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Mass dissemination using a fully automated mass comparator robot system

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Abstract

We used a new mass comparator robot system which covers 6 decades from 1kg down to 1mg in a single system to calibrate an entire 1kg-1mg OIML (5, 2, 2*, 1) weight set against one 1kg reference by subdivision in less than 1.5 days using a fully automatic robot system with no operator action except loading the magazine at begin.

Due to the possibility to perform subdivision comparisons of one weight against combinations of up to four other weights we don't need the "5" check standards. We use one additional "1" check standard and 8 equations for the 5 unknown in each decade. The mean values of the weighing differences of 6 ABBA cycles are used for the adjustment calculation. A detailed description of the weighing design and the adjustment calculation is given.

1. Introduction

The dissemination of the mass scale for E1 weights using a single 1kg reference weight requires mass comparisons between weights and a group of weights (see [2,3,4,7]). If using a robot system this could be done fully automatically. With the Sartorius CCR-10-1000 all necessary weighing equations over six decades from 1kg down to 1mg can be measured. For the weights between 1kg and 10g a 1kg/1µg mass comparator is used. For weights between 10g and 1mg a 10g/0.1µg mass comparator is used. Combinations of weights for the subdivision equations of up to 4 weights are possible and are taken from and loaded to each mass comparator grouped together with a special large gripper. The other gripper is used for the reference weights and can carry one weight or a combination of two weights.

2. Calculations

Due to the possibility to perform subdivision comparisons of one or two weights against combinations of up to four other weights we don't need the "5" check standards. The "5" check standards are necessary if the robot system can only handle combinations of 3 weights (see [3]). We use one additional "1" check standard and 8 equations each decade. The mean values of the weighing differences of 6 ABBA cycles are used for the adjustment calculation. We use the following simple weighing design for the first decade. This weighing design will give us no orthogonal covariance matrix (see e.g. [7] for examples of designs with orthogonal covariance matrix). We have chosen this design nevertheless to keep the calculations for the example as easy as possible.

1kg_ref	↔	1kg								
1kg	↔	500g	+	200g	+	200g*	+	100g		
1kg_ref	↔	500g	+	200g	+	200g*			+	100g_check
500g	↔			200g	+	200g*	+	100g		
500g	↔			200g	+	200g*			+	100g_check
200g	↔					200g*				
200g	↔							100g	+	100g_check
200g*	↔							100g	+	100g_check
100g	↔									100g_check

Table 1: Weighing design for the 1kg-100g decade, for a 10, 5, 2, 2*, 1 set, 1kg reference standard and 100g check standard

This gives us the matrix notation:

$$X_1 = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & -1 & -1 & -1 & 0 \\ 0 & 1 & -1 & -1 & -1 & 0 & -1 \\ 0 & 0 & 1 & -1 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 & -1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix}$$

The last weighing equation of the first decade is the first equation of the 100g-10g decade. Also the last two columns of X_1 are identical with the first two columns the next decade. Repeating this for all 6 decades we get a design matrix with 6*8+1 equations (lines). Normally we would use a mass metrology software like e.g. ScalesNet32, but to keep things transparent for the reader we give the MATLAB code [8] here (see figure 2). Starting with the first decade we add the next decade **Xn** to the actual weighing matrix **Xa** and the required zeros around the X**n**. After repeating this 5 times we got the wanted design matrix.

```
X=[
   1 -1  0  0  0  0  0
   1  0 -1 -1 -1 -1  0
   0  1 -1 -1 -1  0 -1
   0  0  1 -1 -1 -1  0
   0  0  1 -1 -1  0 -1
   0  0  0  1 -1  0  0
   0  0  0  1  0 -1 -1
   0  0  0  0  1 -1 -1
   0  0  0  0  0  1 -1];
Xa=X;
Xn=X(2:end,1:end);
for ii=1:5,
    Xa=[[Xa zeros(size(Xa,1),size(Xn,2)-
2)];[zeros(size(Xn,1),size(Xa,2)-2) Xn]];
End
X=Xa;
```

Figure 1: MATLAB code for assembling the six decades design matrix

With the results of the ABBA mass comparisons in Y we solve the system $X\beta=Y$ in least square sense. We introduce the value for the 1kg reference mass in m_r using the Lagrange multiplier method. The complete adjustment could be done by just four lines of MATLAB code:

```
XTX_L=[X'*X [1 zeros(1,size(X,2)-1)]';[1
zeros(1,size(X,2))]]
XT_L=[X' zeros(size(X',1),1); zeros(1,size(X',2)) 1];
L=inv(XTX_L)*XT_L;
beta=L*[Y;mr]
```

Figure 2: MATLAB code for the least square adjustment calculation

3. MEASURING VALUES

To show how the procedure works we used two 1kg-1mg weight sets of class E2. We have no calibration sheet for the sets. Starting the mass code for the four upper decades requires placing the weights of the set to be calibrated, the 1kg reference weight and the 100g, 10g, 1g and 100mg check standards to the magazine places. Also we have to tell the robot at which magazine place which weight is being placed and what kind of mass comparisons in which order to do. We do this in a simple script that is understood by the robot system control software (see figure 3). After doing the mass comparisons for the 10g and 1g decade on the 10g mass comparator we compare the 1kg and 100g decade on the 1kg mass comparator. For this the robot transfers the 10g weights from place G1 to place C3 and from G11 to C4. No operator intervention is needed after the placement of the weights.

```
COMMAND OPERATION=NEW NAME=massscale_4decades PROFILE=E1
USING place=B1  nominal=1000  diameter=48 height=83 shape=knob id=Ref_1kg
USING place=B5  nominal=100   diameter=22 height=39 shape=knob id=100g_check
USING place=C4  nominal=10    diameter=10 height=19 shape=knob id=10g_check
USING place=G11 nominal=10    diameter=10 height=19 shape=knob id=10g_check
USING place=G12 nominal=1     diameter=6  height=4  shape=knob id=1g_check
USING place=G13 nominal=0.100 diameter=10 height=1  shape=wire id=0.1g_check

USING place=A1  nominal=1000  diameter=48 height=83 shape=knob id=Set1_1000g
USING place=A2  nominal=500   diameter=38 height=66 shape=knob id=Set1_500g
USING place=A3  nominal=200   diameter=28 height=48 shape=knob id=Set1_200g
USING place=A4  nominal=200   diameter=28 height=48 shape=knob id=Set1_200g_dot
USING place=A5  nominal=100   diameter=22 height=39 shape=knob id=Set1_100g
USING place=A6  nominal=50    diameter=18 height=29 shape=knob id=Set1_50g
USING place=A7  nominal=20    diameter=13 height=23 shape=knob id=Set1_20g
USING place=C1  nominal=20    diameter=13 height=23 shape=knob id=Set1_20g_dot
USING place=C3  nominal=10    diameter=10 height=19 shape=knob id=Set1_10g
USING place=G1  nominal=10    diameter=10 height=19 shape=knob id=Set1_10g
USING place=G2  nominal=5     diameter=8  height=15 shape=knob id=Set1_5g
USING place=G3  nominal=2     diameter=6  height=11 shape=knob id=Set1_2g
USING place=G4  nominal=2     diameter=6  height=11 shape=knob id=Set1_2g_dot
USING place=G5  nominal=1     diameter=6  height=4  shape=knob id=Set1_1g
USING place=G6  nominal=0.500 diameter=10 height=1  shape=wire id=Set1_0.5g
USING place=G7  nominal=0.200 diameter=10 height=1  shape=wire id=Set1_0.2g
USING place=G8  nominal=0.200 diameter=10 height=1  shape=wire id=Set1_0.2g_bend
USING place=G9  nominal=0.100 diameter=10 height=1  shape=wire id=Set1_0.1g

COMPARE APLACE=G1  BPLACE=G11
COMPARE APLACE=G1  B1PLACE=G2  B2PLACE=G3  B3PLACE=G4  B4PLACE=G5
COMPARE APLACE=C4  B1PLACE=G2  B2PLACE=G3  B3PLACE=G4  B4PLACE=G12
COMPARE APLACE=G2  B1PLACE=G3  B2PLACE=G4  B3PLACE=G5
COMPARE APLACE=G2  B1PLACE=G3  B2PLACE=G4  B3PLACE=G12
COMPARE APLACE=G3  BPLACE=G4
COMPARE APLACE=G3  B1PLACE=G5  B2PLACE=G12
COMPARE APLACE=G4  B1PLACE=G5  B2PLACE=G12
COMPARE APLACE=G5  BPLACE=G12

COMPARE APLACE=G5  B1PLACE=G6  B2PLACE=G7  B3PLACE=G8  B4PLACE=G9
COMPARE APLACE=G12 B1PLACE=G6  B2PLACE=G7  B3PLACE=G8  B4PLACE=G13
COMPARE APLACE=G6  B1PLACE=G7  B2PLACE=G8  B3PLACE=G9
COMPARE APLACE=G6  B1PLACE=G7  B2PLACE=G8  B3PLACE=G13
COMPARE APLACE=G7  BPLACE=G8
COMPARE APLACE=G7  B1PLACE=G9  B2PLACE=G13
COMPARE APLACE=G8  B1PLACE=G9  B2PLACE=G13
```

```
COMPARE APLACE=G9   BPLACE=G13

COMPARE APLACE=A1   BPLACE=B1
COMPARE APLACE=A1   B1PLACE=A2   B2PLACE=A3   B3PLACE=A4   B4PLACE=A5
COMPARE APLACE=B1   B1PLACE=A2   B2PLACE=A3   B3PLACE=A4   B4PLACE=B5
COMPARE APLACE=A2   B1PLACE=A3   B2PLACE=A4   B3PLACE=A5
COMPARE APLACE=A2   B1PLACE=A3   B2PLACE=A4   B3PLACE=B5
COMPARE APLACE=A3   BPLACE=A4
COMPARE APLACE=A3   B1PLACE=A5   B2PLACE=B5
COMPARE APLACE=A4   B1PLACE=A5   B2PLACE=B5
COMPARE APLACE=A5   BPLACE=B5

TRANSFER FROM=G1  TO=C3
TRANSFER FROM=G11 TO=C4

COMPARE APLACE=A5   B1PLACE=A6   B2PLACE=A7   B3PLACE=C1   B4PLACE=C3
COMPARE APLACE=B5   B1PLACE=A6   B2PLACE=A7   B3PLACE=C1   B4PLACE=C4
COMPARE APLACE=A6   B1PLACE=A7   B2PLACE=C1   B3PLACE=C3
COMPARE APLACE=A6   B1PLACE=A7   B2PLACE=C1   B3PLACE=C4
COMPARE APLACE=A7   BPLACE=C1
COMPARE APLACE=A7   B1PLACE=C3   B2PLACE=C4
COMPARE APLACE=C1   B1PLACE=C3   B2PLACE=C4
```

Figure 3: Script code for the robot, for the subdivision of a weight set over 4 decades using a single 1kg reference and 3 additional check standards

4. RESULTS

After the robot finished we get a summary of all performed mass comparisons:

```
000007 20-09-2007 21:24:22   G1            G11          0.00173          0.18
000008 20-09-2007 22:04:50   G1     G3 G4 G2 G5         0.10378          0.25
000009 20-09-2007 22:50:18   G11  G3 G4 G2 G12          0.07718          0.29
000010 20-09-2007 23:35:33   G2        G3 G4 G5         0.03030          0.14
000011 21-09-2007 00:19:26   G2      G3 G4 G12          0.00539          0.23
000012 21-09-2007 01:03:10   G3             G4          0.02113          0.12
000013 21-09-2007 01:43:28   G3         G5 G12          0.02673          0.15
000014 21-09-2007 02:25:32   G4         G5 G12          0.00533          0.06
000015 21-09-2007 03:07:46   G5            G12         -0.02410          0.07
.
```

Figure 4: Summary result file from the robot system control software

For each mass comparison one single line is given in this summary stating the mean value and the standard deviation of all performed mass comparisons with 6 ABBA cycles. For each of them we get also a detailed result with all the readings, including the climate data for the air buoyancy correction. For the 10g vs. 10g comparison using magazine places G1 and G11 we have e.g. (see next page). A complete 6 ABBA cycle mass comparison on the 10g mass comparator takes approximately 40 minutes including gathering and bringing back the weights. The time is the same if we compare 1 weight against one or against a combination of up to four weights because we collect the weights prior to the mass comparisons at two special grippers. On the 1kg mass comparator a complete 6 ABBA cycle mass comparison takes 60 minutes. The reason for this is that we use there a Centermatic to have the centre of gravity perfectly in the middle and thus no corner load error. With 40 minutes for each of the 17 equations for the two decades on the 10g comparator and 60 minutes for each of the 16 equations on the 1kg mass comparator the 4 decades calibration procedure takes 27.3 hours. A complete calibration of 6 decades would take 38 hours. This is faster than the 61.5 hours given in [3]. Furthermore we need no manual operators work except placing the weights into the magazines because the robot covers 6 decades in a single system and is able to transfer the 10g weights from one mass comparator to the other and link the decades together.

Mass comparison between position A = Pos G1 and B = Pos G11. 6 ABBA cycles

```
                       A      Std.Dev.   B     Std.Dev.   B     Std.Dev.   A     Std.Dev. 2nd Diff.  T1       T2        Hum1       Hum2       P        air-
                     in mg    in µg    in mg   in µg    in mg   in µg    in mg   in µg    in mg    in °C    in °C      in %       in %     in kPa    density
                    -------------------------------------------------------------------------------------------------------------------------------------------------
20-09-2007 21:36:28 2826.28383 (0.23) 2826.28748 (0.22) 2826.28839 (0.22) 2826.28803 (0.24)  0.00200  22.03 (0.00) 22.24 (0.01) 40.3 (0.1)  40.0 (0.00) 96.59 (0.00) 1.1347
20-09-2007 21:41:40 2826.28882 (0.23) 2826.29139 (0.20) 2826.29218 (0.23) 2826.29125 (0.21)  0.00175  22.02 (0.00) 22.24 (0.01) 40.3 (0.0)  40.0 (0.00) 96.59 (0.00) 1.1347
20-09-2007 21:46:51 2826.29145 (0.19) 2826.29402 (0.22) 2826.29439 (0.19) 2826.29322 (0.19)  0.00187  22.02 (0.00) 22.25 (0.01) 40.3 (0.0)  40.0 (0.00) 96.58 (0.01) 1.1346
20-09-2007 21:52:02 2826.29338 (0.22) 2826.29559 (0.25) 2826.29592 (0.22) 2826.29488 (0.22)  0.00163  22.02 (0.00) 22.24 (0.01) 40.3 (0.0)  40.0 (0.00) 96.59 (0.00) 1.1347
20-09-2007 21:57:13 2826.29476 (0.23) 2826.29699 (0.22) 2826.29695 (0.23) 2826.29622 (0.21)  0.00148  22.02 (0.00) 22.24 (0.00) 40.3 (0.0)  40.0 (0.00) 96.60 (0.00) 1.1349
20-09-2007 22:02:24 2826.29612 (0.21) 2826.29821 (0.24) 2826.29834 (0.19) 2826.29709 (0.22)  0.00168  22.02 (0.00) 22.24 (0.00) 40.3 (0.0)  40.0 (0.00) 96.60 (0.00) 1.1349
                    -------------------------------------------------------------------------------------------------------------------------------------------------
                                  Result:                                       Mean    =    0.00173 mg
                                                        Std.Dev. =     0.18 µg
```

Figure 5: Sample result file from the robot system control software

	Δm in mg	Std. dev. in µg		Δm in mg	Std. dev. in µg
A1 vs. B1	-0.56839	1.20	G1 vs. G11	0.00173	0.18
A1 vs. A3+A4+A2+A5	0.18075	4.10	G1 vs. G3+G4+G2+G5	0.10378	0.25
B1 vs. A3+A4+A2+B5	0.78522	8.03	G11 vs. G3+G4+G2+G12	0.07718	0.29
A2 vs. A3+A4+A5	-0.03935	7.62	G2 vs. G3+G4+G5	0.03030	0.14
A2 vs. A3+A4+B5	-0.00561	5.00	G2 vs. G3+G4+G12	0.00539	0.23
A3 vs. A4	0.01192	1.61	G3 vs. G4	0.02113	0.12
A3 vs. A5+B5	0.04899	1.11	G3 vs. G5+G12	0.02673	0.15
A4 vs. A5+B5	0.03317	2.79	G4 vs. G5+G12	0.00533	0.06
A5 vs. B5	0.03703	2.08	G5 vs. G12	-0.02410	0.07
A5 vs. A7+C1+A6+C3	-0.01399	2.62	G5 vs. G7+G8+G6+G9	-0.01184	0.06
B5 vs. A7+C1+A6+C4	-0.04733	2.75	G12 vs. G7+G8+G6+G13	0.01299	0.11
A6 vs. A7+C1+C3	0.00100	1.37	G6 vs. G7+G8+G9	0.02223	0.08
A6 vs. A7+C1+C4	0.00296	0.59	G6 vs. G7+G8+G13	0.02331	0.14
A7 vs. C1	-0.00190	1.51	G7 vs. G8	-0.00289	0.11
A7 vs. C3+C4	-0.05368	1.04	G7 vs. G9+G13	-0.00114	0.06
C1 vs. C3+C4	-0.05299	0.69	G8 vs. G9+G13	0.00157	0.05
			G9 vs. G13	0.00096	0.04

Table 2: Measured mass differences for the four decades

Performing the least square adjustment using the Matlab instructions shown in figure 2 we get values for all the weights in the set and the check standards. This is the desired result if we are doing a calibration. In our example we assume the mass of the 1kg reference to be exactly 1000g.

Table 3
Results for the four decades, using values given in table 2 and a hypothetical 1000g reference for the least square adjustment

B1	Ref_1kg	1000.0000000g	G11	10g_check	10.0000198g
A1	Set1_1000g	1000.0005685g	G2	Set1_5g	4.9999741g
A2	Set1_500g	499.9998894g	G3	Set1_2g	1.9999963g
A3	Set1_200g	199.9999796g	G4	Set1_2g_dot	1.9999751g
A4	Set1_200g_dot	199.9999664g	G5	Set1_1g	0.9999727g
A5	Set1_100g	99.9999839g	G12	1g_check	0.9999970g
B5	100g_check	99.9999480g	G6	Set1_0.5g	0.5000035g
A6	Set1_50g	49.9999993g	G7	Set1_0.2g	0.1999909g
A7	Set1_20g	19.9999875g	G8	Set1_0.2g_bend	0.1999938g
C1	Set1_20g_dot	19.9999890g	G9	Set1_0.1g	0.0999965g
G1	Set1_10g	10.0000218g	G13	0.1g_check	0.0999956g

For this paper we are looking for the residuals of the least square adjustment. We see in figure 6 the standard deviations for all mass comparison carried out. This is shown over the nominal load. Further the residuals after the adjustment *e=β*X-Y* is given with blue circles. We see in the two decades from 1kg down to 10g the residuals are decreasing with the load and well below 3µg. For the lower decades the mass comparisons are carried out on the 10g/0.1µg mass comparator. The residuals are also decreasing with the nominal load and below 0.3µg (see figure 7).

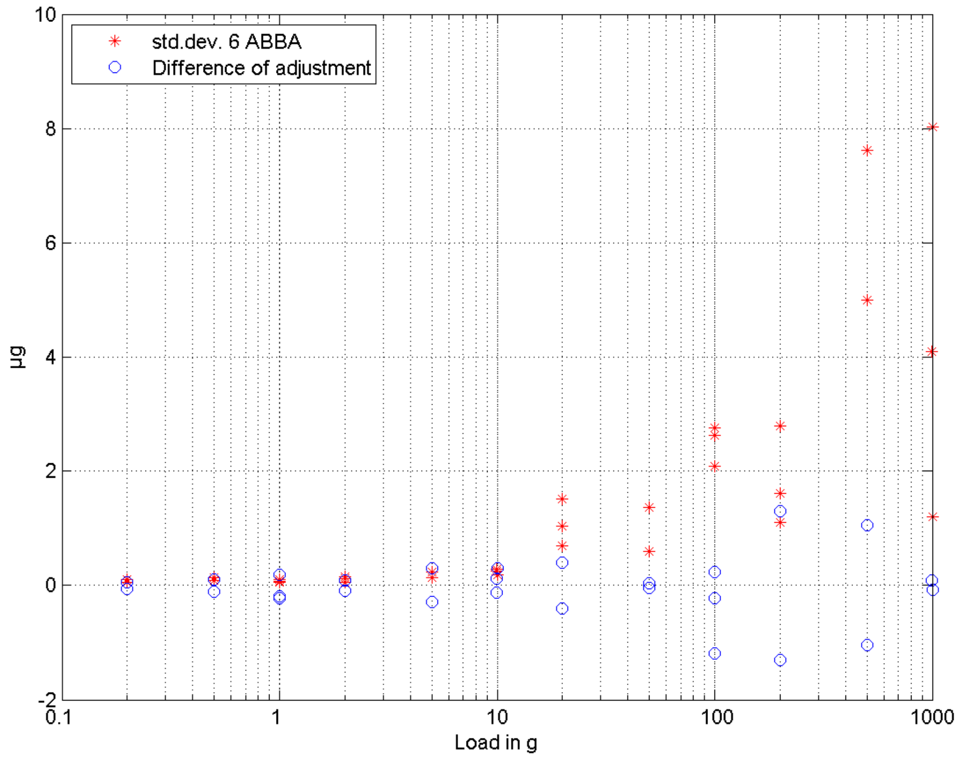

Figure 6: Standard deviations and residuals of the least square adjustment

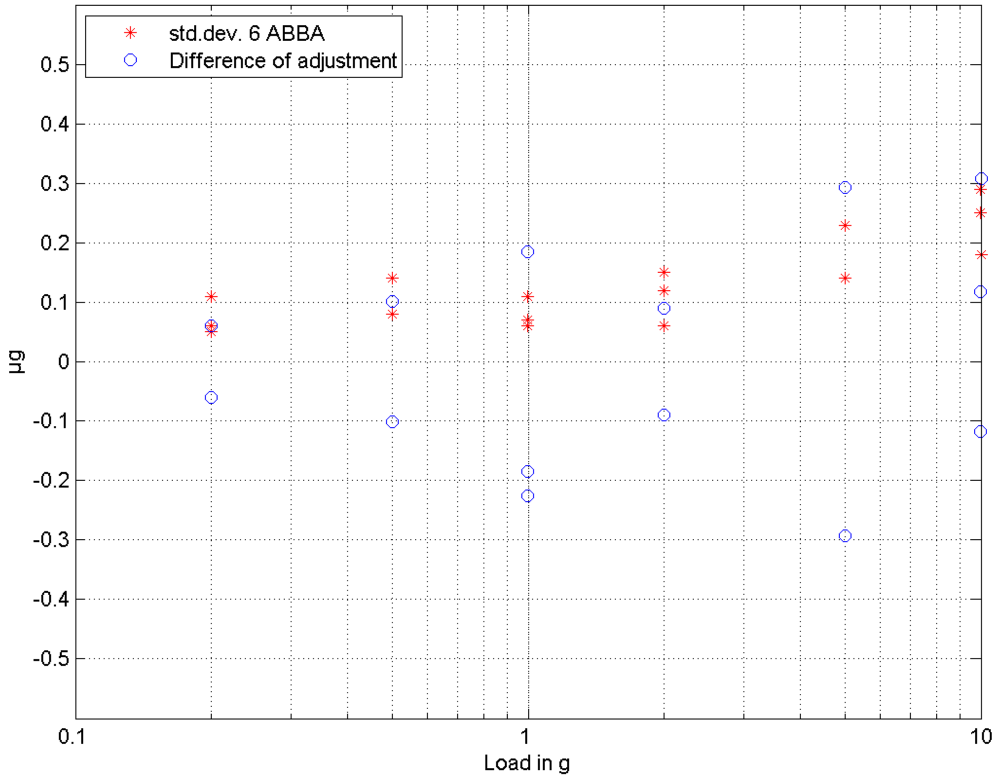

Figure 7: Standard deviations and residuals of the least square adjustment, for the lower decades

7. SUMMARY

It is possible to calibrate an entire 1kg-1mg OIML (5, 2, 2*, 1) weight set against one 1kg reference by subdivision with used new fully automatic mass comparator robot system. This can be done easy and fast. It is sufficient to use only one additional "1" check standard and 8 equations in each decade.

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