

VIBRATIONAL ANALYSIS FOR THE NEXT GENERATION NPL KIBBLE BALANCE

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Abstract:

Low uncertainty Kibble balance measurements require careful consideration of the effect of external sources of vibration. This paper compares vibrational modelling of the machine frame of the National Physical Laboratory's next generation Kibble balance using three different software packages. The output of the modelling on the machine frame showed that the first modes of vibration were all above the design requirement of 60 Hz.

The model was then enhanced to include the moving section of the balance modelled with two types of guidance mechanism. The output from the modelling showed a reduction in the resilience of the balance to lateral vibrations but the linkage type guidance mechanism was still resilient to induced vibrations at frequencies up to 50 Hz.

Keywords: Kibble balance; mass; vibration; modal analysis

1. INTRODUCTION

The unit of mass in the International System of units (SI) is now defined as a fixed numerical value of the Planck constant (h) and as such either a Kibble balance (KB) [1], joule balance [2] or the X-ray Crystal Density Method [3] is required for primary traceability to the SI mass unit. The low mass uncertainties required, in the region of 20 μg at 1 kg, require Kibble balances to have high sensitivity and provisions must be made to prevent external vibration adversely affecting the measurements [4].

The National Physical Laboratory (NPL) is developing a next generation type of KB based on the simplified operating procedure outlined by Kibble and Robinson [5], [6]. The machine frame of this KB has been designed so that all natural modes of vibration are above 60 Hz. This reduces the likelihood that ground vibrations will interfere with servocontrol of the balance as the velocity and weighing servocontrollers are designed to operate at frequencies between 30 Hz and 50 Hz. This paper describes the modal analysis performed on the NPL KB mechanical structure.

2. NEXT GENERATION KB DESIGN

A concept drawing of NPL's next generation Kibble balance design is shown in Figure 1. The machine frame is the static structure that supports all the subsystems of the KB and is made of 6082T6 aluminium except for the three spherical ended feet which are made of hardened steel. The machine frame material properties are listed in Table 1.

Table 1. Machine frame material properties.

Material	6082T6 Aluminium	Hardened Steel
Density, kg m^{-3}	2710	7800
Young's Modulus, Pa	70×10^9	200×10^9
Poisson's Ratio	0.346	0.3

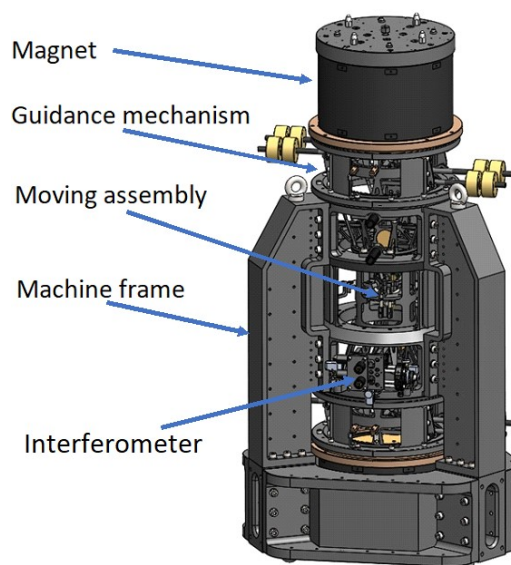


Figure 1. NPL Next Generation Kibble Balance Concept

The inner section containing the coils and mass carriage are referred to as the 'moving assembly' and can only move in the vertical direction as required for KB operation. A cross-section of the central section of the KB with the outer structure removed for clarity and the moving assembly highlighted in orange is shown in Figure 2. The static and moving parts are connected through the guidance mechanism. Two types of guidance mechanism have been designed; flexure type (Figure 3) and watt-linkage type (Figure 4).

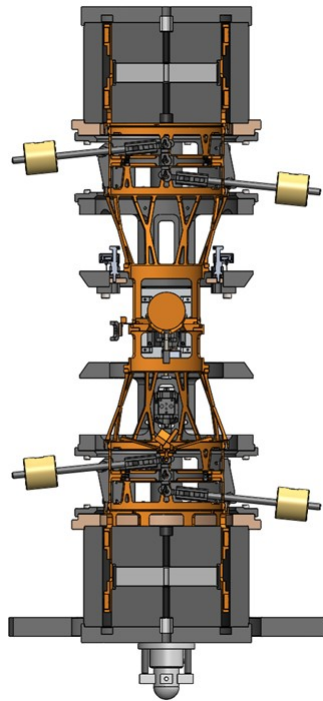


Figure 2. Cross-Section of KB (moving assembly highlighted in orange).

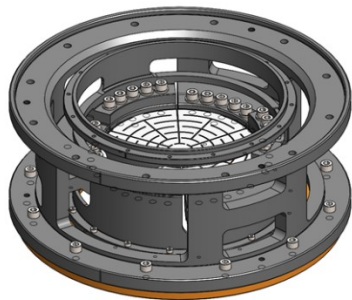


Figure 3. Flexure type guidance mechanism

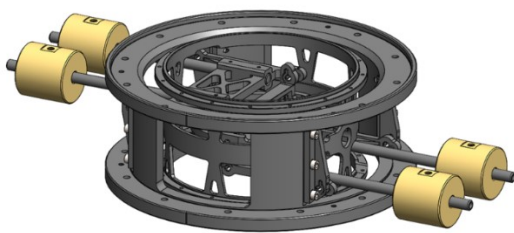


Figure 4. Watt-linkage type guidance mechanism

3. VIBRATION MODELLING

The response of the mechanical structure of the KB to induced vibrations was analysed using four finite element analysis (FEA) packages; Abaqus, COMSOL, CATIA and SolidWorks (SW). To alleviate the complexity of modelling the moving section of the KB, initial modelling of the vibrational response of the structure was performed on just the static machine frame (Part 1). A drawing of the frame is shown in Figure 5. Modelling of the frame in SW was then performed using a vibration

input commensurate with the requirements described by Bessason and Madshus [6] for A-criterion laboratories (Part 2).

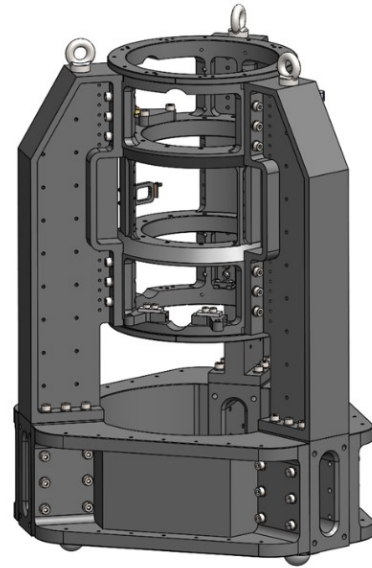


Figure 5. Next Generation Machine Frame Concept

Modelling of the entire system, including a simplified model of the moving mechanism, was then implemented using SW software (Part 3).

3.1. Vibration response of the frame (Part 1)

3.1.1. Machine frame model parameters

A defeatured geometry for the reworked machine frame has been imported in each case from the SW Computer Aided Drawing (CAD) to the FEA packages. Two cylinders were added to the machine frame to represent the magnet assemblies. Each cylinder has the same mass (56 kg), outer diameter and centre of mass as the magnet assembly it represents. Boundary conditions were applied to the nodes at the base of the hemispherical feet of the frame, to allow only radial displacement in the horizontal plane and to prevent vertical displacement.

A range of analyses were run with different element types and mesh densities to explore how much effect these variables have on the model output, and to inform the comparison between the modal analyses. There was a 5 to 10 Hz drop in frequency of the predicted first vibrational mode for the finer meshed models, (more nodes and elements), compared with the courser models with less nodes. The final refined model parameters for Abaqus, COMSOL, CATIA and SW are given in Table 2. Linear tetrahedral elements are used in both the CATIA, COMSOL and SW models with quadratic tetrahedral elements used by Abaqus.

Table 2. Refined model parameters for the FEA packages.

Model	No. of nodes	No. of elements	Mesh density
CATIA	764 599	3 529 449	3 mm
Abaqus	1 309 070	1 251 238	6 mm
COMSOL	2 910 858	800 086	Extra fine
SW	1 945 147	1 218 223	Very fine

3.1.2. Machine Frame modelling results

The results of the Abaqus FEA for the first vibration mode are shown in Figure 6. All the models show a similar distribution of displacements with the largest magnitude occurring at the top magnet. This was anticipated to occur due to the greater rigidity in the structure below the top magnet and the relatively large mass of the magnet (56 kg).

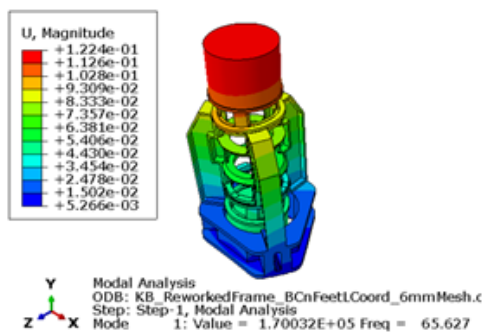


Figure 6. Abaqus modal analysis (first mode of vibration)

The first modes of vibration calculated by the four FEA packages are listed in Table 3.

Table 3. First three modes of vibration of the machine frame

Model	Predicted first mode / Hz
CATIA	81.8
SW	79.3
Abaqus	65.6
Comsol	64.1

There was a 16 Hz to 18 Hz difference between the Abaqus and Comsol predicted first modes of vibration and the CATIA and SW predicted first mode, but even the lowest first predicted mode of 64.1 Hz (Comsol) is above the required 60 Hz target. Although all models used the same SW CAD file and boundary conditions at the feet were applied in similar ways it's possible that the difference in resonant frequencies observed was due to the implementation of those boundary conditions by the different FEA packages.

3.2. Vibration response of the frame (Part 2)

3.2.1. Revised frame model parameters

SW FEA was used to model input parameters obtained from the requirements described by

Bessason and Madshus [6] for A-criterion laboratories. This specifies a peak velocity input of $3 \mu\text{m s}^{-1}$. Assuming a bandwidth of 200 Hz this gives an input to the SW FEA model of $4 \times 10^{-8} \text{ mm s}^{-2} \text{ Hz}^{-1}$. The requirements for A-criterion laboratories specify the peak velocity decreasing at higher frequencies but as the complexity of the model results in a run time analysis of approximately 3 days this was assumed to be constant over the whole 200 Hz. Therefore, the model is likely to deviate from the A-criterion at higher frequencies, but this is still useful from the perspective of a worst-case scenario for the machine frame.

The machine frame has spherical feet which sit on three vee shaped grooves to form a Maxwell kinematic coupling. The SW model restrains movements in the vees but restrains movements in any other directions. Analysis was performed with 2% modal damping which results in a loss of energy of 2% per oscillation. Analysis input was a Power Spectral Density (PSD) random vibration base excitation in the direction of the purple arrow as shown in Figure 7. This was applied in 3 separate analysis, one for each of the principle axis. Since the main mechanism through which transference of ground vibrations could affect KB results is via the interferometer, the software sensor probe was placed on the interferometer platform as indicated by the red arrow in Figure 7. To simplify the modelling both magnets were replaced with remote mass elements of 56 kg centred at 105 mm above and below each mounting face (pink spheres and lines shown in Figure 7)

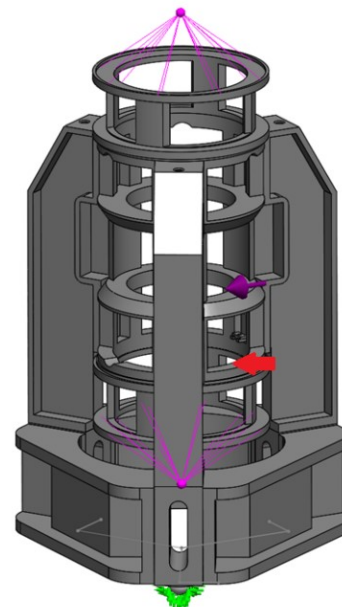


Figure 7. Machine frame model showing analysis input (purple arrow) and analysis output probe (red arrow).

3.2.2. Results of A-criterion induced frame vibration

The resultant displacement PSD for horizontal X direction random base excitation and the PSD displacement frequency domain (measured at the interferometer platform) is shown in Figure 8 and Figure 9 respectively.

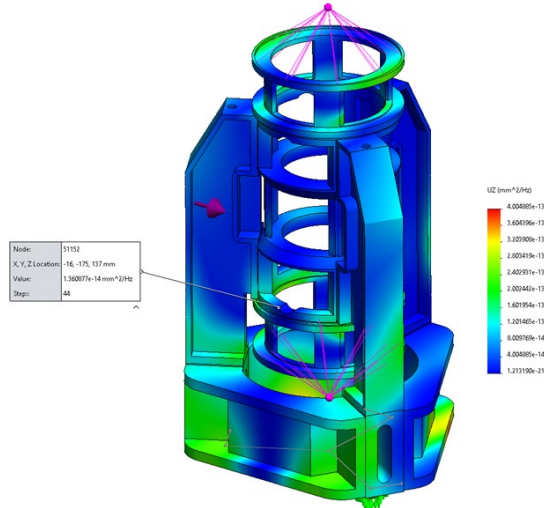


Figure 8. Displacement PSD for horizontal direction excitation

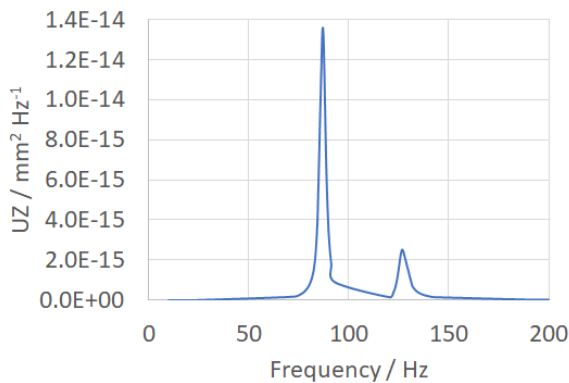


Figure 9. Horizontal PSD displacement frequency domain.

The first vibration mode occurs at a frequency of 86 Hz and results in a maximum displacement of 1×10^{-5} mm at the interferometer platform. The analysis in the orthogonal horizontal direction (Z-direction) produced a very similar result and is not shown here. The KB interferometer is relatively immune to displacements in the horizontal plane, requiring displacements approaching 1 mm in magnitude before the interferometer “lock” would be lost. Therefore, even at the first resonant mode of the machine frame the KB will be unaffected by horizontal displacements.

The resultant displacement PSD for vertical Y direction random base excitation and the PSD displacement frequency domain (measured at the interferometer platform) is shown in Figure 10 and Figure 11 respectively.

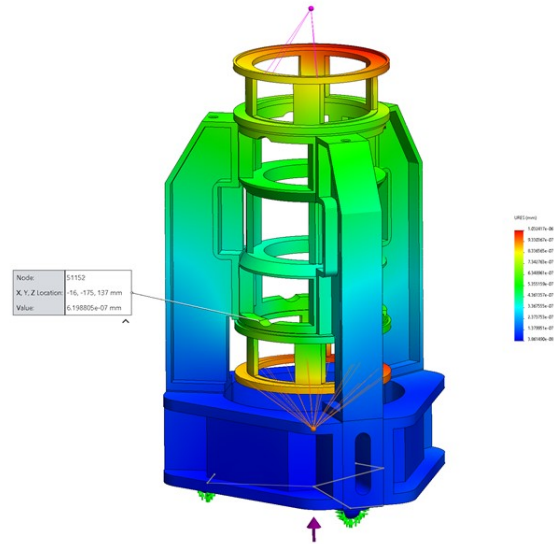


Figure 10. Displacement PSD for vertical direction excitation.

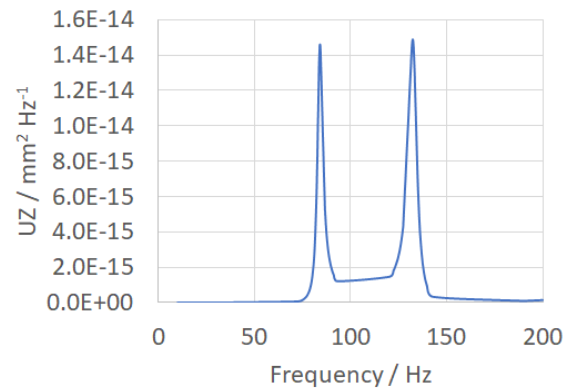


Figure 11. Vertical PSD displacement frequency domain.

Again the first vibration mode occurs at a frequency of 86 Hz but for the vertical direction a maximum displacement of only 6.1×10^{-7} mm occurs at the interferometer platform. Even if KB measurements were made whilst the frame was resonating at 86 Hz this sub nanometre displacement would not be a major contributor to the KB uncertainty budget.

3.3. Vibration response of the Kibble Balance (Part 3)

The aim was to produce a model which simulated the interaction of the machine frame and the moving assembly. These are two separate assemblies which are connected via either the flexure guide (Figure 3) or the watt-linkage guide (Figure 4). Due to the complexities of these guidance assemblies they are not easily included in a full FEA so they are replaced with FEA spring elements. With the SW software this is a “bearing” connection where the axial and lateral stiffness can be set, also allowing self-aligning (therefore not over stiffening the structure). These are the elements

used – the lateral stiffness has been determined by separate linear static FEA of the assemblies with 10N forces applied laterally in horizontal and vertical directions, the predicted displacement at that load will give a value in N/m for each axis. An example of the model is shown in Figure 12 with the machine frame shown in blue and moving assembly shown in red.

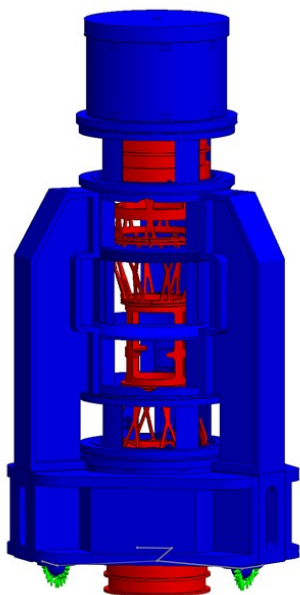


Figure 12. Next generation kibble balance FEA model

The results of the modal analysis for the first ten modes of the flexure and linkage guidance system are given in Table 4.

Table 4. First ten modes of vibration of the kibble balance

Frequency number	Flexure mechanism / Hz	Linkage Mechanism / Hz
1	0.0	0.0
2	0.8	3.8
3	16.4	51.3
4	16.7	61.5
5	23.8	72.5
6	24.0	80.0
7	99.1	101.7
8	99.7	102.5
9	112.7	112.9
10	112.9	113.2

The first two modes of both the flexure and linkage analyses are rigid body modes and can be discarded. The subsequent modes show the flexure configuration possessing much lower lateral stiffness than the linkage configuration and is therefore more likely to be disrupted by external sources of vibration or problems with the servocontrol mechanism.

4. VIBRATION TEST MEASUREMENTS

4.1. Vibration testing of the Machine Frame

The frequency response of the machine frame to induced vibration will be evaluated by using a mechanical shaker device to vibrate the ground beneath the machine frame over a frequency range from 0 Hz to 200 Hz. Resonance modes of the machine frame will be detected using a tri-axial accelerometer. The results of the vibration tests will be compared with output of the modelling.

4.2. Vibration testing of the Kibble balance

The main mechanism through which the transference of ground vibrations could affect KB results is in the interferometer. This measures the velocity of the coil via a retroreflector on the moving assembly and vibration of the interferometer optics could increase the noise in the coil velocity measurements. The tri-axial accelerometer will be placed on the interferometer and the shaker operated over the same 0 Hz to 200 Hz frequency range. Results from the vibration tests will be compared with output from the modelling.

5. SUMMARY

NPL has designed a next generation of Kibble balance based on a seismometer type moving mechanism. Modal analysis was performed on the machine frame of the balance using four FEA software packages. There was a 20 % variation in the predicted first resonance mode between the different software packages but all the predicted first modes were above the 60 Hz requirement for the balance.

Further modelling of the machine frame was done to determine the response of the frame to vibration input corresponding to that of an A-criterion laboratory [7]. This modelling confirmed that the machine frame design is resistant to vibration at frequencies below 80 Hz and even at the first resonant mode the vertical displacement at the interferometer position was less than 1 nm. Modelling of the full Kibble balance (with moving section) was performed in SolidWorks. The results of this modelling showed that the linkage type guidance mechanism is more resilient to induced lateral vibrations at frequencies up to 50 Hz compared with the flexure type mechanism.

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