

# AN ENERGY-BASED DAMAGE INDEX FOR ANALYSIS OF VIBRATIONS FROM ROTATING MACHINERY BASED ON RMS VALUES

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**Abstract:** Focus of the paper is to propose a simpler and intuitive parameter for monitoring and possibly detecting incoming failure conditions of rotating machinery or its components during their stationary operation and subject to misalignment of the rotor (due to increasing eccentricities, uneven mass distribution, propagating cracks, etc.). Such a parameter, which can be considered as a damage index, is obtained by processing the vibration signals acquired from static supporting elements of the machinery. In order to show the feasibility of the idea, the authors performed a series of measurements on a rotating disk driven by an induction DC motor. Here, only some computations have been reported, for compactness purposes.

**Keywords:** RMS, vibrational signal, damage index, rotating element.

## 1. INTRODUCTION

Vibrations affecting rotating machinery may lead to failure its components most under stressing. In addition, whenever mass unbalances, eccentricities, dynamic misalignments occur, the vibrational status is subjected to a steady intensification [1].

An early identification of any anomaly perturbing the dynamics of the machine under observation, is crucial. Indeed, it could be possible to avoid any machine shut-down due to a catastrophic failure occurring whenever a prompt identification of the dynamic perturbation has been not accomplished [1, 2].

The case study reported in this paper, deals with rotating machinery characterized by mass unbalances, due to possible uneven mass distribution in the rotating components. Such dynamical phenomena remarkably influence the vibrational status of the analysed equipment. Thus, in order to detect the dynamical anomaly affecting the machine and ascertain the acceptability of its operating condition, the machine vibration acquisition and its post-processing must be performed [2-4].

Firstly, it is well known that the vibration frequency of a rotor is synchronous with the shaft angular speed (1X component), because the unbalancing force rotates at the shaft running speed. A preliminary analysis of the vibrational signal of unbalanced rotating equipment (by means of the Fourier Transform of the signal itself) shows, in the frequency domain, a series of harmonics of the shaft running speed, i.e. at 1X RPM, 2X RPM, 3X RPM and so forth [5, 6].

Currently, there are many techniques and algorithms, which, applied to the vibrational signal, allow an identification of any anomaly affecting it. These techniques are mainly based on energy or statistical parameters (for analyses performed in the time domain) and signal harmonic content (for analyses in the frequency domain).

In the early studies, the Fourier Transform (FT) has been the dominating signal analysis tool for vibrations due to its implementation simplicity.

However there are some crucial restrictions on the use of the FT. The vibrational signal acquired from the inspected machine has to be linear and temporally stationary; otherwise the related Fourier spectrum has limited physical meaning. In such a case, the harmonic content of the signal is supposed to be spread over the entire time domain, meaning that each detected frequency component holds through the duration of the signal [2, 7, 8].

Other algorithms have been developed for specific cases to avoid the restrictions of the FT use, i.e. [2, 9].

In this paper, the authors provide a simple and intuitive way to easily monitor the operation conditions of a rotating equipment subjected to vibrations caused by the eccentricity of unbalanced masses. The procedure is based on post-processing of vibrational signals sensed on the equipment under analysis by evaluating their RMS parameter or a quantity directly linked to it, called *damage index* here. This statistical quantity can be taken as an energetic index of its status of operation allowing to assess the risk of operating in critical conditions once a threshold value of it has been previously determined by experimentations.

## 2. EXPERIMENTAL SET-UP, MEASUREMENTS AND RESULTS

The experimental set-up used for measurements consists of a DC motor (nominal voltage 24 V, nominal power 140 W @ 3600 RPM), driving an aluminium disk with several bores along a diameter in axisymmetric positions.

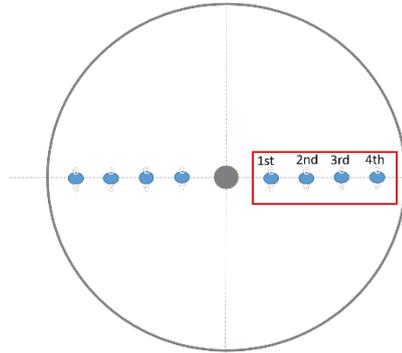
The vibration signals are measured on the surface of the motor housing and acquired by means of a single-point Laser Doppler Vibrometer (LDV), PDV-100, by Polytec GmbH.

Figure 1 illustrates the rotating disk used during the experimental tests. Figure 2 shows the vibrometer targeting the measurement point.

In order to get several configurations characterized by different eccentricities, bolts have been screwed in changed positions on the disk, as described in table 1.

Table 1 – Configuration of the simulated eccentricity

Configuration	
A	without any bolt
B	bolt in 1 <sup>st</sup> hole
C	bolt in 2 <sup>nd</sup> hole
D	bolt in 3 <sup>rd</sup> hole
E	bolts in 1 <sup>st</sup> and 2 <sup>nd</sup> hole
F	bolt in 4 <sup>th</sup> hole
G	bolts in 1 <sup>st</sup> and 3 <sup>rd</sup> hole
H	bolts in 2 <sup>nd</sup> and 3 <sup>rd</sup> hole



Figures 3 and 4 show the D and F configurations, respectively, as example of simulated eccentricities.

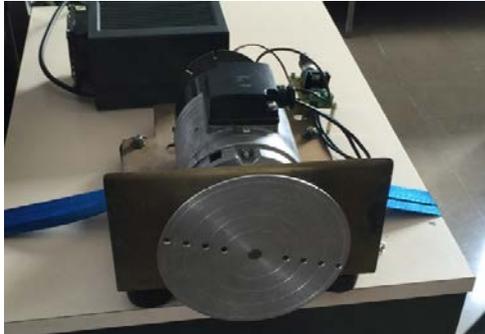


Figure 1 - Experimental set-up



Figure 2 - Measurement point lighted by the laser spot

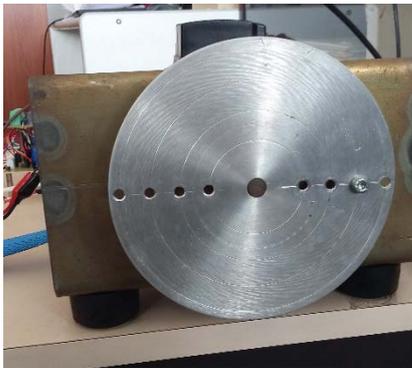


Figure 3 - Example of simulated eccentricity (configuration D)

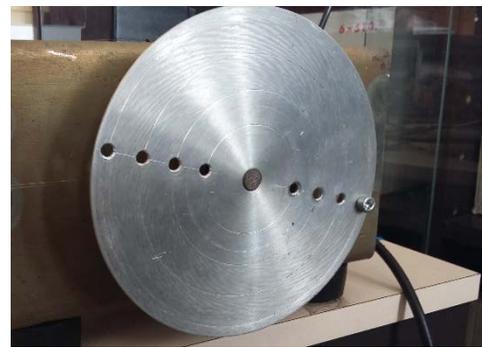


Figure 4 - Example of simulated eccentricity (configuration F)

For each eccentricity configuration, a series of vibrational signals (vibrational velocity) have been acquired on the motor housing for different values of rotational speed  $n$  ranging between 750 and 2250 rpm. For each value of  $n$  and imposed eccentricity, several repetitions have been considered to verify the randomness, the stationarity and the repeatability of the phenomenon under analysis.

A set of observations sampled at a specific time interval is usually denoted as a discrete time series. Such a time series has to be considered as a realization of a specified random variable  $\{X_n, n \in \omega\}$ ,  $\omega$  indicating the acquired discrete set (in this case the values assumed by the vibrational velocity acquired by the vibrometer). Time series stationarity is either strong stationarity or weak (i.e. wide sense stationarity - WSS). If all the statistical properties of the time series are time invariant, the series is said to be strongly stationary [10-13].

A time series is WSS if the only first two statistical moments are time invariant, such that the mean is invariant, i.e.:

$$E(X_{n1} - X_{n1+h})$$

and the covariance only depends on the time lag between two observations, i.e.:

$$Cov(X_{n1}, X_{n2}) = Cov(X_{n1+h}, X_{n2+h})$$

In order to properly define a suitable energy parameter (in order to define a damage index for the characterization of the machinery operating conditions), an assessment of the acquired signals WSS has to be performed. To examine the

time series stationarity in a wide sense, the Reverse Arrangement Test (RAT) is applied [12].

Basically, the RAT is a non-parametric method (i.e. it does not require any *a priori* assumption on the statistical distribution of the signal) and is used to look for possible monotonic trends in the root mean squares (RMS) calculated within non-overlapping intervals.

The test procedure works as follows. First, the signal  $x(n)$  is split into  $N$  time segments of length  $L$ . For each segment, the RMS value is calculated. In relation to each segment, the total number of reverse arrangements is computed. A reverse arrangement occurs when the RMS value of the current frame is larger than the RMS of the subsequent frames. The reverse arrangements relative to each frame are cumulatively added up, to give the value  $A(k)$  relative to the  $k$ -th segment. Then, the parameter  $A_T = \sum_{k=1}^N A(k)$  is computed. The calculated value of the total reverse arrangement  $A_T$  is then compared to the value that would be expected from a realization of a weakly stationary random process. If we considered the sample as weakly stationary, then the expected value of  $A$  has a normal distribution [12] with the mean given by:

$$\mu_A = \frac{L(L-1)}{4}$$

and variance:

$$\sigma^2 = \frac{L(L-1)(2L+5)}{72}$$

The null hypothesis that the random process is weakly stationary is rejected if  $A_T$  falls outside the critical values defined by a significance level  $\alpha$ . Such critical values are calculated by means of the following statistical parameter:

$$\psi_T = \frac{A_T - \mu_A}{\sigma}$$

The critical values of  $\psi_T$  can be defined by  $\psi_{T,1-\alpha/2}$  and  $\psi_{T,\alpha/2}$  with  $\psi_T$  having a normal distribution. At significance level  $\alpha = 0.05$ ,  $\psi_{T,1-\alpha/2} = -1.96$  and  $\psi_{T,\alpha/2} = 1.96$ . According to the  $\psi_T$  value, the following situations are possible:

- $\psi_{T,1-\alpha/2} < \psi_T < \psi_{T,\alpha/2}$ , the null hypothesis that the time series is wide sense or weakly stationary is accepted.
- $\psi_T \geq \psi_{T,\alpha/2}$ , the number of reverse arrangements is larger than expected, implying a downward trend in the signal.
- $\psi_T \leq \psi_{T,1-\alpha/2}$ , the number of reverse arrangements is less than expected, implying an upward trend in the signal.

It follows that when the RAT fails (the null hypothesis is rejected), the data are non-stationary. The probability that a stationary case is tested as non-stationary is the same as the significance level.

The RAT test has been applied on each acquired signal. In this abstract, only the results relative to the F

case at  $n = 2250$  rpm (whose signal is plotted in Figure 5) are reported.

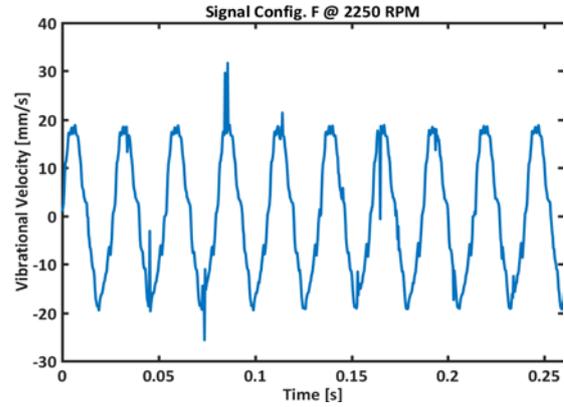


Figure 5 Vibration acquired for configuration F @ 2250 RPM.

The following parameters have been so calculated:

$$\psi_T = 167199, \psi_{T,1-\alpha/2} = 152401, \psi_{T,\alpha/2} = 168384$$

with significance level  $\alpha = 0.05$ .

Once the signal WSS has been assessed, the measurement repeatability (for each eccentricity configuration and tested rotational speed) has been ascertained. Some measurement repetitions have been performed, leading to signal Root Mean Square values normally distributed (within each set) with a mean value and a standard uncertainty. For example, Table 2 shows the mean RMS values with the correspondent standard uncertainty for configuration D and for the tested rotational speeds. Each repetition set contains five signals. In addition, a homoscedasticity test for each test has been evaluated, in order to assess the variances homogeneity (performed by means of the ANalysis Of VAriance). After verifying that the vibrational signals under investigation are stationary (in wide sense) stochastic processes, the RMS mean value (of the repetitions considered) can be easily and fast evaluated and correlated to the value of the eccentricity, without worrying too much about the noise associated to the measurement.

The RMS value here considered, is given by the following relation:

$$RMS = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i)^2}$$

In the previous,  $N$  stands for the number of samples of each signal,  $x_i$  the  $i$ -th signal sample.

Such a parameter can be assumed as an energy level indicator of the whole signal and thus significant of the induced eccentricity.

All the values reported in the following tables and figures are expressed in mm/s, as the quantity acquired is the velocity of the vibration induced onto the support by the eccentricity.

In Figures 6, 7 results are presented for the case of Figure 3 (D configuration, bolt in 3<sup>rd</sup> bore) and Figure 4 (F configuration bolt in 4<sup>th</sup> bore) respectively, in terms of the mean RMS versus the rotational speed  $n$ . Such an analysis have the purpose of showing that a rotating disk exhibiting a

given mass eccentricity, is affected by an increasing vibration energy level as the operation rotational speed increases.

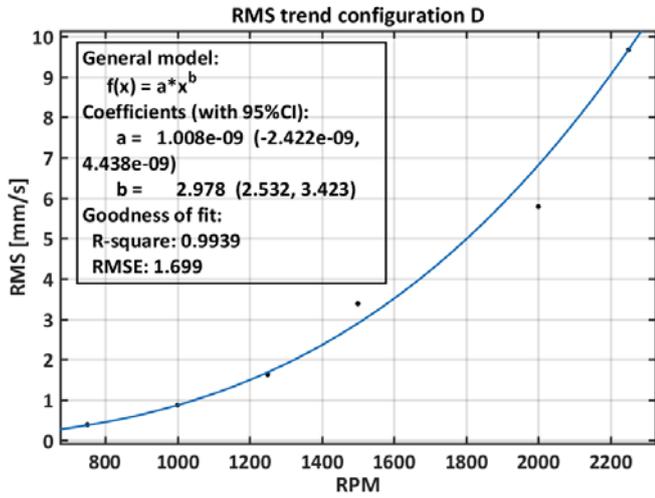


Figure 6 - RMS vs  $n$  for case of Figure 3 (D case)

The fit have been performed (for all tested configurations) through Weighted Nonlinear Least Squares (WNLS) algorithm. In order to improve the fit goodness, each input value (i.e. the mean RMS value computed for each test) is properly weighted with the reciprocal value of the squared standard uncertainty. The employed model (only shown for configurations D and F for compactness reasons) is a cubic power, exhibiting very high  $R^2$ .

In Tables 2 and 3 the values of mean RMS and relative standard uncertainties are reported for D and F configurations.

Table 2 – RMS mean values for D configuration

n (rpm)	750	1000	1250
RMS(mm/s)	0.36±0.07	0.86±0.05	1.66±0.02
n (rpm)	1500	2000	2250
RMS(mm/s)	3.38±0.02	5.78±0.02	9.67±0.03

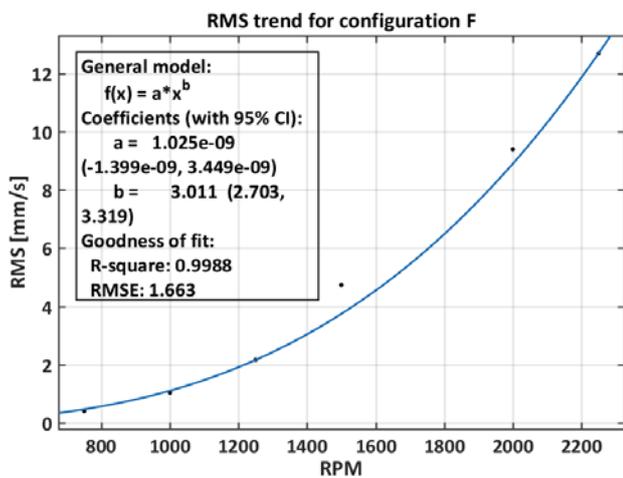


Figure 7 - RMS vs  $n$  for case in Figure 4 (F case)

Table 3 – RMS mean values for F configuration.

n (rpm)	750	1000	1250
RMS(mm/s)	0.48±0.01	1.40±0.01	2.97±0.03
n (rpm)	1500	2000	2250
RMS(mm/s)	6.86±0.01	16.01±0.05	17.33±0.03

From the results obtained, it is easy to observe how the RMS parameter could be considered as an energy level indicator of the stressing eccentricity. By monitoring it during the operation of the mechanical component under observation, it could be possible to state when to intervene in order to avoid irreparable damage to the system, once the threshold value has been established.

Further, for each rotational speed a damage index is possible to define as:

$$\frac{|RMS_{act,n} - RMS_{ref,n}|}{RMS_{ref,n}}$$

where  $RMS_{ref,n}$  is the RMS value in balanced condition, that is a condition which the RMS has the minimum value at, so assumed as reference value (disk without bolts in the present test) and  $RMS_{act,n}$  is the RMS relative to the actual operating condition, whenever the rotating speed of the equipment under analysis is considered constant (as it often occurs). Figures 8 and 9 present the behaviour of the index vs the rotational speed for the case D and F respectively, showing a similar trend already seen for RMS, of course. Also in these cases, the fit has been performed through a cubic power WNLS fit.

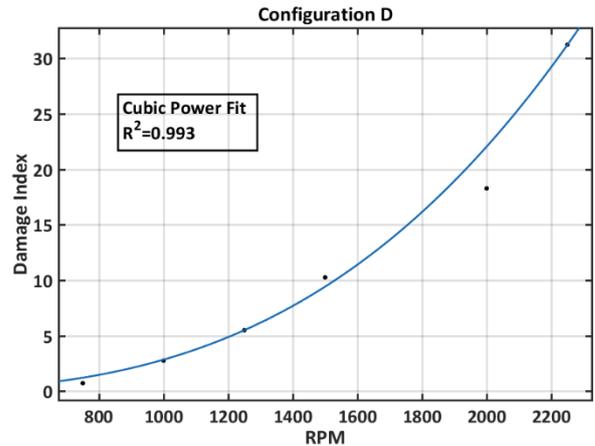


Figure 8 – Damage index vs rotational speed for D case

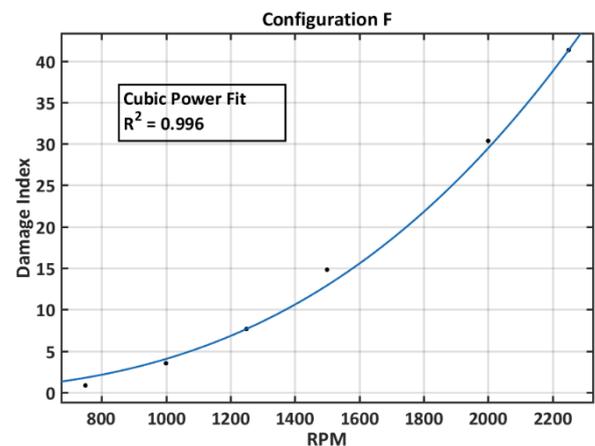


Figure 9 - Damage index vs rotational speed for F case

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