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CENTRE OF GRAVITY MEASURING DEVICE

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Abstract:
This paper describes a new method to measure the height of the centre of gravity of high-sensitive mass artefacts. A measuring device is presented and tested with mass samples of various shapes and densities. The results of the measurements are compared to the geometrical estimated heights of the centre of gravity and further developments of the new instrument are presented.

Keywords: high precision mass determination; mass artefacts; centre of gravity; mass comparator

1. INTRODUCTION

Mass determinations in the highest accuracy range are performed with high precision vacuum mass comparators. The method of mass determination is still relevant despite the redefinition of mass as of 2019 by the Planck constant [1]. High-end mass comparators have a resolution of 0.1 µg which corresponds to a relative mass deviation of 1×10^{-10} for the comparisons of 1 kg artefacts. Mass samples are often made from stainless steel, platinum-iridium (Pt-Ir), or silicon-28 (^{28}Si) but other materials are conceivable. Of course, the mass samples of different materials have different densities, shapes, and volumes, but this does not affect the measurements because the buoyancy effects are eliminated by using vacuum in the mass comparators. However, the different heights of the centre of gravity do affect the measurement results.

This paper describes a method for measuring the height of the centre of gravity of sensitive mass artefacts that avoids the systematic uncertainty influences due to the different heights of the centre of gravity of the mass artefacts. With this method, mass artefacts with unusual shape, inhomogeneous composition, or even hollow mass artefacts with uncertain internal geometry can be compared to high-precision mass standards by eliminating the systematic uncertainty due to the influence of the height of the centre of gravity.

2. STATE OF THE ART AND FUNDAMENTALS

For high precision mass comparisons, the mass artefacts of 1 kg nominal value are usually compared to mass standards in an evacuated vacuum chamber as part of a mass comparator. In an ABBA comparison, the mass standards or artefacts are placed individually on the mass comparator weighing pan. If the mass samples have different densities, the high vacuum avoids inaccuracies due to buoyancy. However, the closer the mass sample is to the Earth's core, the stronger the gravitational field becomes.

In our laboratory at the Technical University of Ilmenau, we perform experiments with a CCL1007 high-vacuum prototype mass comparator built in collaboration with Sartorius AG and the BIPM (see Figure 1). It provides a resolution of 0.1 µg when comparing 1 kg artefacts. It stands on a heavy weighing stone, at the top of which the site-specific gravitational acceleration was measured. The absolute value of gravitational acceleration $\vec{g}$ and the change of gravitational acceleration per metre of altitude $d\vec{g}/dh$ were determined with the absolute gravimeter A10 [2] and the relative gravimeter CG5 [3] (results in Table 1).

Table 1: Absolute gravitational acceleration and gravitational acceleration gradient at the measurement site

Quantity	Value	Standard deviation	Unit
$\vec{g}$	9.810 159 85	1.1×10^{-7}	$m \cdot s^{-2}$
$\frac{d\vec{g}}{dh}$	-3.153×10^{-6}	1×10^{-8}	$m \cdot s^{-2} \cdot m^{-1}$

With the change of gravitational acceleration per altitude metre determined, it becomes clear that it must be considered when comparing mass artefacts with different heights of the centre of gravity. The relative uncertainty contribution when comparing two mass samples of 1 kg with a 10 mm deviation in the height of the centre of gravity is 3.2×10^{-9}, a factor of 32 greater than the resolution of the mass comparator. The requirements for the centre of gravity measuring device are careful handling of mass artefacts and determination of the centre of gravity height to < 300 µm to reach the resolution limit of the mass comparators.

Figure 1: Prototype vacuum mass comparator CCL1007 in the laboratory at the Technical University of Ilmenau

Figure 2 shows several conventional mass artefacts or mass standards. A 1 kg silicon sphere is shown in the upper left corner, which was commonly used in the Avogadro project [4]. In the lower right corner, a 1 kg cylinder from platinum-iridium alloy (90 % Pt, 10 % Ir) is shown. This is a typical example of the International Prototype of the Kilogram (IPK) [5], which defined the kilogram until the redefinition of the International System of Units (SI) in 2019 [6]. Many official copies were also produced, which served as national standards, for example. The two remaining examples on the top right and bottom left represent common mass standards frequently used in industry and research, depending on the accuracy class [7], as calibration standards or as working standards. They are usually made from stainless steel.

For mass artefacts with very simple shapes, such as cylinders or spheres, the centre of mass can be calculated using the geometric dimensions, provided the composition is homogeneous. For other shapes, starting with button weight or mass artefacts with geometrically indeterminate or even unknown geometry, hollow bodies, or mass artefacts with unknown or inhomogeneous composition, it will be very difficult to determine the centre of gravity. Various methods such as 3D scanning, X-ray inspection, etc. are associated with a high level of effort.

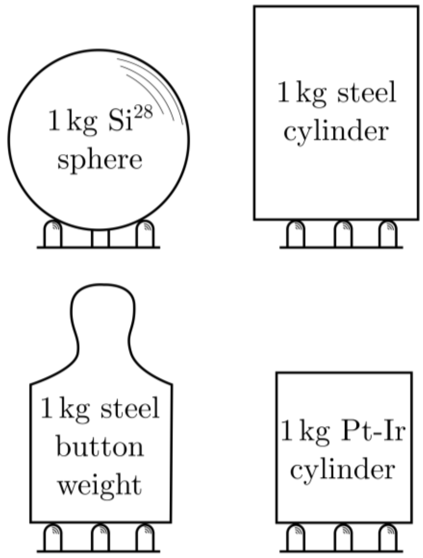

Figure 2: Different kinds of conventional mass artefacts

In the future, it might become more and more interesting to compare mass artefacts with unusual shape, unusual composition or made of new materials. To cover all possibilities, this paper presents a centre-of-mass measuring device designed to reduce the measurement uncertainty when comparing 1 kg artefacts with different centre-of-mass heights.

3. THEORETICAL CONSIDERATIONS

The idea was to develop a device for measuring the height of the centre of gravity of mass artefacts, which has the same contact geometry as the weighing pan of the balance. It consists of a three-point support consisting of three pins arranged at an angle of 3 × 120° on a pitch circle diameter.

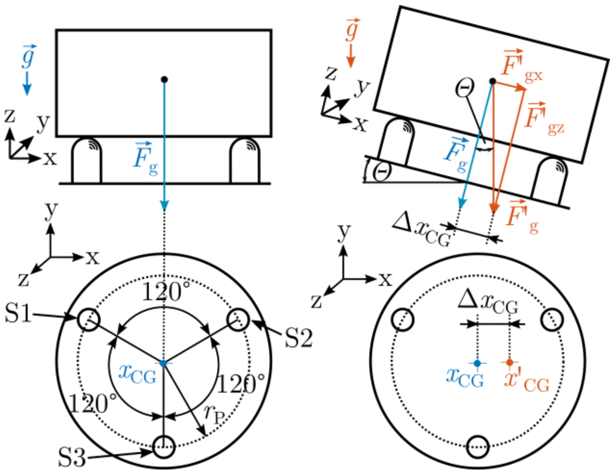

Figure 3: Determination of the height of the centre of gravity of mass artefacts as an example

To measure the height of the centre of mass artefacts, it is necessary to measure the force on each pin. During the force measurements, the base of the three pins needs to be tilted. During the tilting process, the forces acting on the three pins are measured. The measured data can be used to determine the horizontal displacement of the centre of gravity in the $x - y$ plane. Together with the tilt angle Θ and the initial force $F_g = m_s \times g$, the height of the centre of gravity in the z-axis can be calculated, considering the parallelism of $\vec{g}$ and the z-axis (also see Figure 3). Equations (1) to (9) describe the calculation in two 2D-planes and in polar coordinates to account for possible misalignments in the axes of the tilt and the force sensors.

$$x_{CG} = \frac{r_P}{F_g} \sin(60°)(\Delta F_{S2} - \Delta F_{S1}) \tag{1}$$

$$y_{CG} = \frac{r_P}{F_g} (\sin(30°)\,(\Delta F_{S2} + \Delta F_{S1}) - \Delta F_{S3}) \tag{2}$$

with:

$$F_g = \sum_{i=1}^{3} F_{Si} \tag{3}$$

Alternatively described in polar coordinates:

$$r_{CG} = \sqrt{x_{CG}^2 + y_{CG}^2} \tag{4}$$

$$\varphi_{CG} = \arctan\left(\frac{y_{CG}}{x_{CG}}\right) \tag{5}$$

$$\Delta x_{CG} = \frac{r_P}{F_g} \cdot \sin(60°) \cdot (\Delta F_{S2} - \Delta F_{S1}) \tag{6}$$

$$\Delta y_{CG} = \frac{r_P}{F_g} \cdot (\sin(30°) \cdot (\Delta F_{S1} + \Delta F_{S2}) - \Delta F_{S3}) \tag{7}$$

$$\Delta r_{CG} = \sqrt{\Delta x_{CG}^2 + \Delta y_{CG}^2} \tag{8}$$

$$z_{CG} = \frac{\Delta r_{CG}}{\sin(\Delta\Theta)} \tag{9}$$

As can be seen from the equations above, an eccentric position of the mass artefacts on the three-point support is not a constraint for the measurements. Only the forces parallel to the z-axis and the tilt angle are necessary to determine the height of the centre of gravity. With this method, the absolute height of the centre of gravity above the pin tips can be measured.

4. EXPERIMENTAL SETUP

For the first experimental investigation, three 1 kg load cells (like the one shown in Figure 4) were arranged at an angle of 120° (see Figure 5). The load cell transducer is a single pin with a hemispherical tip mounted to each of the load cells.

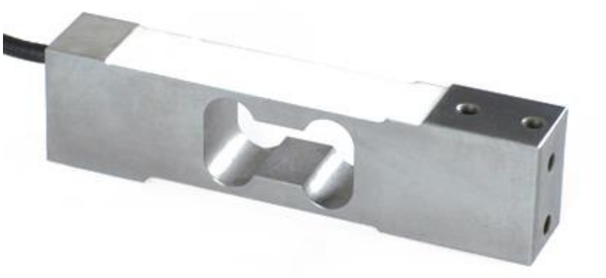

Figure 4: Example of one of the force sensors [8]

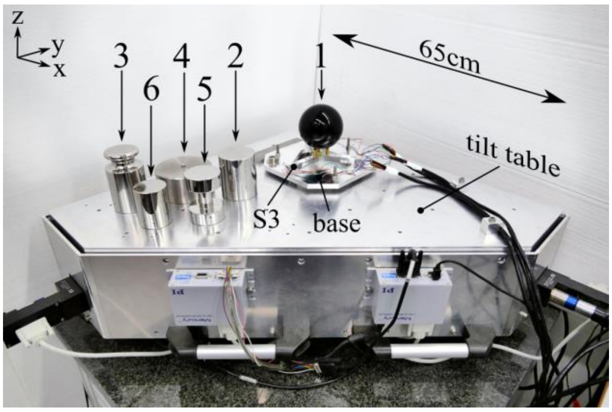

Figure 5: Centre of gravity measuring device with arranged load cells on the dual axis precision tilt stage with various mass samples and a glass sphere in the three-point support

The load cells are calibrated using a glass sphere as it is self-centring in the three hemispherical pin tips. It is ensured that the load acting on each pin is exactly 1/3 of the weight of the sphere. Therefore, the weight of the glass sphere was determined to be 925.189 g with a standard deviation of 0.127 g using a separate balance. Before measurements can begin, the pin tips must be adjusted horizontally. After calibrating the load cells, the base is tilted with a high-precision dual axis tilt stage with a tilt repeatability of less than 0.4 µrad in both axes [9] (also see Figure 5).

The three load cells measure the force exerted by the mass samples on top of the three-points support. Tilting the mass samples by $\Delta\Theta$ = ±15 mrad with the tilt table in each axis changes the eccentricity of the mass samples relative to the three load cells is measured.

Figure 6 shows, as an example from the data analysis, the relative mass change of all three force sensors after tilts according to the specified angles in mrad from equation (10).

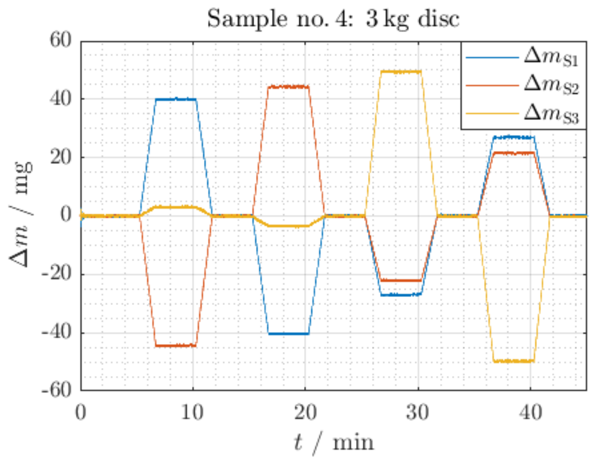

Figure 6: Offset-compensated example raw data from the three force sensors during a measurement process

$$\begin{pmatrix} \Theta \\ \phi \end{pmatrix} = \begin{pmatrix} 0 & 15 & 0 & -15 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 15 & 0 & -15 & 0 \end{pmatrix} \tag{10}$$

For the proof of concept, mass samples of different shapes with known centres of gravity are required. For the investigations, the centres of gravity of the mass samples from Figure 7 were determined geometrically.

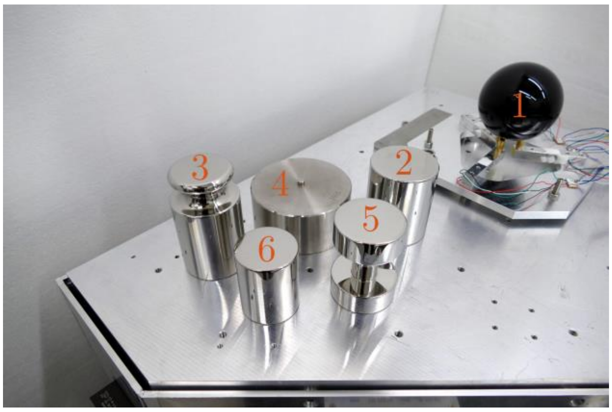

Figure 7: Test artefacts for the comparison of geometrical and measured centre of gravity

The height of the cylinder samples 2 and 6 (cylinder weights) were measured and half of the height was taken as the height of the centre of gravity. The geometries of samples 4 (disc) and 5 (dumbbell weight) were measured and the centre of gravity was calculated. The geometry of sample 3 (button weight) was measured and approximated according to OIML R 111-1 [7]. Based on the measurements, the height of the centre of gravity was calculated using 3D design software. The diameter of sample 1 (sphere) was measured at 80 different spots using an ABBE-comparator and the height of the centre of gravity was taken as the half of the average of the diameter measurements, resulting in a diameter of d_{sphere} = 88.034 1 mm with a standard deviation of 7.16 µm. The results of the geometrical measurements and calculations are shown in column h_{geom} of Table 2 as comparative values for the determined height of the centre of gravity of the measuring device.

For the measurements determining the height of the centre of gravity of the sphere (sample 1), it is not important to know the value of $\Delta z_{\mathrm{sphere}}$, because the geometries of the measuring device are similar to the geometry of the weighing pan in the mass comparator. However, it becomes important for comparisons between the geometrically measured centre of gravity and the height measured by the centre of gravity measuring device.

For the correction of the measured height of the centre of gravity by the measuring device and the geometrical determination of the centre of gravity the immersion $\Delta z_{\mathrm{sphere}}$ of the sphere into the plane, spanned by the three pin tips, can be calculated with the knowledge of the radius of the sphere r_{sphere}, the pitch diameter r_{P} of the pins and the radius of the pin tips r_{pin} according to Figure 8 and equations (11) to (13).

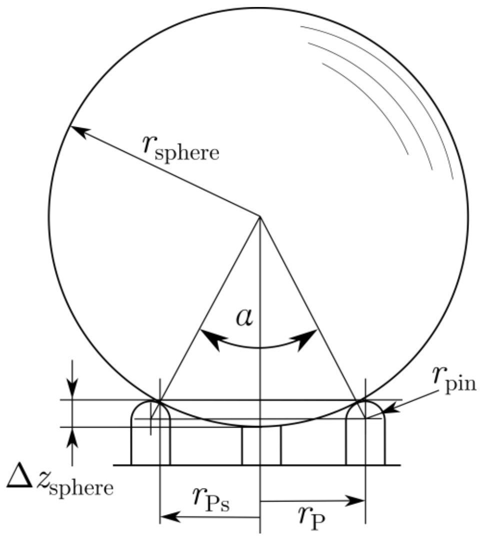

Figure 8: Correction of the measured height of the centre of gravity by geometrical influence

$$\alpha = \arccos\left(\frac{(2 \cdot r_{\mathrm{P}})^2 - 2\left(r_{\mathrm{sphere}} + r_{\mathrm{pin}}\right)^2}{-2\left(r_{\mathrm{sphere}} + r_{\mathrm{pin}}\right)^2}\right) \tag{11}$$

$$r_{\mathrm{Ps}} = r_{\mathrm{sphere}} \cdot \sin\left(\frac{\alpha}{2}\right) \tag{12}$$

$$\Delta z_{\mathrm{sphere}} = r_{\mathrm{sphere}} - r_{\mathrm{sphere}} \cdot \cos\left(\frac{\alpha}{2}\right) \tag{13}$$

5. RESULTS

All samples from Figure 5 and Figure 7 were measured several times and compared with theoretically determined values by the geometry parameters. The results from Table 2 show very good agreement with the theoretically determined parameters.

Table 2: Measurement results of the height of the centre of gravity (h_{CG}) with standard deviation ($\sigma(h_{CG})$) in comparison with the theoretic, geometrically determined values (h_{geom})

Sample number	h_{geom} / mm	h_{CG} / mm	$\sigma(h_{CG})$ / µm	Δh_{CG} / %	Δh_{CG} / µm
1	44.02	44.168	24	0.34	148
2	39.15	39.279	116	0.33	129
3	46.40	46.758	61	0.77	358
4	24.50	24.518	54	0.07	18
5	40.50	40.751	123	0.62	251
6	32.00	31.738	142	-0.82	-262

When comparing the geometrically determined heights h_{geom} and the measured heights h_{CG} from Table 2, the target of an absolute deviation of $|\Delta h_{CG}| < 300$ µm was not achieved in the case of sample 3 (button weight), but it should be noted that the geometrically determined values are not the actual values of the centre of gravity height. They are an approximate estimate that allows conclusions to be drawn about the newly developed measuring device. Nevertheless, with a standard deviation of less than 150 µm ($k = 1$), the results are very promising for a further development of the device into a stand-alone device.

6. OUTLOOK

Since the first prototype shows very good results, it is desirable to develop the device into a stand-alone device. The tilt table is a very precise but heavy and expensive device, which is also much more accurate than necessary. Therefore, it should be replaced by two linear stages replacing two of the three adjustable feet of the base (see Figure 9). In this way, the two linear stages can be used to tilt the force sensors. Either the linear stages should be calibrated, or a small inclinometer is placed under the base plate of the force sensors.

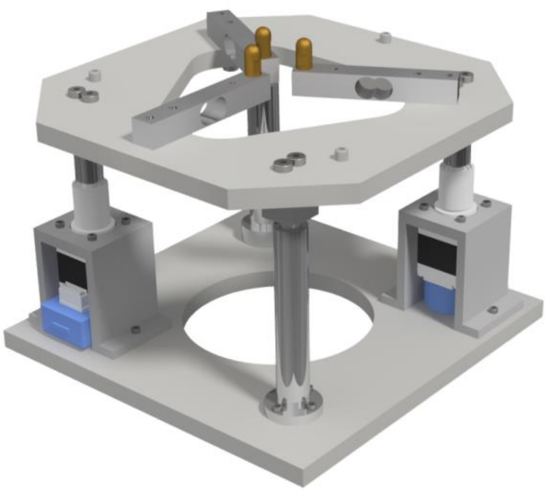

Figure 9: Outlook for a stand-alone device

To further reduce measurement uncertainties, an automatic load changer can be installed for automatic ABBA calibration of the force sensors. The automatic calibration and evaluation algorithm guarantees calibration immediately before a high-precision measurement is performed.

7. SUMMARY

The new centre of gravity measuring device presented in this paper measures the height of the centre of gravity of sensitive mass artefacts with a standard deviation of less than 150 µm. Thus, the remaining relative uncertainty contribution is as small as 4.8×10^{-11} or 48 ng, which is beyond the readability of commercially available mass comparators. With this method, mass samples with unusual shapes or hollow mass standards with uncertain internal geometry can be compared with high-precision mass standards, reducing the systematic uncertainty contribution due to different heights of the centre of gravity of mass artefacts.

8. REFERENCES

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