COMPENSATION OF INTEGRATION INTERVAL INACCURACY IN SAMPLING METHODS FOR AC MEASUREMENTS

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Abstract – *A calculating procedure for neutralizing integration interval inaccuracy in sampling methods for AC measurements is described. The procedure involves one approximation, but very accurate one, so its contribution to the uncertainty of the amplitude of the measured signal can be neglected. The efficiency of the proposed method has been proven by a series of experiments.*

Keywords - *AC signal, sampling, high accuracy*

1. INTRODUCTION

All sampling methods for sinusoidal wave measurement have the same problem. Derived signal (i.e. waveform formed by samples [1,3]) is theoretically pure sinusoid, but imperfection of measurement system introduces a number of different errors in samples, that is, in the final result. Besides all errors that show some kind of random character, there are two particularly inconvenient systematic errors. One is inaccuracy of the sampling frequency, or generally inaccuracy of the ratio of sampling and signal frequency, and the other is inaccuracy of the integration interval. While sampling frequency inaccuracy changes only the frequency of derived sinusoid and can be relatively easy completely (mathematically) neutralized as described in [1], inaccuracy of integration interval appears in both amplitude and phase of samples, and can not be eliminated without some approximations. In this paper we show the origin of the problem, its consequences on the real measurements and solution, together with results of real laboratory measurement obtained in Croatian Primary Electromagnetic Laboratory (CPEL).

2. ORIGIN OF THE PROBLEM

By sampling sinusoidal signal we obtain points of derived signal. If the original signal is pure sine wave, derived signal will theoretically also be clean sinusoid of the same frequency as the original one, but of less amplitude. The amplitude of the derived signal is [1,3]

$$A_s = A\frac{T}{\pi T_m}\sin\frac{\pi T_m}{T} \qquad , \qquad (1)$$

where A_S is the amplitude of derived sinusoid, A is the amplitude of the original sinusoid, T is period of the original signal and T_m is the integration interval. If the sampling device was perfect, i.e., if the integration interval during sampling was exactly T_m as we wanted it to be, and if the measured signal was time invariant (constant period T), the amplitude of the original signal could be found very simply from (1), because we can calculate A_S from samples very accurately [1], and T as well as T_m are known quantities. But even with the best available instruments (in our work we use HP3458A Digital Multimeter as sampling device), integr-ation interval will not be exactly the one we assigned. If during sampling real integration interval was $T_m' = T_m + \Delta T_m$, instead of T_m, and we do not account that fact in our calculation, we shall not obtain accurate result. Error can be few tenths ppm to few thousands ppm, depending on the T_m (i.e., on the ratio T_m/T).

The extent of destruction can be proven by very simple experiment. We can measure well known and time invariant sine wave assigning different integration intervals and watch how the final results change. Simple measurement system for such experiment, which we used in our research, is FLUKE 5200 AC Calibrator, whose voltage is sampled by HP3458A. Typical result (obtained for the calibrator voltage 7 V_{rms} and frequency 50 Hz) contain Table 1 and Figure 1, which is graphical presentation of table data.

Table 1 - Results of measurement of calibrator voltage without compensation of Δt_m.

T_m / T	0,05	0,1	0,15
U / V	6,999230	6,998275	6,996677

Table 1 - Continuation.

T_m / T	0,2	0,25	0,3
U / V	6,994408	6,991410	6,987575

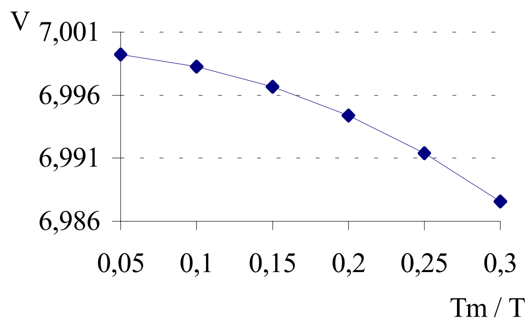

Figure 1 : Graphical presentation of data in Table 1.

We see that destructive effect of unknown ΔT_m can easily reach order of magnitude of few percents and make high accuracy measurement completely impossible. Obviously, if

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we need uncertainties of few ppm, dependance of the measurement result on the integration interval T_m must be overcome, that is, we must know the integration interval error ΔT_m and neutralize it in the final result.

3. SOLUTION

If we account ΔT_m in calculation, mathematical form of samples is defined by integral

$$U_i = \frac{1}{T_m + \Delta T_m} \cdot \int_{iTs}^{iTs+Tm+\Delta Tm} A \sin(\omega t + \varphi) \cdot dt \quad , \qquad (2)$$

where U_i is value of i^{th} sample[1], T_s is period of sampling and the other symbols have the same meaning as before. Solving this integral we find mathematical form of each sample as function of its index i

$$U_i = \frac{A \cdot T}{\pi (T_m + \Delta T_m)} \sin \frac{\pi (T_m + \Delta T_m)}{T} \sin\left(\frac{2\pi T_S}{T} i + \frac{\pi (T_m + \Delta T_m)}{T} + \varphi \right)$$

and we see that formula (1) becomes

$$\begin{aligned} A_s &= A \frac{T}{\pi (T_m + \Delta T_m)} \sin \frac{\pi (T_m + \Delta T_m)}{T} \\ &= A \frac{T}{\pi T_m (1 + \Delta t_m)} \sin \frac{\pi T_m (1 + \Delta t_m)}{T} \end{aligned} \quad , \qquad (3)$$

where Δt_m is relative integration interval error $\Delta T_m / T$. Now, instead of (1) where we had one unknown variable A, in (3) we have two unknown quantities A and Δt_m in a single equation. Without knowing Δt_m, A_S is not enough for calculating A any more. But if we could found Δt_m, we would be able to calculate A much more accurate then by neglecting it as in (1). The solution lies in the fact that, if we work with quality digital instruments (and HP3458A is certainly such a one), relative integration interval error Δt_m is constant, independently of duration of integration T_m. Under this assumption, we perform two measurements with two different integration intervals (T_{m1} and T_{m2}) and from samples we calculate two different amplitudes of derived signal (A_{s1} and A_{s2}). According to (3), their values will be

$$\begin{aligned} A_{s1} &= A \frac{T}{\pi T_{m1} (1 + \Delta t_m)} \sin \frac{\pi T_{m1} (1 + \Delta t_m)}{T} \\ A_{s2} &= A \frac{T}{\pi T_{m2} (1 + \Delta t_m)} \sin \frac{\pi T_{m2} (1 + \Delta t_m)}{T} \end{aligned} \quad . \qquad (4)$$

Taking their ratio, we eliminate the amplitude of the original signal and obtain

$$\frac{A_{s1}}{A_{s2}} = \frac{\sin \frac{\pi T_{m1} (1 + \Delta t_m)}{T}}{\sin \frac{\pi T_{m2} (1 + \Delta t_m)}{T}} \cdot \frac{T_{m2}}{T_{m1}} = \frac{\sin \alpha}{\sin k\alpha} \cdot k \ ; \qquad (5)$$

$$\alpha = \frac{T_{m1}}{T} (1 + \Delta t_m) \quad , \quad k = T_{m2} / T_{m1} \ .$$

In (5) there is again only one unknown quantity - relative integration interval error Δt_m, that is, the whole argument α of the *sin* function. Because we need Δt_m, our next problem is the mathematical form of (5), i.e., it is transcendent equation and can not be explicitly solved for α (Δt_m). Obviously, we have only two possibilities: either we shall approximate this function with some other one for which we can calculate the argument for known function value, or we must find Δt_m with some iterative routine.

3.1 *Approximate method for calculating* Δt_m

In order to get any idea of the function that would be an acceptable approximation of (5), first we shall plot (5) versus α for a wide interval, much wider than values of T_{m1}/T and Δt_m in real measurements would give (Fig.2). We see that (5) is a kind of "pulsing" function, with frequency of pulses proportional to k. At the first glance, second order polynom would be appropriate approximation. However, an enlarged plot of the central area of Fig.2, that is, plot of (5) for the interval $\alpha \in [-0{,}4 \ , \ 0{,}4]$ shows that the edges of pulses are more exponential than parabolic (Fig.3).

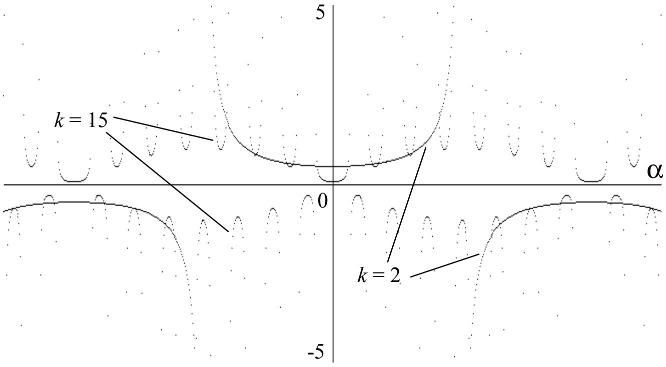

Figure 2- Values of sin(α)/sin(kα), for T_{m1}/T = 0,01 , α∈[-8 , 8], k = 2 and k = 15.

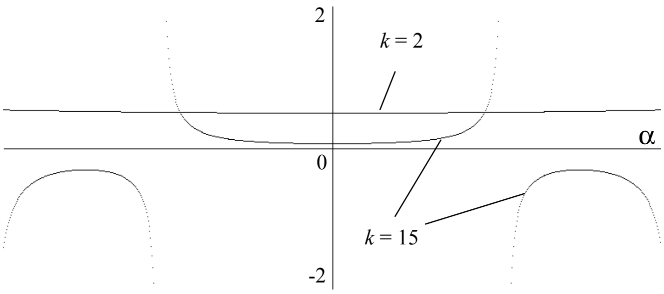

Figure 3- Values of sin(α)/sin(kα), for T_{m1}/T = 0,01 , α∈[-0,4 , 0,4], k = 2 and k = 15.

Nevertheless, we do not need to worry about the edges, because $\alpha \in [-0{,}4 \ , \ 0{,}4]$ is still few tenths times wider interval than we have in reality. For example, T_{m1}/T = 0,01 and Δt_m = 0,01 (what is much too much) give α = 0,0317, that is, 12,6 times lower value than α = 0,4. Unfortunately, more precise plots (i.e. for much narrower intervals) are not useful, because it is optically impossible to perceive anything, but the straight lines, although a slight winding really appears. Parabolic course of the curves at the Fig.3 around the origin of the coordinate system can be proven by calculating the growth of (5), i.e.,

$$\Delta y = sin(\alpha + step) / sin(k(\alpha + step)) - sin(\alpha) / sin(k\alpha) \ ,$$

[1] Boundaries of integral (1) are set assuming i = 0, 1, …, N-1, where N is overal number of samples.

which is shown at the Fig.4 for $\alpha \in [-0{,}031 \ , \ 0{,}0318]$ and *step* = 10^{-5} (all ordinate values must be divided by 10^7 in order to obtain real value).

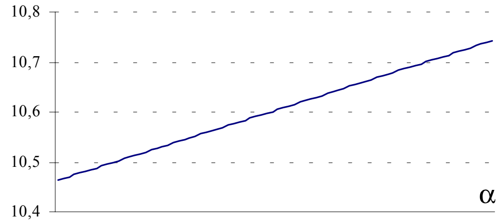

Figure 4- Growth of $\sin(\alpha)/\sin(k\alpha)$ for T_{m1}/T = 0,01 in the interval $\alpha \in [-0{,}031 \ , \ 0{,}0318]$.

We see that growth of (5) is a straight line, so it follows that (5) has parabolic course. Now it is clear how we can calculate Δt_m. Substituting at least three different α (or directly Δt_m) into (5), we can find second order approximation polynom for the function of ratio $A_{s1} \ / \ A_{s2}$. Knowing the ratio $A_{s1} \ / \ A_{s2}$ obtained by measurement, we can calculate the argument of approximation curve. That argument is (very accurate) approximation of the real unknown α and once we get α (i.e., Δt_m), it is easy to calculate the amplitude of the original signal from any of two relations (4). The choice of points through which the approximation polynom is to pass depends on the expected Δt_m, as well as on two ratios T_{m1}/T and T_{m2}/T for which we sample the measured signal. In any case, approximation error is much lower than any other error in measurement system, and it can be neglected in calculation of the final uncertainty of the amplitude. It would be too extensive to prove that statement in this paper for all possible cases, but as an illustration, Fig.5 shows relative approximation error for $T_{m1}/T = 0{,}01$, $T_{m2}/T = 0{,}1$ (k = 10) and $\Delta t_m \in [-10^{-2} \ , \ 10^{-2}]$.

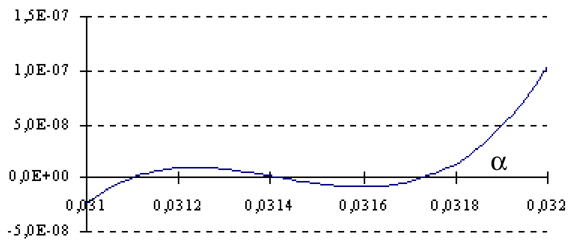

Figure 5- Relative approximation error for T_{m1}/T = 0,01, T_{m2}/T = 0,1 (k = 10) and $\Delta t_m \in [-10^{-2} \ , \ 10^{-2}]$.

3.2 *Iterative method for calculating* Δt_m

Although, in theoretical sense, approximation method described above is "more intelligent" and straightforward, for software realization it is perhaps more suitable to apply an iterative procedure. It is based on the fact that we now interval of possible values of α (i.e. Δt_m), so we can seek through that interval to find α which corresponds to the given (obtained by measurement) ratio $A_{s1} \ / \ A_{s2}$. If we assume that Δt_m will never exceed boundaries $\pm 10^{-2}$, what is really enormous value for all laboratory measurements, we have already seen (from (5)) that (for T_{m1}/T = 0,01) α will be within [0,0311 , 0,0317]. Generally, for any T_{m1}/T we can calculate interval of possible α and check that interval for the appropriate value. The iteration itself can be realized in many ways and its algorithm is completely irrelevant for our considerations, so it will not be discussed here.

4. CONCLUSION

The efficiency of the proposed procedure for compensation of the integration interval inaccuracy can be best proven by experiments. With the same measurement system we used in the first experiment (Table 1) and for any combination of T_{m1} and T_{m2} we obtain practically the same amplitudes (rms value) of the original signal (Table 2 and Figure 6), within the accuracy (voltage stability) limits of FLUKE 5200 and our measurement system. It is obvious that the main problem, i.e., dependence of the result on the integration interval T_m, has been successfully solved.

Table 2 - Results of measurement of calibrator voltage with compensation of Δt_m.

T_{m1}/T; T_{m2}/T	0,05 ; 0,1	0,05 ; 0,25	0,05 ; 0,3
U / V	6,999477	6,999460	6,999479

Table 2 - Continuation.

T_{m1}/T; T_{m2}/T	0,1 ; 0,3	0,15 ; 0,3	0,15 ;0,25
U / V	6,999490	6,999471	6,999465

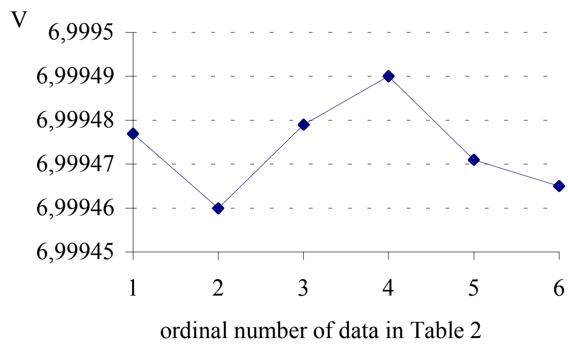

Figure 6 : Graphical presentation of data in Table 2.

Data in Table 2 are typical results we obtain in Croatian Primary Electromagnetic Laboratory (CPEL). The method presented here has been tested up to frequencies of about 500 Hz (with the same success) and is nowadays standard method for neutralizing described errors in sampling measurements of sinusoid signals in CPEL. What is most important, it is suitable for application in more complex measurements such as measurements of phase difference [2] or power.

5. REFERENCES

[1] N. Hlupic , M. Kresic and I. Lenicek : Comparison of two sampling methods for sine wave measurement, XVI IMEKO World Congress 2000, Proceedings Volume IV, p. 131-135
[2] N. Hlupic, J.Butorac : A sampling method for high accuracy phase difference measurement, IMEKO 2001
[3] V.Bego, K.Poljancic, D.Hrsan : Precise measurement of power and RMS values of AC voltages and currents with digital multimeters, IMEKO TC-4, 7th International Symposium, Part 3, p. 639-644