AXISYMMETRICAL ELASTIC ELEMENTS FOR VERY LARGE FORCE TRANSDUCERS

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ABSTRACT

Measuring of large forces up to 10 MN with high accuracy is a complex subject, blending theory, practice and applications, with a special accent in the field of Metrology. It is simply to generate complex shapes of mono-block elastic structures starting from simple shapes of strain gage measuring sections, the axisymmetrical ones being the best suited in this respect. The body of the force transducer is easy to design by 2D axisymmetric FEM and easy to manufacture. The paper have in view a unified approach of square (type A) and rectangle (type B). Square section is among the few with an analytical but complicated formula, having two strain gauges tangentially located on the outside of the ring torsion and the other two strain gauges, which complete the Wheatstone bridge, being diametrically opposed. The rectangular section (with different ratios between the two sides but without analytical formula), is obtained by a minimum modification of the previous one (making two slots), the strain gauges being circumferentially located on the upper and the lower faces. We formulate a standard FEA procedure for axisymmetrical elastic elements of strain gauged force transducers using ANSYS Mechanical program. The starting model is very “flexible”, so that, changing one by one different parameters (e.g. modifying the keypoints coordinates), a lot of variants could be studied. A special attention is necessary to obtain proper strain diagrams on the superior, lateral and inferior sides of the elastic element measuring section, because it is essential to compare these diagrams in order to establish the best strain gauges positioning. In this respect, appropriate paths were conceived and plotted each time on graph, more precise and suggestive than plotting on geometry. An original grouping of all kinds of strain diagrams on the same plot of the deformed and undeformed bodies is presented. A lot of interpretations are possible based on the multitude of data and having in view: the influence of the axisymmetrical elastic element shape (varying different parameters) on the strain gauge sensitivity; the best combination of conflicting design criteria: strain, stress (determining the overload) and displacement (determining the stiffness); the possible correlation between the nominal load and the dimensions of a particular variant, e.g. with square measuring section. In the next stages will be very useful to unite FEM and CAD, following the elastic elements parametric modeling for their best constructive optimization.

1. INTRODUCTION

The challenge to measure greater forces, up to 10^7 N, constitutes a development trend together with the opposite one, that of the measuring methods for very small forces (micro- and nano-range). In different industries (ship or aerospace building, rolling mills, civil engineering) large forces measuring with high accuracy is a proper subject for harmoniously blending theory, practice and applications, with a special accent in the field of Metrology [1].

As visiting researcher in the Force Laboratory of KRISS, Dr. D.M. Stefanescu has developed a design technique for strain gage type force transducers using the finite element method (FEM) having in view new transfer standards. Design specifications of the present work are the following:
- Capacities: 1 MN and 7 MN, overload: 200 %;
- Average strain at nominal load under each strain gauge: $\varepsilon = \pm$ 1000 µm/m to $\pm$ 1500 µm/m;
- Maximum displacement under nominal load < 0.4 mm;
- Height: as small as possible, not recommended column type sensing element;
- One-piece solution, maybe a new and aesthetic one.

Among the various flexible structures studied only few types are suited for measuring forces in the meganewtons range [2]: classical tensioned/compressed column (cylinder or tube), shearing structures, membrane, compressed torus or sphere, and other axisymmetrical shapes. Their maximum strain gauge (tensometrical) sensitivity is ensured by loading in shearing or bending.

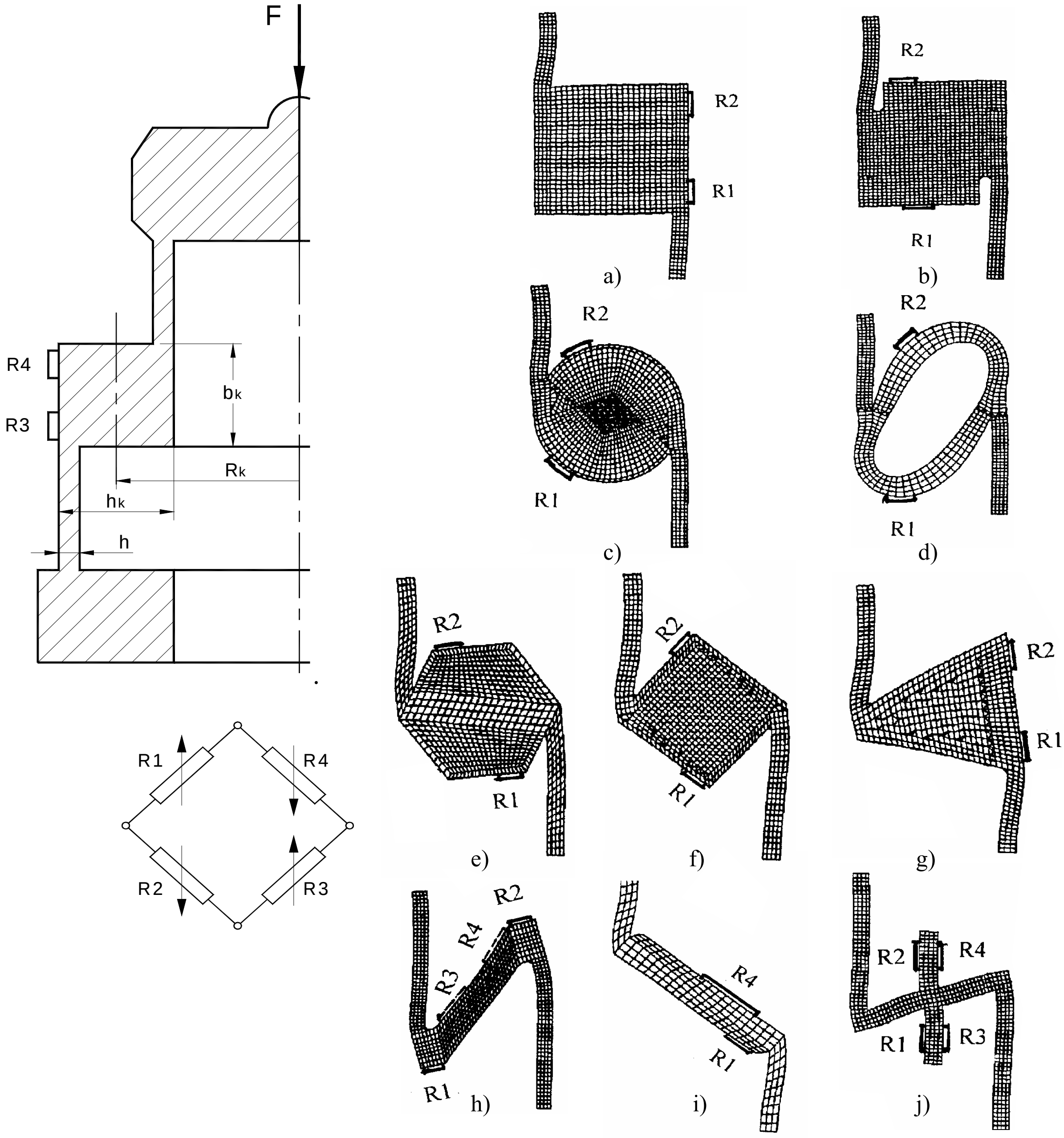

Figure 1: Axisymmetrical sections of different geometrical shapes.

$$\varepsilon_{max} = \frac{\varepsilon_0 \cdot (1-\nu^2) \cdot \beta_k^3 \cdot k^2}{(1-\nu^2) \cdot \beta_k^3 \cdot k^2 + 0{,}5 \cdot \sqrt[4]{[3(1-\nu^2)]^3} \cdot \beta_k^2 \cdot k_1\sqrt{k_1} + \sqrt{3(1-\nu^2)} \cdot \beta_k \cdot k_1 \cdot k_2 + \sqrt[4]{3(1-\nu^2)}\sqrt{k_1} \cdot k_2^2}$$

$$\varepsilon_0 = \frac{3 \cdot F}{\pi \cdot E \cdot \beta_k^2 \cdot h_k^2} \qquad \beta_k = \frac{b_k}{h_k} \qquad k = \frac{h_k}{2R_k} \qquad k_1 = \frac{h}{R_k} \qquad k_2 = \frac{h}{h_k}$$

To generate complex shapes of mono-block elastic structures starting from simple shapes of the strain gage measuring sections, the axisymmetrical ones are the best suited in this respect. The body of the force transducer is easy to design by 2D axisymmetric FEM and easy to manufacture. Ten sections of various geometrical shapes (Figure 1) were studied in a previous paper [3]: square, rectangular, circular, toroid, hexagonal, rhomboid, triangular, N-shaped, parallelogram (rotated as a truncated cone) and ribbed membrane. They have similar configurations being made up of two thin and short tubes of different diameters joined by the strain gauged measuring section. The axial load was in all cases F = 50 kN, to make a proper comparative analysis of the strain diagrams (ε), either longitudinal (l) or tangential (t) ones, for the ten analyzed section-types.

Now, an increased measurement range, i.e. a possible extension up to 10 MN, is considered. We follow a unified approach of square (type A) and rectangular (type B) sections. Square section is among the few with an analytical but complicated formula, having the strain gauges R_1 and R_2 tangentially located on the outside of the ring torsion and the other two strain gauges, which complete the Wheatstone bridge, being diametrically opposed. The rectangular section (with different ratios between the two sides but without analytical formula), is obtained by a minimum modification of the previous one (adding two slots), the strain gauges being located on the upper and the lower faces of the measuring section, respectively, and has an industrial use.

Both above-mentioned strain gauged measuring sections belong to the generic category RING TORSION, also named BENDING RING, which is associated with special spiral strain gauges without end loops. So, the tangential/circumferential strain of the bending ring is transferred into such a spiral without any shear stress, since the spirals are, in this sense, endless [4]. This elastic structure is insensitive to the interference effects by eccentric loading as well as to the side forces and momenta.

COMPUTATION MODEL

For these axisymmetrical elastic elements with square (both equal sides) or rectangle section an analytical formula is given by Malikov et al [5], please see under Figure 1. It is a complicated, non-linear relationship from the Strength of Materials field, referring only to the tangential/circumferential strains on the lateral side of the measuring section.

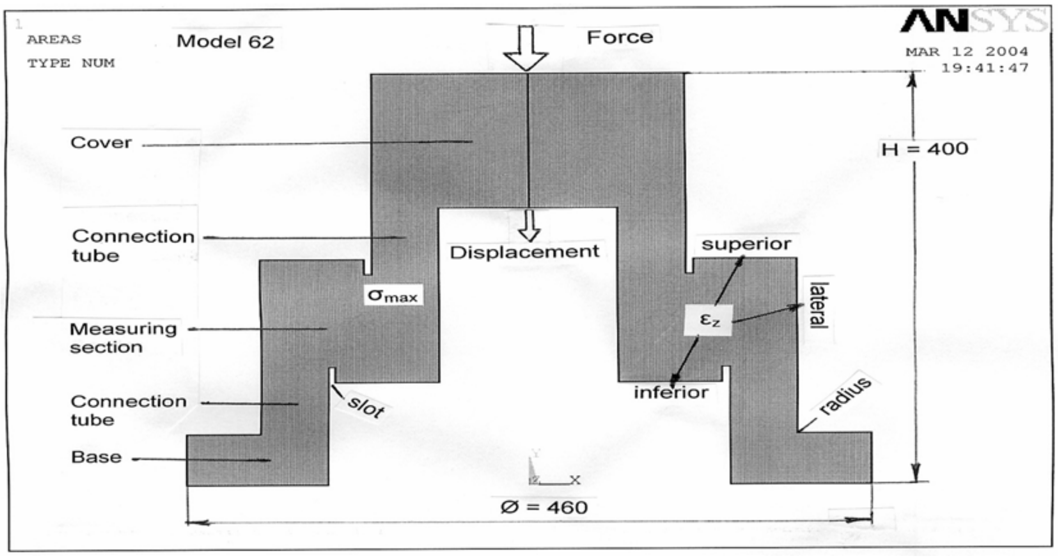

Figure 2: Computation model for square/rectangular axisymmetrical sections.

We choose, for F = 7 MN, a first model having the following geometrical parameters, considered as design variables: h = 25 mm, R_k = 120 mm, h_k = 90 mm, b_k = 54 mm, resulting

$\beta_k = 0.6$. The influences of vertical dimensions (such as length of the two connection tubes, height of the cover or height of the base portion) or other shape factors (like the fillet / corner radius) are not estimated analytically but are best suited for FEM analysis! We can calculate only the maximum circumferential strain (1421 μm/m) on the lateral side of the measuring section, but predimensioning is very complicate because of the multiple cross-influences between the geometrical parameters as defined by Malikov. Substantial estimations and improvements are possible by using the finite element analysis and developments of new designs based on these, so that very little prototyping to be required.

We define a complete model representing the geometrical parameters from the Malikov's formulae (Figure 1) as well as other parameters not included in this analytical relation (Figure 2). Material properties (elasticity moduli) are the same for all variants and are the classical ones for steel: Young's modulus E = 210000 MPa (N/mm^2) and Poisson's ratio $\nu = 0.3$. This starting model is very "flexible", so that, changing one by one different parameters (e.g. modifying the keypoints coordinates), a lot of variants could be studied. The strain gauges will be placed on areas with high strain gradients, which make the gauge outputs very sensitive to placement errors. Therefore, it is important to generate high strain over a sufficiently large area. This objective is difficult to formulate analytically, but it can be achieved by finite element method.

A general procedure for finite element analysis of the force transducers elastic elements do not exist. Designers are working based on their experience and intuition in this multi- and inter-disciplinary field [6]. We formulate a standard FEA procedure for axisymmetrical elastic elements of strain gauged force transducers using ANSYS 6.1 – Mechanical program [7]. As finite element we choose Quad 42 which is a 2D quadratic and low order structural element. We create a bottom-up model, starting with keypoints, from which we build-up lines, areas and, by rotating the axisymmetrical section, the solid model. We may consider the model being fixed (UX and UY are constrained) or supported on its base (only UY is constrained), there are not significant differences in strain diagrams and the displacement on the base is negligible (d_x = 0.05 mm).

A special attention is necessary to obtain proper strain diagrams on the superior, lateral and inferior sides of the elastic element measuring section, because it is essential to compare these diagrams in order to establish the best strain gauges positioning. In this respect, appropriate paths for ε_Z (STRAIN Z) were conceived and plotted each time on graph, more precise and suggestive than plotting on geometry! It is very important to define these paths in the same conditions for all variants of axisymmetrical elastic elements. A great importance has also the precise numerical computation of maximum stress and maximum displacement having in view the design criteria fulfillment.

The above-mentioned "standard" FEA procedure has been applied on 82 different 2D axisymmetrical models of square or rectangular sections in three great stages:

I – *Determining the tangential / circumferential strains on the lateral side of the type A measuring section in order to compare with the analytical calculus.*

Remember that for the first variant of rectangular section the maximum circumferential strain analytically computed was: 1421 μm/m. By finite element analysis we obtain the maximum value ε_Z = 1276 μm/m. We may consider a difference of 10 % as a good agreement between analytical and numerical computations. Unfortunately, this kind of strain gauge arrangement is not useful because of large displacements of the elastic element under the rated load: about 1 mm for the lower point from the OY axis belonging to the model.

II – *Determining the tangential / circumferential strains on the upper and lower sides of the type A measuring section in order to improve the strain gauge sensitivity.*

For this kind of strain gauge positioning we have not analytical solution to compare with, but we observe a great improvement of the tensometrical sensitivity comparing with lateral emplacement of the strain gauges: up to 70 % for the negative values of ε and more than doubled for the positive values of ε. The displacement values of the elastic element under the rated load remain large (about 0.8 mm for the lower point from the OY axis), so a third stage of investigations has been started.

III – *Determining the tangential / circumferential strains on the upper and lower sides of the type B measuring section (with slots), continuing to improve the strain gauge sensitivity.*

Also, for this kind of strain gauge positioning we have not analytical solution to compare with, but we observe a supplementary improvement of the tensometrical sensitivity comparing with the previous case: approximately doubled for the negative values of ε and up to 35% for the positive values of ε. In this last case, the displacement values of the elastic element under the rated load were practically half (about 0.4 mm for the lower point from the OY axis), but, unfortunately, the stress values were seriously increased, reaching 1000 MPa for some variants. So, a study for diminishing the stress concentrators becomes necessary.

The results of the three investigation stages conduct to the idea of a unified FEA approach of the rectangular axisymmetrical sections in order to determine the influence of different parameters on strain gauge sensitivity. At the same time, it is very important to establish the best combination of design criteria: strain, stress (determining the overload) and displacement (determining the stiffness). Quoting Farhad Aghili, consultant with the Canadian Space Agency in Montreal: “The sensor design must optimize, and trade off among several conflicting design criteria. Also, many design iterations are required to arrive at a final design. Despite this complexity, it is possible to arrive at a novel basic sensor design.”

Changing different parameters (e.g. modifying the keypoints), a lot of variants are possible, excepting the circular ones. So, it is possible to compare with various axisymmetrical measuring sections studied in other works or to create new shapes based on the results of such complex comparisons:

- Two connected tubes (the outside diameter of the first tube is equal to the inside diameter of the second one) extending the shapes of axisymmetrical structures within the present study, and also including the primary shape of the tube. Tubular structures have the smallest axial displacements; the best way improve their sensitivity is to profile the axisymmetric section of the tube by milling, knowing that greater the fillet radius smaller stress concentrators;
- A model with nearer slots, which modify the deformation mode of the elastic structure and suggest other types of measuring sections with longer and larger slots and increased radius in order to reduce the stress concentrators and the displacements;
- A model without the connection tubes, having in view the low profile measuring section characteristics and attempting to harmonize the sensitivity and the strength criteria.

2. RESULTS ANALYSIS

The results obtained, and grouped in a great synoptic table containing all the variants, conduct to the idea of a unified FEA approach of the rectangular axisymmetrical sections in order to investigate the influence of different parameters on strain gauge sensitivity, on all sides of the measuring section. A lot of interpretations are possible based on this multitude of data and having in view:

- the influence of the axisymmetrical elastic element shape (varying different parameters) on the strain gauge sensitivity;
- the best combination of conflicting design criteria: strain, stress (determining the overload) and displacement (determining the stiffness);

- the possible correlation between the nominal load and the dimensions of a particular variant, e.g. with square measuring section;
- the optimum strain gauge emplacement solution for a chosen model of axisymmetrical elastic element, e.g. variant 62 fine, presented in Figure 2.

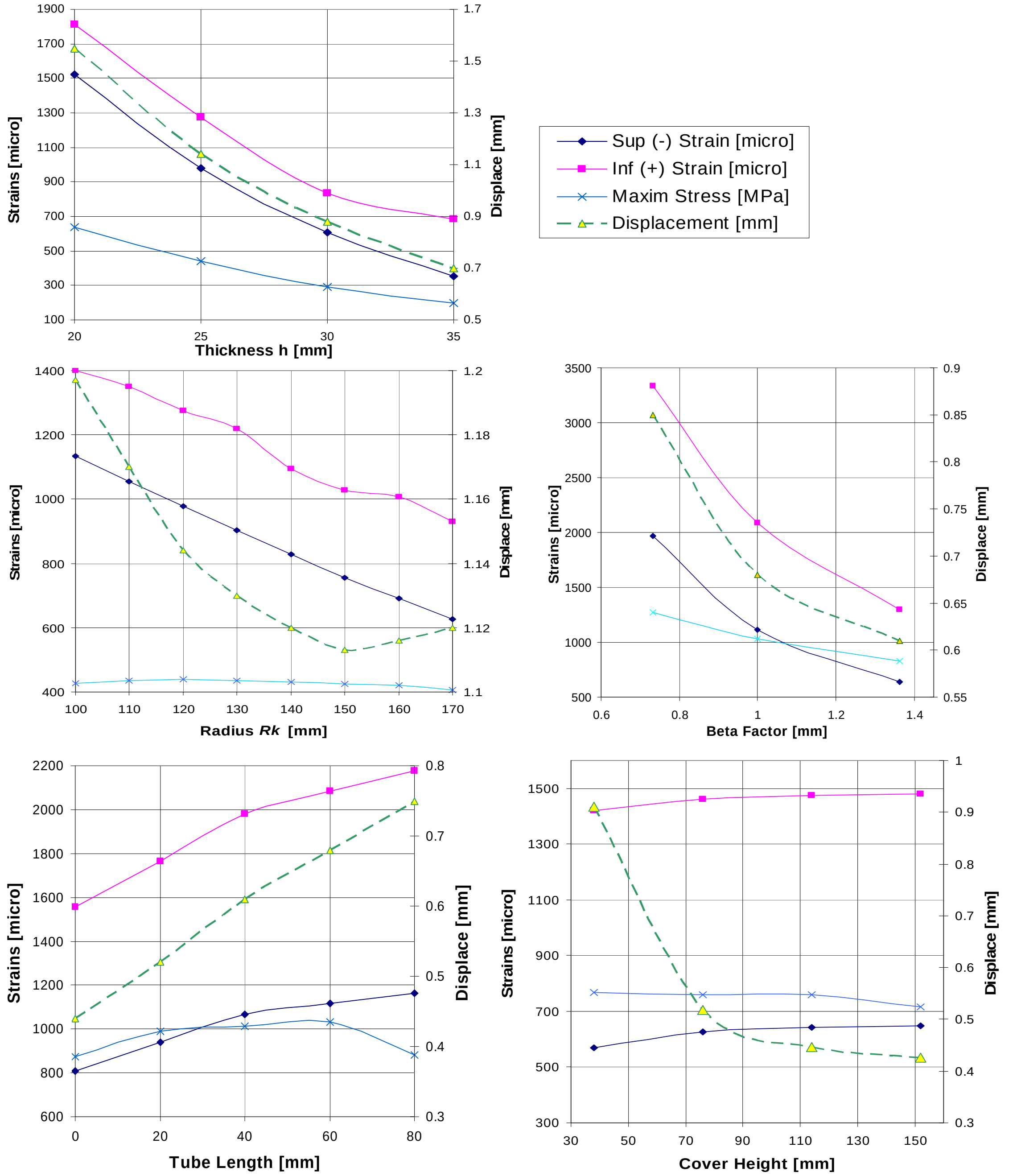

Figure 3: Influence factors on strain, stress and displacement.

Five of the most important strain gauge sensitivity factors were chosen to be graphically represented in Figure 3: h – thickness of the two connection tubes; R_k – distance from the symmetry axis to the measuring section center; β_k – square / rectangle shape factor, as ratio between b_k – height and h_k – width of the measuring section; length of the two connection tubes; height of the cover. “Active” data in Excel are Strain and Stress (connected by the Hooke’s law), represented on the left axis, and Displacement - on the right axis. Other useful data are mentioned in the following Table:

Represented parameter	Design variables, in mm, excepting β_k factor [-]				
	R_k	h_k	b_k	β_k	h
Thickness h	120	90	54	0.7	20 - 35
Radius R_k	110 - 170	90	54	0.7	25
Shape factor β_k	120	110 ; 150	150 ; 110	0.73–1.36	25
Tube length: 0 – 80 mm	120	110	110	1	35
Cover height: 38–152 mm	120	110	110	1	45

Comparing the diagrams, strain and displacement have the same type of variation (partially excepting the cases of parameter R_k, which has not very significant variations, and of the cover height, only if it is too small). So, it is necessary to establish priorities, like this concerning the shape factor:

- if a greater strain gauge sensitivity is preferred, the measuring section is placed horizontally;
- if a smaller displacement is preferred, the rectangular measuring section is placed vertically.

The square section is a particular case of the rectangle one and offers a multitude of dimensioning possibilities, because of its combined vertical and horizontal influences! Increasing progressively the dimensions is possible to increase the applied load. Negotiating the best combination of conflicting design criteria (strain, stress and displacement) three variants are proposed, all having R_k = 120 mm, for using at different nominal loads, the difficult problem remaining the high stress. Maybe, a simple nomogram would be useful to start the design of the force transducer elastic element. After that, many design iterations via the finite element analysis are required to arrive at the final solution.

Rated load F [MN]	Dimensions [mm]		Strain ε_Z [μm/m]		Displacement d [mm]	Stress σ [MPa]
	Thick h	Square side	- Superior	+ Inferior		
3	25	86	834	1349	0.41	658
5	35	110	781	1460	0.48	723
7	45	120	693	1508	0.42	764

The elastic element material is the limiting factor for sensitivity while the stiffness depends linearly on the Young's modulus. By virtue of Hooke's law (stress is linearly related to strain via the modulus of elasticity), one can conclude that high sensitivity and stiffness are achievable simultaneously only by use of a high-strength material, hardened to a yield strength of 1500 MPa or higher.

Strain is the essential quantity of every strain gauged force transducer. After resolving stress and displacement constraints, it is very important to have the best strain estimations, in our case all kind of circumferential strains: superior, inferior and lateral (the interior emplacement of strain gauges is not usual). We propose a unified FEA approach of the rectangular axisymmetrical sections, comparing a model without slots (Figure 4) with a model with slots (Figure 5), the last being more sensitive from the tensometrical point of view. It is an original representation grouping all kinds of ε_Z on the same plot of the deformed and undeformed axisymmetrical elastic element. The deformation mode of the square section under 7 MN is very clear while at the models tested, in the year 2000, under 50 kN was visible the deformation mode of the connection tubes! And, the option for strain gauges emplacement on the superior and, respectively, the inferior sides of the square measuring section is imperative.

Variant number	Circumferential strains ε_Z [μm/m]				Displacement d [mm]
	– Min Lateral	+ Max Lateral	– Superior	+ Inferior	
27	572	921	883	2282	0.84
33	1165	1037	1974	3277	0.99

The final stage of this very complex numerical procedure is the establishment of the optimum strain gauge emplacement solution for a chosen model. Special paths were designed to ensure the best tensometrical view, this operation is not yet automatically made. As we see on the inferior path plot, a strain gradient is present, so it is necessary to estimate carefully the strain gauges position, depending on their dimensions. Maybe, a special design of double spirals (each

having two windings), not the same on the superior and, respectively, inferior sides of the square measuring section, will be recommended to achieve the maximum strain gauge sensitivity.

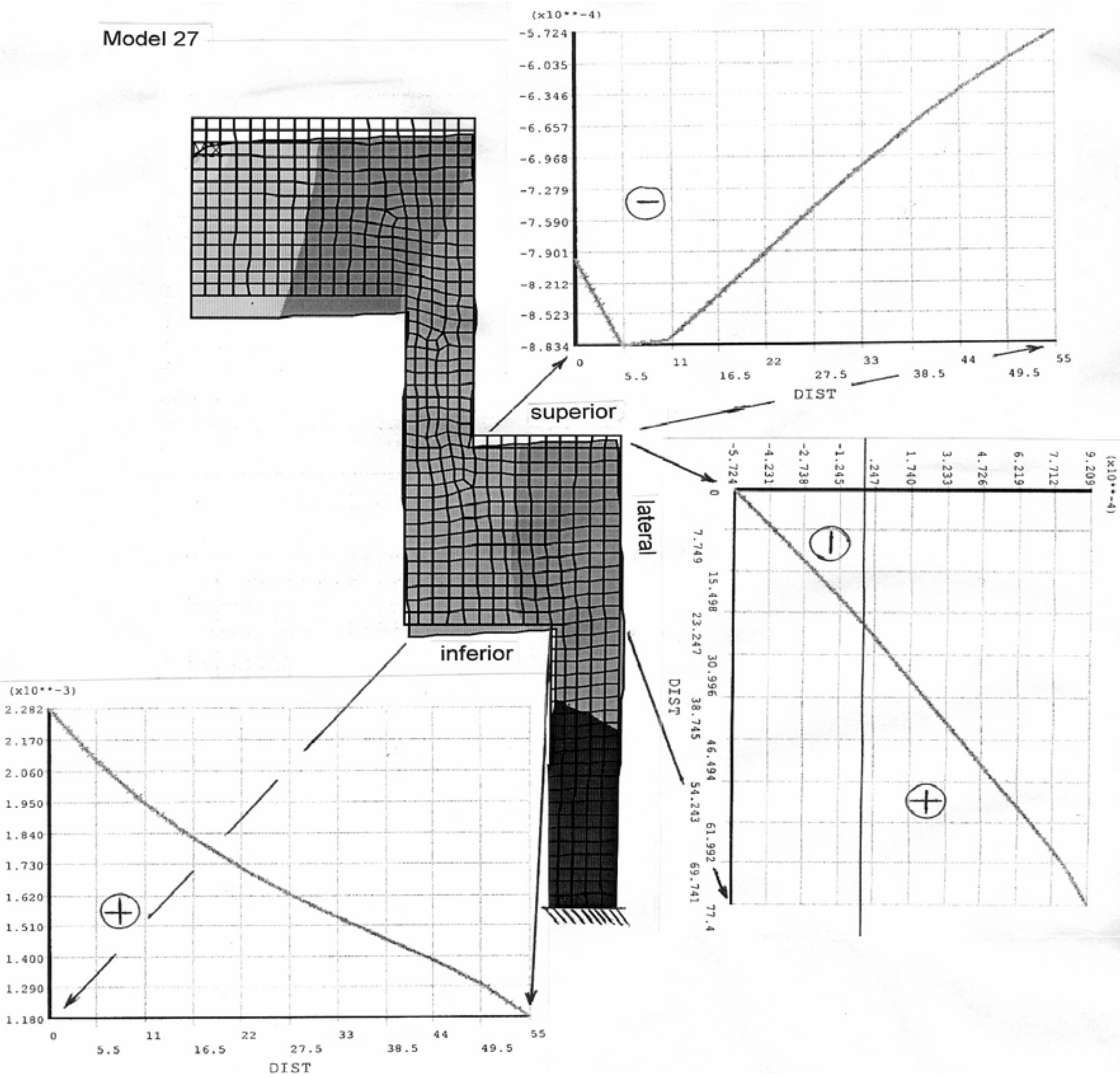

Figure 4: Rectangular axisymmetrical section without slots.

3. CONCLUSIONS AND FURTHER PROSPECTS

In order to develop an innovative design technique for strain gage type force transducers using the finite element method (FEM) important stages have been fulfilled, part of them representing original contributions: standardized presentation of 12 types of elastic elements, choosing criteria for elastic elements in the meganewton range and decision to use complex shapes generation of mono-block elastic structures, known as bending rings, achievement of a standard FEA procedure for axisymmetrical elastic elements of strain gauged force transducers, definition of a unique and complete model representing all geometrical parameters.

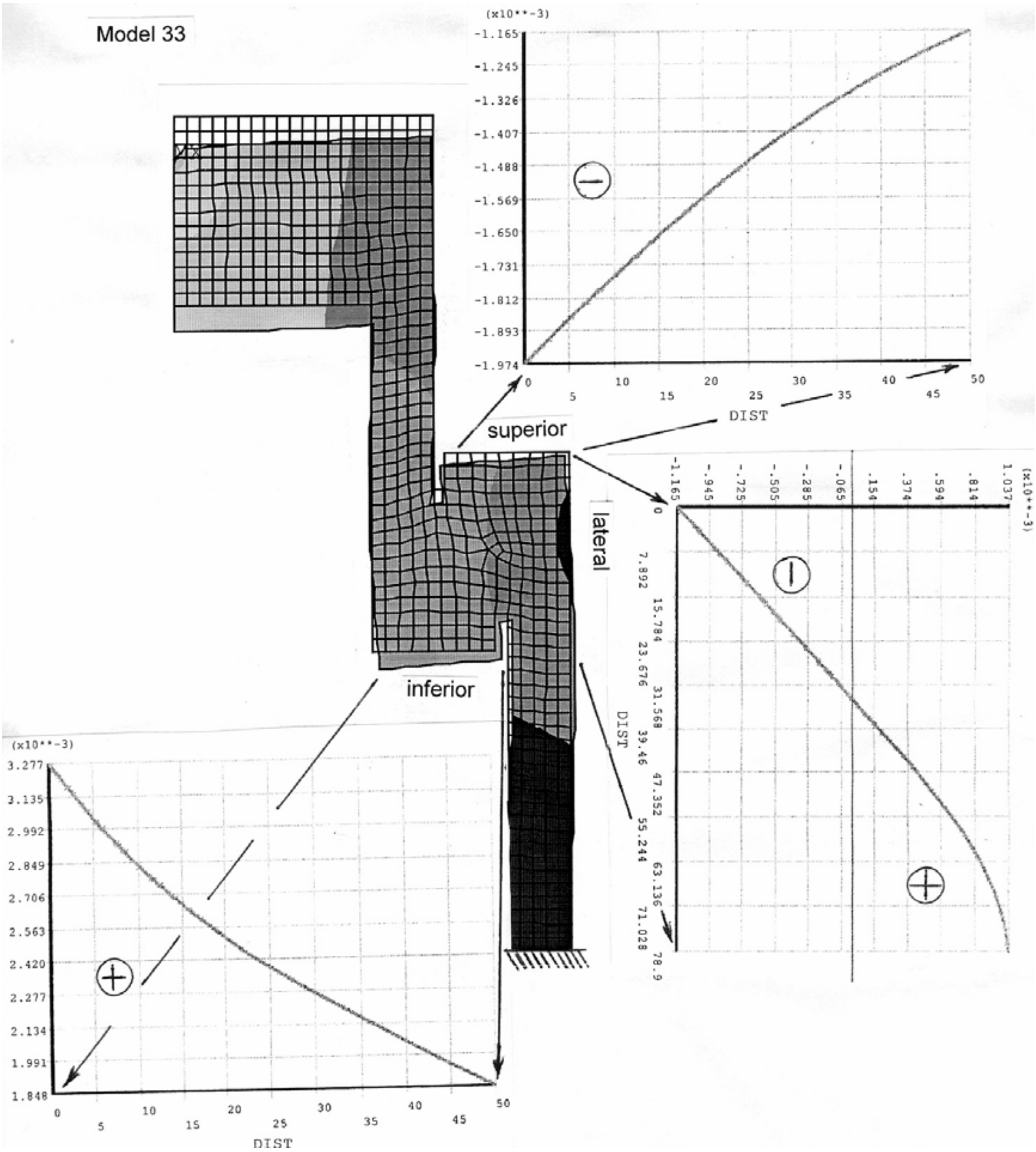

Figure 5: Rectangular axisymmetrical section with slots.

The obtained results permit the detailed investigation of different parameter influences on strain gauge sensitivity, on all sides of the measuring section and having in view some contradictory design criteria: strain, stress (determining the overload) and displacement (determining the stiffness). The particular idea of a unified FEA approach of the rectangular axisymmetrical sections is illustrated comparing two identical models, but having a single difference: with or without slots. An original grouping of all kinds of ε_Z (superior, lateral and inferior) on the same plot of the deformed and undeformed axisymmetrical elastic element is presented. The option for strain gauges circumferential emplacement on the superior and, respectively, the inferior sides of the square measuring section becomes imperative. In the next stages will be very useful to unite FEM and CAD, having in view the parametric modeling of force transducers elastic elements for their best structural optimization [8].

It is very difficult to fulfill simultaneously two optimization conditions, if they are antagonistically. To resolve the classical dilemma "sensitivity or stiffness", i.e. greater strains or smaller displacements, a careful negotiation of behavior restrictions is necessary. Of course, first priority is the maximum <u>strain</u> at the nominal load. For deciding between strength (maximum stress) and stiffness (minimum displacement) constraints, two new and very different research directions are envisaged, as follows:

Minimum displacement (< 0.3 mm) for axisymmetrical elastic structures is ensured by tubes, but their strain gauge sensitivity is reduced, due to Poisson's coefficient for two of them. The best way to improve their tensometrical sensitivity (up to 25 %) is to "transform" tension / compression in bending after profiling the tube section (by milling), knowing that greater the fillet radius smaller stress concentrators. Unfortunately, due to space restrictions, the overload is limited. But, the Saint Venant's principle becomes invalid and the height of the tubular structure may be substantially reduced.

Maximum admissible stress is supported by complex elastic structures subjected to very large forces (up to 10 MN) if they have pairs of deeper slots with larger radii, inspired by genetic algorithms [9]. Such configurations are obtained starting from very flexible structures, like membranes (Fig. 1,j) or N-shaped (Fig. 1,h) elastic elements, and strengthening them by vertical ribs, respectively, thickening their sides, so that the displacement is also reduced. First example is, in fact, the GTM/PTB solution of ring torsion, having four symmetrical slots near the intermediate thick rib, while the second is a new one, based on an oblique bending ring resulted by cutting only two skew-symmetric slots. Optimal design should be applied to satisfy reasonably strain, stress and displacement conditions.

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REFERENCES

[1] Kang D.-I. et al, "Traceability of large force standards in Korea", *Proceedings of the XVIIth* IMEKO World Congress *Metrology in the 3rd Millenium*, Dubrovnik/Croatia, 2003, pp. 393-396.

[2] Stefanescu D. M. and Kang D.-I., "Selection Criteria for the Elastic Elements of Very Large Force Transducers", *Proceedings of the 6th Asia-Pacific Symposium on Measurement of Mass, Force and Torque* (Editors Y. Zhang et al), Shanghai/China, APMF 2003, pp. 95-100.

[3] Stefanescu D. M., "FEM and strain gauge analyses for axisymmetric elastic elements of force transducers*", ACTA 5th Asia - Pacific Symposium Measurement of Force, Mass and Torque* (Editors T. Tojo and K. Ohgushi), Tsukuba/Japan, 2000, pp. 85-90.

[4] Allgeier Th., Gassmann H. and Sawla A., "Load application and measuring behaviour of a bending ring force transfer standard", *Proceedings of the IMEKO 16th TC3 in Parallel with APMF'98 Conference Force, Mass and Torque Measurements - Theory and Practice* (Editor M. S. Chung), Taejon/Korea, 1998, pp. 112-117.

[5] Malikov G.F., Sneiderman A.I. and Sulemovici A.M., Rasciotî uprughih tenzometriceskih elementov, Moskva, Izdat Masinostroienie, 1964.

[6] Sandu M. and Sandu A., Captoare cu traductoare rezistive – proiectare si aplicatii (Romanian), Bucuresti, Printech Publishing House, 1999.

[7] *** Introduction to ANSYS (UL registered ISO 9001:1994 Company), Training stage. Tae Sung Software & Engineering, Inc., Seoul, Republic of Korea, 18-22 August 2003.

[8] Stefanescu D. M., Dolga L. and Korsten M., "About the Parametrical Design of Strain Gauged Pressure and/or Force Transducers", *Proceedings of the 7th International Research/Expert Conference "Trends in the Development of Machinery and Associated Technology"*, Lloret de Mar/Spain, TMT 2003, pp. 617-620.

[9] Baumgartner A, Harzheim L. and Mattheck C., "SKO (soft kill option): the biological way to find an optimum structure topology", *International Journal of Fatigue*, **14**, 1992, pp. 387-393.

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