IMEKO 24th TC3, 14th TC5, 6th TC16 and 5th TC22 International Conference
11 – 13 October 2022, Cavtat-Dubrovnik, Croatia

ENHANCED METHODS FOR CALCULATING UNCERTAINTY ENVELOPE FUNCTIONS FOR FORCE RANGES

D. Röske

Physikalisch-Technische Bundesanstalt (PTB), Braunschweig, Germany, dirk.roeske@ptb.de

Abstract:
In static force and torque calibrations, the quantity is applied to the given measuring instrument in discrete steps to form a measurement series that usually consists of five to ten values. The corresponding calibration results and uncertainties are then acquired through calculation. During normal use of the instrument, the values measured are often distributed within a range and do not align with the calibrated values. A fitting function is then applied to the result that allows the mechanical quantity to be calculated from the indicated deflection value. The uncertainty associated with this calculation must likewise be calculated using a function defined over the range. This article presents enhanced methods for calculating this function.

Keywords: static force; static torque; discrete calibration; continuous application; uncertainty function

1. INTRODUCTION

Static force and torque calibrations are often carried out according to the procedures described, for example, in ISO 376 [1] for force and DIN 51309 [2] or EURAMET cg-14 [3] for torque. During such a calibration, discrete values ('steps') of the quantity are applied to the measuring instrument under test to form a measurement series normally consisting of five to ten values. Some series are repeated under unchanging conditions while others are done with the instrument placed in different rotational positions. Both types of series may include only incremental steps or both incremental and decremental steps. The results and the associated uncertainties are calculated for each step following the guidance of the above-mentioned standards. In the end, fitting functions are determined that depict the functional dependence of the deflection X, calculated from the measured indications, on the applied force F or, vice versa, the dependence of force F on the measured deflection X. This last function, $F = F(X)$, is necessary so that the instrument can be used even if information about the acting force is lacking and only indicated values are available to determine the (measured) deflection. Both results, $X = X(F)$ and $F = F(X)$, must be provided with the corresponding uncertainties.

The uncertainties are first calculated for every single load step. DIN 51309 offers no further guidance on calculating an uncertainty function over a range. But section C.1.10.2 of ISO 376 contains details on how the fitting function for the range should be determined (it can be applied to torque as well). Linear, polynomial, and exponential functions are recommended as possible function types. The result should not be too optimistic by underestimating the uncertainty; on the other hand, the user of a measuring instrument wants to draw maximum benefit from the instrument's measuring capabilities.

2. ENHANCED CALCULATION METHODS

In this work, enhanced methods for the calculation of the fitting functions are described. The list of possible functions has been extended to include power and logarithmic functions as well as polynomial functions, the latter of which are used for both absolute and relative uncertainties.

These functions are always calculated as envelope functions, meaning that none of the calculated 'step' uncertainties may be larger than the value of the uncertainty function for that step. This is not a requirement prescribed by a guideline but rather a rule applied to our laboratory practices.

Another enhancement involves the way the functions are determined. A value that is a slightly larger than the others in relative terms can be treated as a kind of 'outlier'. As such, it would make sense not to use this value for the calculation of the fitting function. In some cases, this will lead to a more 'natural' function that can then be shifted to include the 'outlier' by adding a constant term. There is much discussion about outliers and how to define and identify them. The method proposed here is especially useful if the 'outliers' are considered normal values that, though slightly larger, show no evidence of a being a true outlier. For this work, a combinatorial approach was chosen in which every single value or group of values is temporarily considered such an outlier. The fitting function is then calculated from the remaining values (those not considered outliers), and the final functions acquired after the shift to include the outliers are compared with one another. When a comparison shows that the residual (the sum of the squared deviations of the function values from the step values) of a function is smaller than that of the former function, then the new function is taken for the next comparison.

This calculation is repeated for the complete set of outlier combinations to ultimately yield the optimum function for the given function type. In a final step, the different function types are compared to see which one best represents the data. This function is given in the calibration certificate.

3. EXAMPLE OF A LINEAR FUNCTION

The principle can be demonstrated using a linear envelope function. Figure 1 shows uncertainty values in a force range obtained in a real measurement. Here and in the following, all expanded uncertainty values are calculated with a coverage factor of $k = 2$.

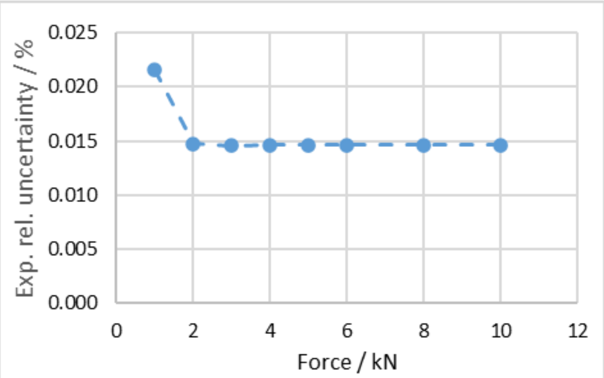

Figure 1: Uncertainty values in a force range

Following the procedure described above, a linear regression function was calculated and is shown in Figure 2.

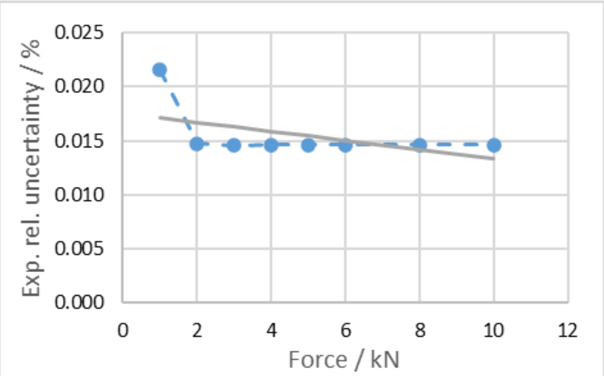

Figure 2: Linear regression function (grey) for the uncertainty values from Figure 1

An envelope function can be found by shifting this function (adding a constant value) until all measured values are ‘below’ the envelope, shown in Figure 3. A good parameter Q for the quality of an envelope function is the square root of the sum of all squared deviations between the function values and the corresponding uncertainty values. This parameter is closely related to the mean squared error (also known as mean squared deviation).

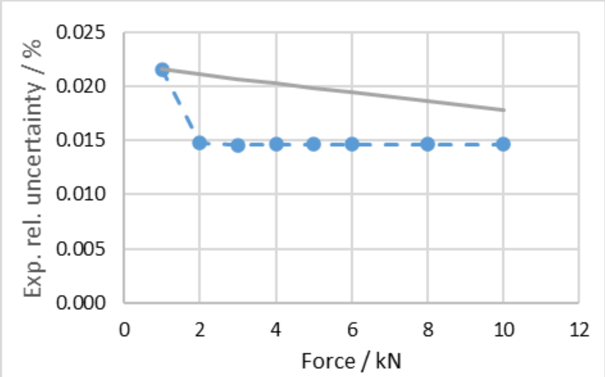

Figure 3: Linear envelope function for the uncertainty values from Figure 1; quality parameter Q = 0.0136 %

Obviously, the function shown in Figure 3 is not optimal. The best envelope function is shown in orange in Figure 4; it is just a straight line connecting the first and the last points.

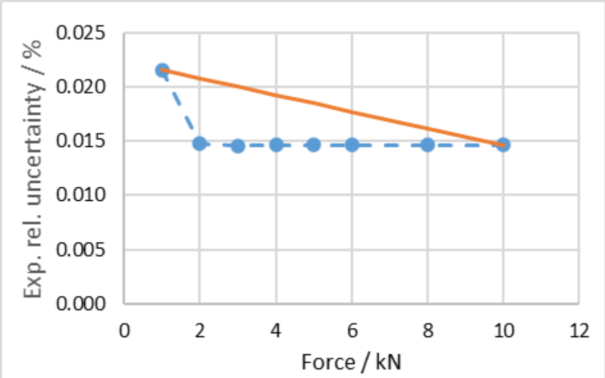

Figure 4: Optimal linear envelope function (orange) for the uncertainty values from Figure 1; Q = 0.0106 %

The described method works well in the case of monotonically falling uncertainty values as shown in Figure 1. An example of a case where the values do not meet this condition is shown in Figure 5. Here, the 8 kN value is slightly larger than the two neighbouring values and a straight line going through the first and the last points would not be an envelope function, see Figure 6.

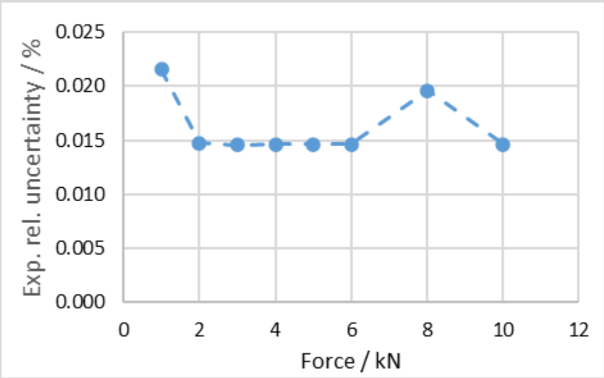

Figure 5: Non-monotonically falling uncertainty values in a force range

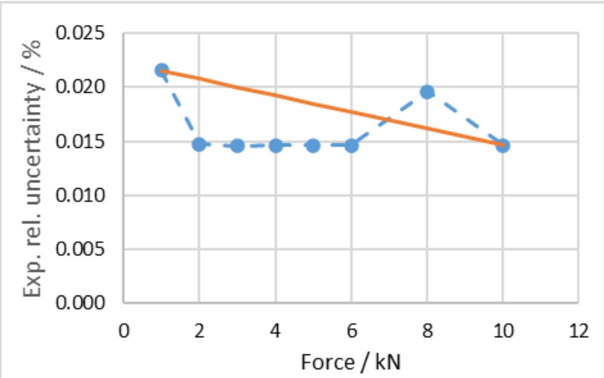

Figure 6: Straight line (orange) connecting the first and the last points in the set of uncertainty values

The problem could be solved either by shifting the line in Figure 6, as seen in Figure 7, or by applying the procedure described at the beginning of this section to calculate the linear regression function from all values and then shifting this function, see Figure 8. This figure reveals that the obtained linear function is again not optimal. The envelope could be improved by connecting the 1 kN and 8 kN points, see Figure 9. A comparison of the quality parameter values Q given in the captions of Figure 7 and Figure 9 shows that these values become smaller in each step.

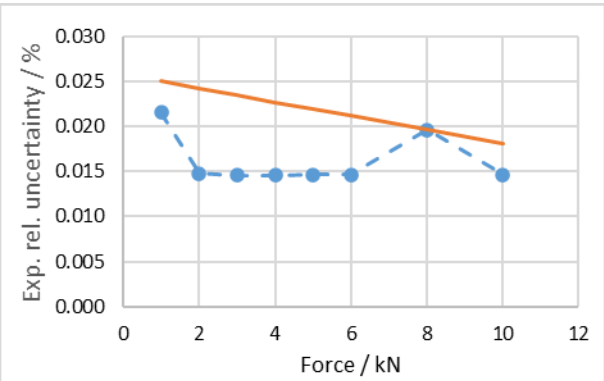

Figure 7: Straight line from Figure 6 shifted to form an envelope (orange); Q = 0.0188 %

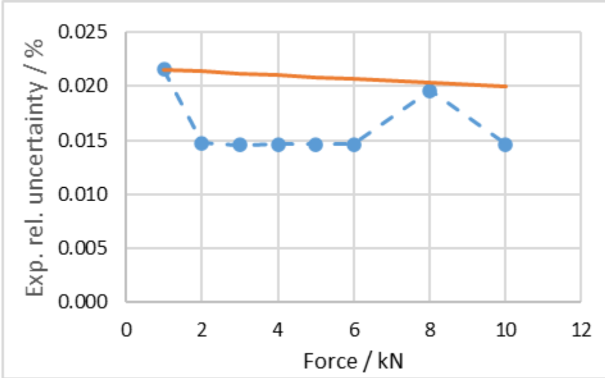

Figure 8: Linear regression function from all uncertainty values shifted to form an envelope (orange); Q = 0.0152 %

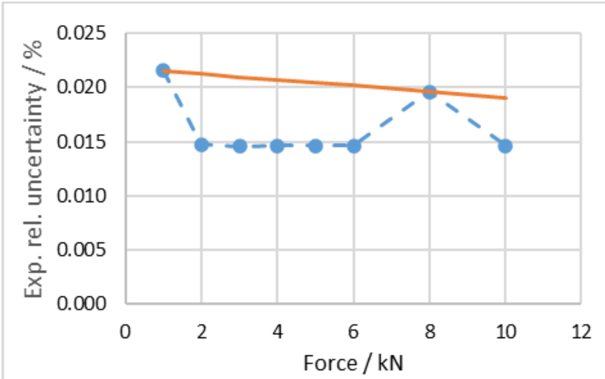

Figure 9: Optimised linear envelope function (orange) for the uncertainty values from Figure 5; Q = 0.0143 %

For an automated evaluation of measurement results, a calculation method is required that determines the best envelope function for all possible cases. Let there be N uncertainty values (in the examples above, $N = 8$). Since at least two points are necessary to calculate a linear regression function, there are $N!/[2!\,(N-2)!]$ combinations of two points that can be selected from a set of N points. But one may also select three, four, or more points – up to a maximum of N points – for the determination of the corresponding regression function. In general, the number of possible combinations of m points in a set of N points (where $m \leq N$) can be calculated from $N!/[m!\,(N-m)!]$. The number of combinations and the cumulative sum for a set of eight points are given in Table 1. It turns out that 247 different functions can be considered in this case. According to the procedure, the regression functions need to be shifted to form an envelope and the parameter Q calculated for each function and compared with that of the other functions to find the best envelope for the given function type.

Table 1: Number of envelope functions for N = 8

Number m of selected points	Number of combinations	Cumulative sum
2	28	28
3	56	84
4	70	154
5	56	210
6	28	238
7	8	246
8	1	247

4. FURTHER FUNCTION TYPES

Linear envelope functions are good for showing and investigating the effects of regression, interpolation and shift. Unfortunately, they are usually not well suited for describing the data shown in Figure 1 and Figure 5. If the residual deviations between envelope function and measured values need to be kept small, then other function types should be used.

As mentioned in section 2, the procedure includes polynomial and exponential functions as well as power and logarithmic functions. In the case of polynomial functions, both the absolute and the relative uncertainties are considered. It must be noted that for polynomial functions of second (quadratic) and third (cubic) order, the number m of selected points must be greater than two, so the total number of calculated functions decreases slightly. In the final evaluation, the best (lowest) values of the quality parameter Q for the different function types are compared and the function with the lowest Q value is chosen as the best envelope function.

5. PRACTICAL REALISATION

The spreadsheet software Excel is used for the calculation of the calibration results in our laboratory. For this reason, the calculation of the uncertainty functions and the determination of the best value of Q for every function type was also realised in Excel using functions programmed in Visual Basic for Applications (VBA). Due to the rather large number of functions – more than 1000 for N = 10 – and the limited speed of VBA, this calculation is not carried out every time the spreadsheet is updated. The calculation usually takes a few seconds, so to ensure fluid work with Excel it is only performed on request, i.e., when a 'Calculate' button is pressed or when the calibration certificate is saved in a PDF file.

For the comparison of the different function types, standard Excel functions are used. The user is shown the results as well as which function type was selected as the optimal envelope function. The main results important to customers are the type and the coefficients of the regression function $F = F(X)$ and the type and coefficients of the uncertainty function $W = W(F)$. Laboratories often use a fixed numbers of decimal places for the coefficients of the two functions, i.e., four for higher order coefficients and six for the linear part. But it can be shown that in some cases, in particular for the higher order coefficients in polynomials, one or two decimal places are enough. How many significant decimals are sufficient is calculated in Excel taking into account the measurement uncertainty of the calibration standard used.

Complementing the work on the functions, we have also changed the form in which they are presented in our calibration certificates. Now we use exponents, set variables in italics, and include all units as well as parentheses where necessary to obtain a more professional representation of the equations, see Figure 10.

Fitting functions X = X(F)

The balanced mean X_a values stated in the table above have been calculated with the following function:

$X_a = [+ 0{,}002000545 \cdot F / \mathrm{kN} + 4 \cdot 10^{-12} \cdot (F / \mathrm{kN})^2 + 3 \cdot 10^{-14} \cdot (F / \mathrm{kN})^3]\ \mathrm{mV/V}$

Considering the reversibility v the following function results for decremental forces:

$X_f = [+ 0{,}002001178 \cdot F / \mathrm{kN} - 5{,}17 \cdot 10^{-10} \cdot (F / \mathrm{kN})^2 - 1{,}29 \cdot 10^{-13} \cdot (F / \mathrm{kN})^3 + 3{,}9 \cdot 10^{-5}]\ \mathrm{mV/V}$

Fitting functions F = F(X)

The calculation of the force F_a related to a measurement signal X is carried out using the function:

$F_a = \{ + 499{,}8639 \cdot X / (\mathrm{mV/V}) - 5 \cdot 10^{-4} \cdot [X / (\mathrm{mV/V})]^2 - 0{,}0019 \cdot [X / (\mathrm{mV/V})]^3 \}\ \mathrm{kN}$

Considering the reversibility v the following function results for decremental forces:

$F_f = \{ + 499{,}7058 \cdot X / (\mathrm{mV/V}) + 0{,}0645 \cdot [X / (\mathrm{mV/V})]^2 + 0{,}0081 \cdot [X / (\mathrm{mV/V})]^3 - 0{,}02 \}\ \mathrm{kN}$

Envelope functions for measurement uncertainties

The expanded relative uncertainties W_{Xa} stated in the table above can be approximated with:

$W_{Xa} = + 0{,}0127 / (F / \mathrm{kN}) + 8{,}13 \cdot 10^{-5} + 1{,}08 \cdot 10^{-7} \cdot F / \mathrm{kN} - 5{,}63 \cdot 10^{-11} \cdot (F / \mathrm{kN})^2$

The expanded relative uncertainties W_{Xv} stated in the table above can be approximated with:

$W_{Xv} = + 0{,}00313 / (F / \mathrm{kN}) + 5{,}92 \cdot 10^{-4} - 7{,}97 \cdot 10^{-7} \cdot F / \mathrm{kN} + 3{,}47 \cdot 10^{-10} \cdot (F / \mathrm{kN})^2$

Figure 10: Examples of fitting and envelope functions from a calibration certificate for a force transducer

6. DISCUSSION

Although the enhanced methods will not transform a bad transducer into a good one, we are certain that they can in some cases deliver improved results. This is especially important when it comes to classification, where strict limits are set on uncertainty contributions. The representation of functions in the calibration certificate is now much clearer and serves to avoid misunderstandings. In the near future, the transmission of results (functions and coefficients) will be one of the essential tasks performed by the coming digital calibration certificates (XML).

7. SUMMARY

In static force or torque calibrations, the calculation of uncertainty functions for a range is not very standardised. This gives researchers the opportunity to develop their own procedures and methods, as was done in this work. Enhanced calculation methods based on a combinatorial approach allow optimised functions to be found.

The result is that users of a calibrated instrument can benefit from the lower uncertainties generated.

8. REFERENCES

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