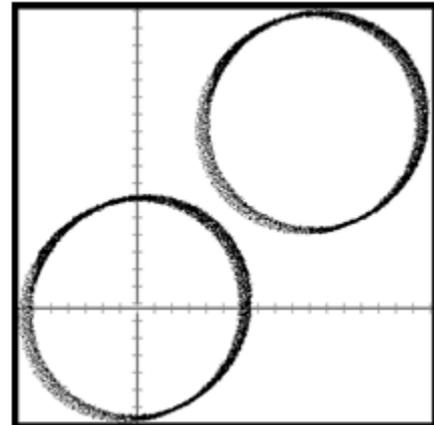


**Figure 3.** The realized experimental set up at NIMT



**Figure 4.** Examples of quadrature output signals from the photo detectors (top) and that from a UUT pickup (bottom)

Examples of the two quadrature signals before and after correction are illustrated in Fig 5. (1 kHz, 10m/s<sup>2</sup>)

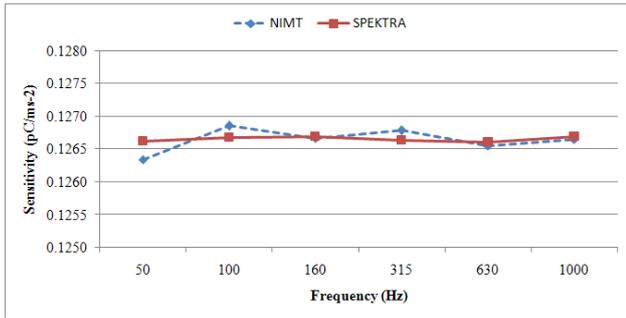


$$\alpha = 0.026491, p = 0.518460, q = 0.545688, r = 1.013144$$

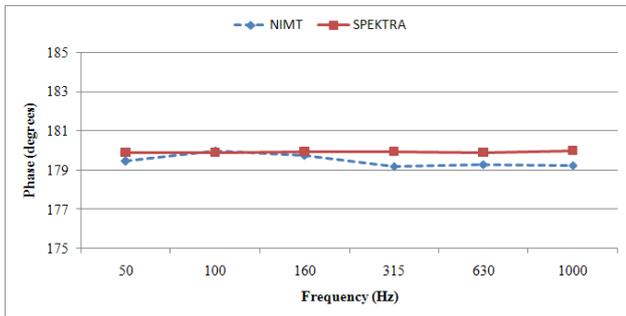
**Figure 5.** Scattering plots, showing examples of quadrature signals before (top right) and after (bottom left) corrections.

To validate this preliminary experimental, the results acquired and processed were compared against those obtained from the SPEKTRA calibration system in the middle frequency range, 50 Hz – 1 kHz and are presented in Fig 5 and Fig 6.

The output amplitude of accelerometer,  $\hat{U}$  and the initial phase angle of acceleration,  $\varphi_u$  was estimated by using (13) and (14). Finally, all parameters required to determine the magnitude and phase shift of the complex sensitivity of the accelerometer was obtained by using the equations (15) and (16). The measured magnitude and phase shift of sensitivity are given in Figs. 6 and Figs. 7 respectively.



**Figure 6.** Comparison result of Charge Sensitivity for the accelerometer type 8305 on NIMT experimental system and the SPEKTRA calibration system



**Figure 7.** Comparison result of Phase Sensitivity for the accelerometer type 8305 on NIMT experimental system and the SPEKTRA calibration system

An estimated error was computed by averaging the differences between the measured shown in equation (4) and estimated modulating phase in equation (7) and normalised by its magnitude over the specific time period, *i.e.*, from 10000 to 50000 samples, covering at least 1 cycles of the exciting signal. The process was repeated 6 times for each vibration frequency, where an averaged error (mean) and respective standard deviations are computed. The following table shows these statistics assessed by varying the vibrating frequency from 50 Hz to 1 kHz.

Frequency (Hz)	Estimation Error (%)	S.D.
50	1.31E-06	9.12E-07
100	7.89E-06	5.42E-06
160	2.69E-06	1.49E-06
315	1.87E-06	2.04E-04
630	2.41E-06	1.99E-04
1000	4.77E-06	2.04E-04

**Table 1** The estimation Error

#### 4. CONCLUSION

The preliminary experiment based on sine approximation method has been presented and the result on complex sensitivity was validated with SPEKTRA calibration system. In our findings, the consistency between the two systems is clearly evident. The deviation of sensitivity and the difference of phase shift over 160 Hz will be evaluated and improved. The future investigations will focus on expanding the magnitude and frequency range with the improvement on accuracy and the uncertainty evaluation.

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