ERRORS DUE TO MAGNETIC EFFECTS IN 1 kg PRIMARY MASS COMPARATORS

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ABSTRACT

A revised draft of *OIML R111* has been approved recently. Among the many notable changes to the previous edition are specific requirements for both the magnetic susceptibility and residual magnetization of weights used in legal metrology. The highest class of such weights is E_1, for which a 1 kg weight has a manufacturing tolerance of ± 0.5 mg. It therefore follows that better magnetic properties might be needed for weights having mass uncertainties substantially better than E_1 tolerances. The work presented below shows how this problem was approached at the BIPM and supports our conclusion that alloys meeting the new Class E_1 specifications for 1 kg weights have magnetic properties sufficient for the needs of the BIPM.

1. INTRODUCTION

In mass metrology, the possibility of errors due to magnetic weights has been known for a great many years. Recently, however, interest in this subject has been revived due to the ubiquitous use of very precise balances operating with electromagnetic servocontrol. A formalism for describing the interaction of a magnetic weight with a balance was introduced by Davis, who also proposed a method for quantifying the magnetic properties of weights [1]. However, [1] is largely mute concerning the equally important need to characterize the magnetic environment of modern balances. This problem was addressed by Gläser [2] so that it was finally possible to estimate reasonable upper bounds to weighing errors due to weakly magnetic weights that are used in modern electromagnetic balances. This topic has been reviewed in a recent publication [3].

A new edition of *OIML Recommendation 111* [4], drawing on ideas presented in [2], sets limits on the permanent axial magnetization (M_Z) and volume magnetic susceptibility (χ) for weights used in legal metrology. According to [4], 1 kg weights of Class E_1 should have the following magnetic properties:

Polarization ($\mu_0 M_Z$) within the range ± 2.5 μT;

Volume magnetic susceptibility ($\chi < 0.02$).

Reference [2] suggests limits to axial magnetization and susceptibility based on the magnetic environment of a particular balance combined with the principle that magnetic errors should be less than 1/10 of the Maximum Permissible Error (MPE) of the weight. The final limits recommended in [4] are more stringent, by approximately a factor of two, compared with those proposed in [2]. Nevertheless, one might infer that these limits allow the possibility of a 1 kg Class E_1 weight to have an error of as much as ±25 μg due to magnetic behaviour (1/20 MPE).

Note that Class E_1 weights are intended for linking national mass standards (traceable to the international prototype of the kilogram [5]) and weights of Class E_2 or lower. Thus we may ask whether the magnetic properties of 1 kg Class E_1 weights might lead to observable errors during calibration with respect to 1 kg national prototypes. Additionally, we may ask whether Class E_1 weights are appropriate transfer standards for mass comparisons among national metrology institutes (NMIs). Although undefined, one occasionally encounters terms such as Class "E_0" or "better than Class E_1". Is there a need for stainless steel weights with better magnetic properties than the limits set for Class E_1? We have attempted to answer this question, at least for the 1 kg mass comparators currently used for calibrations at the BIPM.

2. THEORY

A change in the vertical force F on the pan of a balance results in a change in the balance reading. Ideally, the relation between these two quantities is linear. As was pointed out in [1], it can be useful to think of F as derived from the change in mechanical potential energy U of the weight on the pan if it is moved infinitessimally in the vertical (Z) direction:

$$F = -\frac{dU}{dZ} \tag{1}$$

For instance, the gravitational potential energy U_g of a weight placed on the balance pan is simply

$$U_g = -mgZ\,, \tag{2}$$

where m is the mass of the weight and g is the local acceleration of gravity. The added complication of air buoyancy has been ignored.

If U_g represents the total potential energy, then $F = mg$ and there are no errors from magnetic forces. If, on the other hand, there is a magnetic potential energy U_m such that

$$\frac{dU_\mathrm{m}}{dZ} = F_\mathrm{m} \neq 0 \tag{3}$$

then the balance pan will also be subjected to a magnetic force which may be significant. The magnetic force F_m will be interpreted by the balance as being a mass equal to F_m/g.

It is customary to distinguish two contributions to the magnetic potential energy due to the weight on the balance pan. One of these depends on the volume magnetic susceptibility χ of the weight and the second on its axial polarization $\mu_0 M_Z$, where μ_0 is the magnetic constant ($4\pi \times 10^{-7}$ N/A^2) and M_Z is the permanent axial magnetization.

If we take an extremely simple model [1], the vertical component B_Z of the ambient magnetic flux density in the vicinity of the balance pan is $B_Z = B_{0Z} + b_Z Z$, where B_{0Z} and b_Z are constants. The coordinate system is fixed to the laboratory frame of reference and the origin is taken to be the centroid of a weight placed on the balance pan. A cylindrical weight of volume V will then be subject to two possible mass errors, e_1 and e_2 :

$$e_1 = \chi B_{0Z} \frac{b_Z V}{\mu_0 g} \quad , \tag{4}$$

which depends on the susceptibility of the weight and

$$e_2 = \mu_0 M_Z \frac{b_Z V}{\mu_0 g} \quad , \tag{5}$$

which depends on the axial polarization of the weight. The susceptibility is a positive scalar for stainless steel. Whether M_Z is positive or negative depends on the orientation of the weight with respect to the coordinate system defined above. The model assumes that the remaining components of M and B may be neglected.

This simple model appears to be sufficient to explain many of the experimental results shown in the following section. Note that, according to the model,

1. If a weight is turned upside down and remeasured, then e_1 should be unchanged in both sign and magnitude. By contrast, the sign of e_2 will change but not its magnitude;

2. If $\mu_0 M_Z$ is known from other measurements, then b_Z can be determined from a measurement of e_2;

3. If, in addition, B_{0Z} and χ are known from other measurements, then [1]

$$\frac{\chi B_{0Z}}{\mu_0 M_Z} = \frac{e_1}{e_2} \ . \tag{6}$$

We conclude this section by mentioning a subtlety: As in [2], we have assumed that B_Z as measured in our defined coordinate system is not a function of the angle of the balance beam. In practice, this means that we have assumed that the source of b_Z is the servocontrol magnet fixed to the balance housing. If, instead, the ambient flux density were invariant when measured in a coordinate system fixed to the balance pan, then the derivative on the left-hand-side of (3) would vanish and both e_1 and e_2 would be zero. This absence of magnetic weighing error may occur in very special cases; for example, when the source of b_Z is a magnetic pan suspension [6].

3. EXPERIMENTAL

Four 1 kg cylindrical mass standards constructed from type 316L stainless steel were selected for use. Rods of this particular grade of steel often have mediocre magnetic properties. We will refer to the four cylinders as HK1, HK2, HK3 and HK4. All surfaces of the weights were machined but not polished. Each weight is identified by a number engraved on the curved surface. The volume of each weight was found to be 126.25 cm^3, as determined from mass measurements at reduced air pressure [7].

The experiment proceeded in two steps. First, the susceptibility and axial polarization were determined for all four weights using the BIPM susceptometer [1,3]. The results are shown in Table 1, where the numbers in parentheses are the numerical values of the combined standard uncertainties expressed in the units of the quoted results. Since the polarization measurements are model dependent [1,3], it is necessary to note that the susceptometer magnet was positioned 26 mm below the weight. We see in Table 1 that the susceptibilities are essentially homogeneous from top to base and are approximately the same for all weights. This nominal susceptibility is only about a factor of 1.6 greater than the recommended limit for 1 kg Class E_1 weights [4]. However, weights HK1, HK2 and HK4 all exceed the recommended limit for polarization [4] by factors ranging from 3 to 15. If the axial polarization were really uniform throughout each weight, then the values found for the top and base of each weight would be equal in magnitude but opposite in sign. This is approximately the case for HK1 only. However, the difference in polarization from top to base of the weights is more uniform in magnitude. This difference is shown in the final column of Table 1 and is our estimate of the theoretical parameter $2M_Z$ with the weight in its normal orientation. Additional measurements made with a Hall-probe gaussmeter [3] placed near the surface of each weight confirm that the permanent magnetization of each of the HK weights is not purely axial and the axial magnetization is not homogeneous on the top and bottom surfaces. As is often the case, the real magnetization is too complicated to model [3] so we are forced to see how far an admittedly oversimplified model will take us.

Table 1: Magnetic properties of test weights as determined by a susceptometer.

		volume magnetic susceptibility, χ	polarization $\mu_0 M_Z$ / µT	difference $\mu_0 M_Z$ / µT
HK1	Base	0.031(0.002)	−20(2)	−33
	Top	0.031(0.002)	+13(2)	
HK2	Base	0.031(0.002)	+ 7(2)	−31
	Top	0.034(0.002)	+38(2)	
HK3	Base	0.032(0.002)	+ 0.5(0.3)	−1.4
	Top	0.032(0.002)	+ 1.9(0.3)	
HK4	Base	0.032(0.002)	+32(2)	+21
	Top	0.033(0.002)	+11(2)	

In the second part of the study, the weights were compared among themselves using two different commercially available comparators. One, produced by Metrotec, has a standard deviation of about 0.1 µg and the second, a Mettler HK1000MC, has a standard deviation of about 0.5 µg. When small systematic effects seen in both balances are taken into account or averaged out there is no significant difference in the Type A uncertainties of the two balances, both being less than 1 µg. Both balances are housed in airtight enclosures thus making it too difficult to map the ambient magnetic flux density in the volume normally occupied by a 1 kg weight on the balance pan. Both balances have a suspended pan that is relatively distant from the servocontrol magnet, which is located at the level of the beam on the counter-weight side.

Using each of the balances, the four HK weights were calibrated in two orientations: normal and turned upside-down. Initially, each weight was loaded into the balance so that its identification number faced away from the centre of the carrousel (weight changer). Weights were always turned so that the identification number still faced away from the centre. This method of positioning the weights should make the results more repeatable in the presence of radial magnetization, which has been neglected in the theory. The apparent mass of the HK weights was determined with respect to stainless steel working standards whose magnetic properties are excellent.

The changes in apparent mass that were observed as a consequence of turning the four weights are shown in Table 2. The results are given in micrograms and are expected to correspond to the theoretical parameter $2e_2$ with the weight in its normal orientation. The Metrotec data are the average of two completely independent series of measurements. Based on the replicate measurements, we find that the standard deviation of a single result is 2 µg with four degrees of freedom.

Table 2: Change of apparent mass (in micrograms) for each weight after turning.

	HK1	HK2	HK3	HK4
Metrotec	–8.5	–7.0	–1.0	+3.5
HK1000MC	+15.1	+15.2	+3.6	–9.5

According to the simple model, e_2 is eliminated by averaging the apparent mass obtained from the two orientations of the weight. If e_1 is negligible, then these averages should be independent of the balance used. As discussed, below, this hypothesis could not be tested definitively.

4. DISCUSSION

The results are in semi-quantitative agreement with the model presented in Section 2. Specifically, We would infer from Table 1 that the effect of turning the weights should be roughly the same in sign and magnitude for weights HK1 and HK2. The sign should be reversed for HK4 and the magnitude slightly less compared to HK1. For HK3, we expect a result at least an order of magnitude less. Table 2 confirms these predictions except for HK3 where the experimental result could be due to inhomogeneous axial polarization or to radial polarization. In addition, we infer that the magnitude of b_Z for the HK1000MC balance is roughly twice that of the Metrotec environment. We also see that b_Z has opposite sign in the two balances.

From the theoretical model, $b_Z \approx -50$ µT/m for the HK1000MC balance. While we are unable to measure this quantity directly, the result nevertheless appears to be similar to the smallest values obtained in the study of five other balance models [2]. Thus, if we wish $|e_2| \leq 1$ µg, then the magnitude of the axial polarization of a 1 kg standard should be less than 1.2 µT.

Taking 50 µT/m as the magnitude of b_Z, we may then use (4) to estimate that a constant ambient field of B_{0Z} = 62 µT would produce e_1 = 1 µg for the HK weights. Although the ambient value of B_{0Z} is not known accurately, it is of the order of 50 µT. This is also the ambient value elsewhere within the laboratory. Therefore, the effect of magnetic susceptibility cannot account for the any of the results presented in Section 3. Rather, any results not accounted for by (5) must be attributed to other sources (inadequacy of the theoretical model, the instability of the HK weights).

The question of whether averaging the apparent mass over the two orientations of the weight eliminates e_2 as the model suggests cannot be answered by this study. The long-term stability of the HK weights is questionable and there are other significant sources of uncertainty as well. If the HK weights were real mass standards, we would obviously reject their use rather than try to average over their magnetic imperfections.

5. CONCLUSION

We have determined the apparent mass of 1 kg standards whose magnetic properties are inferior to those recommended for Class E_1 standards and have developed a simple model to paramaterize our results. Although the polarization of the weights is not as simple as we assume and the model is far from perfect, we can explain semi-quantitatively our most important experimental results. Thus we may draw some useful conclusions.

The aim of this study was to determine if the limits set for Class E_1 are sufficient to avoid errors greater than 1 µg when using our two mass comparators devoted to calibrations. We find that the recommended limits [4] are appropriate for the stated purpose of Class E_1 standards but are barely adequate for our own needs. We must maintain a set of 1 kg working standards made of stainless steel with mass traceable, at minimum uncertainty, to the international prototype of the kilogram.

Nevertheless, it is our experience that the magnetic properties of weights currently produced by several different manufacturers are superior to the limits set by legal metrology. Specifically, weights having a magnetic susceptibility less than 0.005 and an axial polarization less than 0.5 μT in magnitude are commercially available as Class E_1. If such weights were not readily available from commercial sources, then the BIPM and doubtless many national metrology institutes would be greatly inconvenienced.

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