MAGNETIC PROPERTIES OF OIML SHAPED WEIGHTS

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ABSTRACT

The work presents an improved approach to the determination of magnetic properties of weights, which have a shape according to OIML R111. The main stress is laid on a measurement of the volume magnetic susceptibility and an evaluation of its uncertainty. The method is presented, how the volume magnetic susceptibility of the OIML weight can be determined taking into account an actual shape of the weight instead of using only an approximation of its shape with "outer" and "inner" cylinders. Monte Carlo simulation was used to evaluate the measurement uncertainty. The susceptibility of 1 kg weight was calculated by the method of "outer" and "inner" cylinders and the method presented in the article. Results of both methods are compared and discussed.

1. INTRODUCTION

Magnetic interaction forces may lead to an error in the process of weight calibration, because they cannot be distinguished from other forces, which affect weighing. In order to quantify such problems it is necessary to characterise magnetic properties of weights.

In Appendix C of [1] calculation of the susceptibility for the OIML weight is presented, which is based on an approximation of the shape of the weights with "outer" and "inner" cylinders and a method of the superposition (Davis's method). The final result is the interval where the actual value of the susceptibility can be found. In some cases this interval can be several times larger than the measurement uncertainty of the susceptibility. The interval is especially large if the OIML weight is turned upside down.

This work discusses theoretical and practical aspects of the determination of the volume magnetic susceptibility of the weights with a shape according OIML recommendation R111 [2] and when their actual shape is taken into account. The theory of BIPM susceptometer was adopted according to [1, 3, 4] and a similar susceptometer was constructed at MIRS. Then the mathematical model was developed, which takes into the account actual shape of the weight without approximation of the shape. Measurement uncertainty was evaluated using Monte Carlo simulation (MCS). Results show that the method can significantly improve a confidence in the result of susceptibility measurement. That is especially important when the susceptibility of the weight is near the limit of the maximum susceptibility from OIML R111.

2. MODEL FOR MAGNETIC FORCE IN MASS METROLOGY

In order to determine the magnetic properties of weakly magnetic bodies such as stainless-steel mass standards, the susceptometer was developed by Davis and was described in details in [1, 3], but it is useful to review a basic mathematical model. If a weakly magnetized body is placed in a magnetic field of the susceptometer's permanent magnet, the vertical force on the body is given by

$$F_z = -\mu_0 \chi \frac{1}{2} \int \frac{\partial H^2}{\partial z} \mathrm{d}v - \mu_0 \chi H_{ze} \int \frac{\partial H_z}{\partial z} \mathrm{d}v - \mu_0 M_z \int \frac{\partial H_z}{\partial z} \mathrm{d}v , \qquad (1)$$

where μ_0 is the magnetic constant ($4\pi \cdot 10^{-7}$ N/A^2), χ is the volume magnetic susceptibility of the body, H is the field strength in A/m due to the magnet, H_{ze} is the field strength of ambient (e.g. earth magnetic field), H_z is the vertical component of H, M_z is the vertical magnetization of the body and the integrals are taken over its volume. To make the calculations manageable, the forces between the magnet and the body are calculated under several simplifying assumptions [1, 3].

Values of the susceptibility and the permanent magnetization cannot be determined only from one measurement of the magnetic force and use of (1). The measurement of the magnetic force shall be repeated when the magnet is reversed, where the first term of (1) does not change a sign while the second and the third do. The first term of (1) can be calculated from the first and the second measurement of the force as $F_a = (F_{1z} + F_{2z})/2$ and from the force F_a the susceptibility of the body can be calculated. With a known value of the magnitude m of the magnetic moment $\boldsymbol{m}$ of the permanent magnet and the assumption that it behaves as a point dipole [1, 3, 4], where the magnetic field of the magnetic dipole moment can be found in [5], allows us to write the next force F_a:

$$F_a = \frac{3\mu_0 \chi\ m^2}{4\pi^2} \int_V \frac{z^3}{(r^2 + z^2)^5} \mathrm{d}v = \frac{3\mu_0 \chi\ m^2}{64\pi} G\,, \tag{2}$$

where the G is geometrical term. The susceptibility of the body of an arbitrary shape can be calculated from (2). A calculation of the susceptibility of the sample of cylindrical shape, which can be obtained analytically, is shown in [1, 3]. The next section shows how the calculation of the susceptibility for the OIML weights is handled. The method can be easily used also for calculation of the susceptibility of the OIML weight, which is turned upside down.

3. CALCULATIONS FOR THE OIML SHAPED WEIGHTS

In order to find the force F_a according to (2), the OIML weight was divided into six significant parts, as shown on the right side of Figure 1.

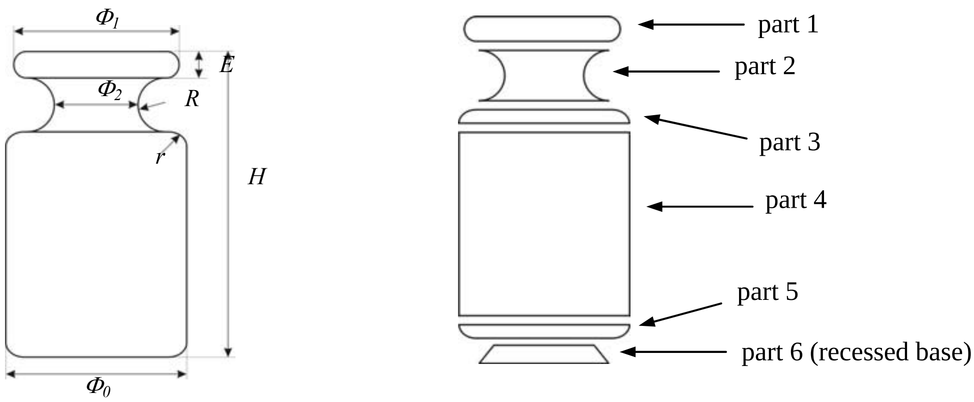

Figure 1: OIML weight with its characteristic dimensions (left) and divided into six significant geometrical parts (right)

For each part of the weight the force $F_{a,i}$ was determined according (2), where the index i refers to the geometrical parts as shown on Figure 1. With the method of superposition, force F_a can be written as

$$F_a = F_{a,1} + \ldots + F_{a,5} - F_{a,6} = \frac{3\mu_0 \chi\ m^2}{64\pi} (G_1 + \ldots + G_5 - G_6)\,. \tag{3}$$

To determine the force $F_{a,i}$ and the geometrical term G_i for each part of the weight, the parts are treated as individual bodies placed into the magnetic field of the permanent magnet of the susceptometer as shown on Figure 2. Only parts 1, 2 and 6 are shown on Figure 2, because parts 3 and 5 are half parts of part 1. Part 4 has a shape of the cylinder and the calculations can be found in [1, 3].

A body with the shape of part 1 is placed into the magnetic field of the magnetic point dipole moment m as shown on the left side of Figure 2. For the calculation of the force $F_{a,1}$ (2) is used, but rewritten as

$$F_a = \frac{3\mu_0\chi\, m^2}{4\pi^2}\int_0^{2\pi}\int_{z_1-R_1}^{z_1+R_1}\int_0^{r_1}\frac{z^3}{(r^2+z^2)^5}\, r\,\mathrm{d}r\,\mathrm{d}z\,\mathrm{d}\varphi = \frac{3\mu_0\chi\, m^2}{64\pi}G_1 . \qquad (4)$$

According to Figure 2, the geometrical term G_1 of (4) equals

$$G_1 = 4\int_{z_1-R_1}^{z_1+R_1}\left[\frac{1}{z^5}-\frac{z^3}{\left(z^2+(\phi_1-R_1+\sqrt{R_1^{\,2}-(z_1-z)^2})^2\right)^4}\right]\mathrm{d}z . \qquad (5)$$

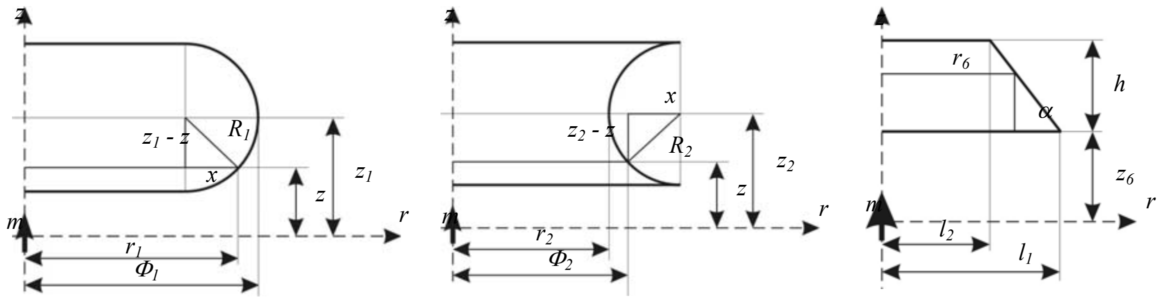

Figure 2: Part 1, 2 an 6 of the OIML weight placed into magnetic field of magnetic point dipole moment m

The same procedure is used for parts 2 and 6. According to Figure 2 and with a change of the borders of the integration with r in (4), the geometrical terms G_2 and G_6 are equal:

$$G_2 = 4\int_{z_2-R_2}^{z_2+R_2}\left[\frac{1}{z^5}-\frac{z^3}{\left(z^2+(\phi_2+R_2-\sqrt{R_2^{\,2}-(z_2-z)^2})^2\right)^4}\right]\mathrm{d}z \qquad (6)$$

and

$$G_6 = 4\int_{z_6}^{z_6+h}\left[\frac{1}{z^5}-\frac{z^3}{\left(z^2+(l_1-\frac{(z-z_6)(l_1-l_2)}{h})^2\right)^4}\right]\mathrm{d}z . \qquad (7)$$

An integration of geometrical terms of parts 1, 2 and 6 in (5), (6) and (7) and consequently of geometrical terms of parts 3 and 5 cannot be solved analytically but only numerically. A numerical integration with Simpson's integration formula was used [6].

It is useful to review the principle of calculation of the susceptibility presented in Appendix C of [1]. The calculation is based on an approximation of the volume of the weights with "outer" and "inner" cylinders and the method of superposition. The result is an interval where the actual value of susceptibility can be found. Shown approximation is applicable only to a normal position of weight on the susceptometer and the approximation for weight turned upside down is shown on Figure 3.

A principle of the calculation is similar to those stated in Appendix C of [1] and it is not repeated.

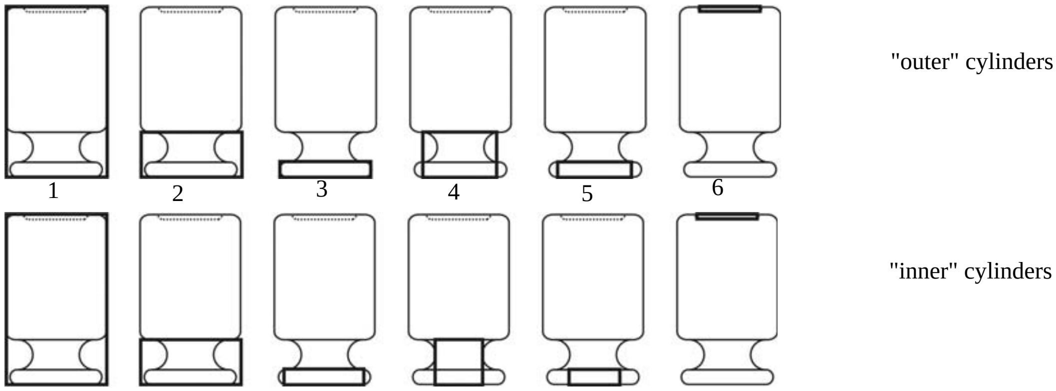

Figure 3: Outer and inner cylinders used to approximate OIML weight turned upside down

4. MODES OF OPERATION OF THE SUSCEPTOMETER AND ITS TRACEABILITY

The absolute and the comparison method of operation of the susceptometer can be used. For both methods traceable measurements are established: the magnitude m of the magnetic moment is determined as described in [1, 4] and calibrated in the laboratory, distance between the permanent magnet and the weight and outer dimensions of the weight were measured with Vernier calliper traceable to National Length Laboratory, forces are calculated from the balance readings, traceable to National Mass Laboratory, multiplied by a value of local gravitational acceleration.

Using the absolute method, the unknown susceptibility is determined directly from (3). Using the comparison method, the reference magnetic susceptibility standard, calibrated at NPL, is used. Its dimensions were measured with Vernier calliper. Unknown susceptibility is determined by the measurement of the weight and the reference standard and use of the following equation:

$$\chi(x) = \chi(s)\frac{F_a(x)\cdot G(s)}{F_a(s)\cdot G(x)}, \qquad (8)$$

where x relates to the weight of unknown susceptibility and s to the reference standard. When comparison method is used, the distance between the permanent magnet and the weight is calculated from results of measurement with the reference standard [4].

5. MEASUREMENT UNCERTAINTY

The Guide to the Expression of Uncertainty in Measurement (GUM) [7] is widely used and accepted for an estimation of measurement uncertainty. However it cannot be practically applied to estimate the uncertainty of susceptibility based on the model described above. Therefore numerical approach based on Monte Carlo simulation (MCS) was used, where only evaluation of the model is performed, without calculation of rather difficult partial derivatives of the model. An evaluation of the uncertainties according MCS and its operations can be found in [8, 9]. MCS can be applied more generally then GUM, because GUM has some limitations as presented in [8, 9].

6. RESULTS

The susceptometer of MIRS consists of Metler Toledo AT 261 balance with *Max* = 62 g / 210 g and a readability d = 10 μg / 100 μg. The permanent magnet on the balance is a neodymium-iron-boron cylinder (Vacuumschmeltz, type 335HR). The reference standard is a low permeability reference material of cylindrical shape produced and calibrated at NPL at magnetic field strength 5 kA/m. Sample weight was 1 kg OIML weight of class E2. Table 1 shows relevant experimental data with their standard uncertainties (k = 1) for calculating the susceptibility of the OIML weight and its uncertainty.

Table 1: Experimental data and their standard uncertainties used for calculation of the susceptibility of 1 kg OIML weight

	estimation	standard uncertainty
Φ_0	48,00 mm	± 0,10 mm
Φ_1	43,00 mm	± 0,10 mm
Φ_2	27,00 mm	± 0,15 mm
H	81,33 mm	± 0,02 mm
E	8,00 mm	± 0,15 mm
R	7,00 mm	± 0,15 mm
r	2,00 mm	± 0,10 mm
m_{ax}	2,145 mg	± 0,040 mg
g	9,806 m/s^2	± 0,001 m/s^2

	estimation	standard uncertainty
m	0,08533 Am^2	± 0,00075 Am^2
l_1	36,36 mm	± 0,02 mm
l_2	27,62 mm	± 0,02 mm
h	1,03 mm	± 0,02 mm
χ_s	0,0272	± (0,025 % · χ_s / 2)
m_{as}	6,509 mg	± 0,040 mg
h_s	81,33 mm	± 0,004 mm
d_s	81,33 mm	± 0,004 mm

The data from Table 1 were used also for the calculation of the susceptibility of the weight turned upside down. Table 2 shows results for susceptibility of 1 kg OIML weight class E2, measured according comparison method, outlined in Section 4, which gives smaller uncertainties then absolute method. First two rows of Table 2 show values of the susceptibility determined according approximation of weight with Davis's method for inner and outer cylinders (χ_{out} and χ_{inn}) and the last row shows result of the susceptibility χ obtained by the method presented in the article.

Table 2: Susceptibility of 1 kg OIML weight

normal position		
	estimation	standard uncertainty
χ_{out}	0,01092	± 0,00022
χ_{inn}	0,01116	± 0,00022
χ	0,01107	± 0,00022

upside down position		
	estimation	standard uncertainty
χ_{out}	0,00873	± 0,00018
χ_{inn}	0,00945	± 0,00018
χ	0,00904	± 0,00018

In addition, next theoretical test were performed. With all experimental data from Table 1 unchanged, except the balance reading m_{ax}, susceptibilities χ_{out} and χ_{inn} from Table 2 where recalculated. Balance reading m_{ax} has changed from zero up to values where the calculated susceptibilities were 0,12, what is the upper limit of the model as stated in draft of OIML R111. Then differences between χ_{out} and χ_{inn} were calculated. These differences and uncertainties of susceptibilities for a normal and upside down position of the OIML weight are shown on Figure 4.

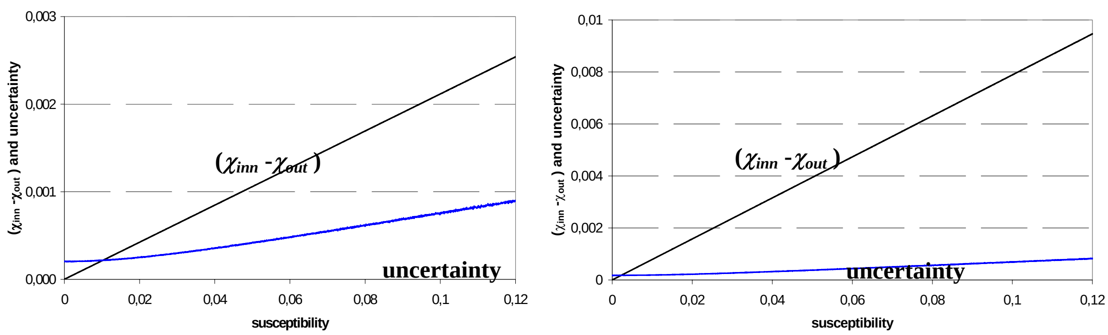

Figure 4: Differences between χ_{inn} and χ_{out} and uncertainties for normal and upside down position of OIML weight

7. CONCLUSION

The susceptibility χ calculated with the method presented in the article actually lays on interval between χ_{out} and χ_{inn}. For presented experimental results and the normal position of OIML weight on the susceptometer the difference between χ_{out} and χ_{inn} is little larger than the value of estimated uncertainty, and in the case of the OIML weight turned upside down the difference between χ_{out} and χ_{inn} is more than four times value of stated uncertainty. If the susceptibilities of OIML weights are under the maximum susceptibilities by the OIML R111 or we do not need higher accuracy, then the determination of susceptibilities with approximation of its shape with "outer" and "inner" cylinders is sufficient. With presented method better accuracy can be achieved, and if the susceptibility is at the limit of the maximum susceptibilities by the OIML R111, we have more suitable decision.

It can be also seen from Figure 4 that the difference between χ_{out} and χ_{inn} increases faster than their uncertainties. This is especially obvious for the OIML weight turned upside down. The differences between χ_{out} and χ_{inn} become more evident at larger values of uncertainties, while at low value of susceptibility the differences are smaller than their uncertainties.

Applying GUM to presented model or similar pretentious models can be difficult because of an algebraic complexity of the partial derivates, which are needed to obtain sensitivity coefficients. In other hand, uncertainty of susceptibilities obtained by Davis's method can also be estimated with MCS.

REFERENCES

[1] R. S. Davis: *Determing the magnetic properties of 1 kg mass standards*, Journal of Research of the National Institute of Standards and Technology 100 (1995),pp. 209 – 225.

[2] *OIML R 111, Weights of classes E1, E2, F1, F2, M1, M2, M3*, International recommendation, OIML, Paris, 1994.

[3] R. S. Davis, *New method to measure magnetic susceptibility*, Measurement Science and Technology 4 (1993),pp. 141 – 147.

[4] J. W. Chung, J. W. Ryu, R. S. Davis, *Uncertainty analysis of the BIPM susceptometer*, Metrologia 38, 2001.

[5] A. R. Sinigoj, *Osnove elektrotehnike*, Fakulteta za elektrotehniko, Ljubljana 2002.

[6] Z. Bohte, *Numerične metode*, Društvo matematikov, fizikov in astronomov Slovenije, Ljubljana 1991.

[7] *Guide to the Expression of uncertainty in measurement*, International Organization for Standardization, Switzerland, 1995.

[8] M. G. Cox, M. P. Harris, *Measurement uncertainty and the propagation of distuributions*, NPL, 2001, downloaded at http://www.npl.co.uk/ssfm/

[9] M. G. Cox, M. P. Harris, *GUM Supplements*, NPL, 2001, downloaded at http://www.npl.co.uk/ssfm/

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