A SAMPLING METHOD FOR HIGH ACCURACY PHASE DIFFERENCE MEASUREMENT

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Abstract – *Sampling methods are nowadays commonly accepted for measurements of various signals and here we present our achievements in measurement of pure sine wave at low frequencies (up to about 500 Hz), that is, measurement of phase difference between two sinusoids. Firstly, we briefly discuss the mathematical form of samples and its implications on the calculations needed to determine the phase difference. We continue with deep explanations of an alghoritm for very accurate calculation of one parameter of the measured signals, the most important one for phase difference calculation. At the end, we present results of real laboratory measurements as the final proof of declared accuracy.*

Keywords – phase difference, sampling, high accuracy.

1. INTRODUCTION

Since few recent years we have developed a sampling method for high accuracy sine wave measurement in order to implement it in electrical power measurement. Nowdays, the method is finished in all its aspects and we are now able to measure all parameters of a sine wave with accuracy of few ppm to few tenths ppm [1,2,3]. Especially interesting are our results in phase, i.e., phase difference measurements. Without rigorous demands on the measurement system, we achieve absolute error of about 60 µrad for phase difference measurement. Of course, specified error is maximum possible absolute error and uncertainty is much lower. In this paper we explain the theoretical background of our method and provide results of real measurements performed in Croatian Primary Electromagnetic Laboratory (CPEL), which clearly confirm declared accuracy.

2. THEORETICAL BASIS OF PHASE DIFFERENCE MEASUREMENT

As we already know [1,3], ideal samples of perfect sine wave also form a sinusoid, which we named as "derived signal" or "derived sinusoid". As every other sinusoid, derived signal is completely described with three parameters managed by formula

$$U_i = A_s \cdot \sin(xi + y), \tag{1}$$

where symbols have the following meaning: U_i is the value of i^{th} sample (assuming that $i = 0, 1, \ldots, N-1$; where N is overall number of samples), A_s is the amplitude of the derived sinusoid, parameter x is its frequency and parameter y is its phase. Parameters of derived signal (A_s , x and y) are firmly related to the parameters of the original (measured) one, depending on the sampling frequency, i.e., on the ratio of sampling and signal frequency. Relations among them are given by [1,3]:

$$A_s = \frac{A}{\pi}\frac{T}{T_m(1+\Delta t_m)}\sin\frac{\pi T_m(1+\Delta t_m)}{T}$$
$$x = 2\pi\frac{T_s(1+\Delta t_s)}{T}\quad , \quad y = \pi\frac{T_m(1+\Delta t_m)}{T} + \varphi \tag{2}$$

where symbols have the following meaning: A is the amplitude of the original sinusoid, T is period of the original sinusoid, φ is phase of the original sinusoid, T_s is period of sampling, T_m is the integration interval, Δt_s is relative error of sampling period [1] and Δt_m is relative error of integration interval [2]. For phase (phase difference) measurement, the most important parameter is y, because it is the only parameter of derived signal that contains information on the phase φ of the original sine wave. If we were able to calculate y enough accurate, and under condition that we knew Δt_m, we could find the phase φ. We have developed an algorithm for extremely accurate calculation of parameter y (that is topic of the next passage of this text), but in reality we can calculate only the phase difference of two sine waves, rather than absolute phase of one signal. Namely, formulae (2) are valid only for a fictive sampling device that would have an instantaneous (infinitely short) response to the signal for starting measurement and to the signals for starting and stopping integration. Real sampling device will express a short delay after any excitation and neglecting this imperfection leads to completely wrong results [3]. We shall denote "dead time" after the signal for starting of measurement as T_o (**o**ffset time) and "dead time" after the signals for starting and stopping integration as T_d (***d**elay time*). Accounting this two new quantities into the boundaries of the integral which defines the samples [1,3], we obtain

$$U_i = \frac{1}{T_m + \Delta T_m}\cdot\int_{T_o+T_d+i(T_s+\Delta T_s)}^{T_o+T_d+i(T_s+\Delta T_s)+T_m+\Delta T_m} A\sin(\omega t+\varphi)\cdot dt\ , \tag{3}$$

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where ΔT_m is absolute error of integration interval, ΔT_s is absolute error of sampling frequency and the other symbols have the same meaning as before. Solving this integral, yields the mathematical form (4) of samples. Comparing (4) and (2), we see that imperfections of sampling device directly influence the most critical parameter *y*, that is, they act as new, virtual time shift between original and derived signal, according to (5).

$$U_i = \frac{A}{\pi}\frac{T}{T_m + \Delta T_m}\sin\frac{\pi\left(T_m + \Delta T_m\right)}{T} \cdot \sin\left(2\pi\frac{T_s + \Delta T_s}{T}i + 2\pi\frac{T_o + T_d}{T} + \pi\frac{T_m + \Delta T_m}{T} + \varphi\right) \quad (4)$$

$$y = 2\pi\frac{T_o + T_d}{T} + \pi\frac{T_m\left(1 + \Delta t_m\right)}{T} + \varphi \quad (5)$$

Unfortunately, we can not calculate neither T_o , nor T_d from samples, so we are not able to determine the absolute phase φ of the original signal. However, absolute phase is not something we frequently need. The more common needs are related to the phase difference between two sine waves, and that is something we are able to calculate, in spite to the fact that we can not determine T_o and T_d.

Our method for phase difference calculation is based on one assumption that greatly depends on hardware of the sampling device and dictates its quality. Namely, if we use quality digital instruments (in our work we use HP3458A Digital Multimeter as the sampling device), it is justified to assume that they have very repeatable response in two or more sampling cycles. The benefit of such assumption is that additional phase shift of derived signal due to T_o and T_d will always be the same. In that case, by sampling two sine waves consecutively[1], we obtain two parameters y_1 and y_2.

$$y_1 = 2\pi\frac{T_o + T_d}{T} + \pi\frac{T_{m1}\left(1 + \Delta t_m\right)}{T} + \varphi_1$$

$$y_2 = 2\pi\frac{T_o + T_d}{T} + \pi\frac{T_{m2}\left(1 + \Delta t_m\right)}{T} + \varphi_2 \quad (6)$$

Because we know T_{m1} and T_{m2}, and are able to calculate Δt_m as explained in [2], second term in formulas (6) is known and can be subtracted. We shall name the remainder as *reduced* parameter *y* and denote it as *Y*. So, after subtracting of known quantities we have

$$Y_1 = 2\pi\frac{T_o + T_d}{T} + \varphi_1 \quad \text{and} \quad Y_2 = 2\pi\frac{T_o + T_d}{T} + \varphi_2 \ . \quad (7)$$

It is obvious that the difference of reduced parameters *Y* exactly matches the phase shift between two sine waves.

$$Y_1 - Y_2 = \varphi_1 - \varphi_2 \quad (8)$$

Here we come to the core of our method. From (8) it is obvious that the accuracy of phase difference is equal to the accuracy of difference of reduced parameters *Y*, and the accuracy of $(Y_1 - Y_2)$ depends on accuracy of y_1 and y_2. For achieving an uncertainty of few ppm, we must calculate parameter *y* very accurately. The basic formula for its calculation [1,3]

$$y = arcctg\frac{U_{i+1} - U_{i-1}}{2\sin x \cdot U_i} - xi \quad (9)$$

is not enough, because random errors of samples and, even more, imperfections of the sampling device lead to overall systematic (usually parabolic) parameter *y* dependence on index *i*, that is, on time in which particular sample was measured. This time-dependance significantly contributes to the uncertainty of parameter *y* and, without compensation described latter in this text, hinders us to determine it with error less than about two times the overall change. The assertion that the change of all parameters *y*, which we calculate from samples, is generally parabolic can be mathematically proven, and we shall do that soon, but as an illustration of that fact we provide Fig.1, which is a typical result of real measurement.

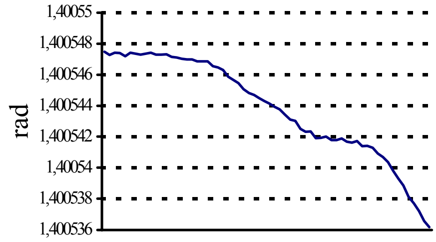

Figure 1 : An illustration of the change of parameter *y* value when calculated from samples taken in different moments during an uninterruptedly performed sampling. The abscissa represents the (time) duration of sampling.

Looking at the Fig.1, we better understand the effect of parameter *y* time-dependance, and we can not get rid of question: "Where is the right *y* value?". As we can see, the overall change of parameter *y* does not have to be too large (in our example it is only few ppm), although it can, depending on the quality of measurement system. However, the fact that the curve is so expressively parabolic imposes conclusion that this change in value from the beginning to the end of measurement is an inherent (systematic) property of parameter *y*, rather than merely a consequence of random influences on the measurement system. A short calculation explains this parabolic change and confirms that the curve is really (in ideal case) a parabola, and not something similar.

Let us start with samples themselves, which are defined by formula (1). It follows that parameter *y* is

$$y = \arcsin\frac{U_i}{A_s} - xi = konst. \quad (10)$$

and it is (physically must be) a constant, regardless on the manner of calculation or different values U_i of samples (change in U_i is compensated with change of indexes *i*). That is an ideal case, i.e., we have assumed that we know the exact value of the frequency of derived signal. However, frequency *x* is something we must calculate from samples too, and we never know it perfectly accurate. Moreover, we must take into account the possibility that the ratio of sampling and signal frequency during measurement changes. Random changes of that ratio should not be problematic, because they are expected to neutralize each other in a larger number of

[1] It is understood that we must ensure synchronisation between signals and sampling device in order to maintain relative phase relationships.

final results (parameters y) [3]. Problems arise when the change of ratio of sampling and signal frequency is systematic. If we work with quality instruments, drifts of their characteristics will be very slow. This means that in a short time interval during our measurement, we may expect a continuous and uniform change of the frequency ratio, so we can suppose it to be the first order polynom (straight line). Mathematically, instead of a constant frequency x, we shall in reality have $x = f(i) = a_x i + b_x$. From samples we can calculate many values for x, and till now we have been taking their mean value to be "the right" one. Assuming that x is not a constant, but first order polynom, we must calculate the regression line through all obtained values of x, and take the points of that line to be "the right" values of x for different indexes i. If the real frequency drift was straight line $x = a_x i + b_x$, substituting points of that polynom into (10) would give always the same and exact value of y. Of course, we shall never be able to calculate the nature of the frequency drift (a_x and b_x) perfectly accurate, but we shall obtain some A_x and B_x, which will generally contain errors Δa_x and Δb_x , so we can write $A_x = a_x + \Delta a_x$, $B_x = b_x + \Delta b_x$. It follows that we shall, instead of $x = a_x i + b_x$, have wrong values $x'(i) = A_x i + B_x = (a_x + \Delta a_x)i + (b_x + \Delta b_x)$. Substituting these wrong values into (10) yields

$$y' = \arcsin\frac{U_i}{A_s} - (A_x i + B_x)i = \arcsin\frac{U_i}{A_s} - [(a_x + \Delta a_x)i + (b_x + \Delta b_x)]\cdot i$$

$$= \arcsin\frac{U_i}{A_s} - [(a_x i + b_x) + (\Delta a_x i + \Delta b_x)]\cdot i = \arcsin\frac{U_i}{A_s} - xi - \Delta a_x i^2 - \Delta b_x i$$

$$= y - \Delta a_x i^2 - \Delta b_x i \quad , \qquad (11)$$

that is, we obtain the pure parabola. Formula (11) explains why we in reality (as on Fig.1) notice parabolic course of parameter y change and is exact, mathematical proof that this change is an inherent property of a real (quality) measurement system. Because (11) is parabola only due to errors of frequency drift calculation, we named its parabolic term $\Delta a_x i^2 + \Delta b_x i$ as "parabola of errors".

But, formula (11) is not only the proof of the problem. It is also, in the same time, its solution. It answers two main questions - where is the right value of y and how to get it. Obviously, the right value of y is the one which we would obtain from the first taken sample, i.e., from the sample whose index $i = 0$. Unfortunately, we need three samples (see formula (9)) to calculate one y and, with such calculation, we can not even theoretically obtain y from the first sample. Therefore, we must pass a parabola, according to the least squares theory, through all parameters y. That parabola will, in fact, be the formula (11). Once we get the coefficients of approximation parabola, according to (11) we know that the right value of y should be equal to the least significant coefficient of that parabola. That is certainly the most straightforward way to calculate the right y. But, as always when we use computer, we must consider the consequences of its limited ability for storing numbers and perform floating-point calculations. Direct least square approximation can result with errors of the same order of magnitude as the frequency shift itself and is mostly not enough accurate [3]. In order to overcome this limitation of ordinary PC computers, we can use the fact that the right y value will, as we see from (11), be the remainder of every single calculated y, after we subtract from it the parabola of errors $\Delta a_x i^2 + \Delta b_x i$ (i.e. just a part of approximation parabola). This fact enables us to obtain many "candidates" for the right y, as many as much parameters y we calculated at all. Then we can take the mean value of all "remainders" as the right value and it will be really very accurate. Nevertheless, it is still not the best we can do. For the best achievable accuracy, we must perform an iterative procedure as follows:

1) Calculate approximation parabola. Let us denote its coefficients as $y' = C_1 i^2 + C_2 i + C_3$. With such notation, parabola of errors will be $C_1 i^2 + C_2 i$.
2) Subtract parabola of errors $C_1 i^2 + C_2 i$ from all obtained parameters y. If we calculated approximation parabola perfectly, the remainders will be randomly scattered (because of random errors of samples) around regression line, which would be a constant, i.e., the right value of y.
3) If, because of calculation errors, we do not get exact values for $C_1 i^2 + C_2 i$, the remainders after subtraction in step 2) will still express a kind of parabolic course. Therefore, we repeat this procedure from step 1), but we calculate approximation parabola through the remainders obtained in step 2) until we obtain (in step 2)) remainders whose regression line will be a constant.

Usually, already a few cycles of iteration are enough (under condition that we have at least few tenths parameters y) that parabola of errors becomes neglectable (C_1 and C_2 less than 10^{-12}), and such procedure completely overcomes eventual calculation errors, so accuracy of parameter y depends exclusively on samples, that is, on measurement itself.

2. CONCLUSION

When we are able to calculate parameter y enough accurate, measurement of phase difference means just one subtraction more, as stated by (8). Efficiency of proposed manner for calculation of parameter y is illustrated by Fig.2, which shows the example in Fig.1, but after neutralization of parabola of errors applying the iterative procedure explained above.

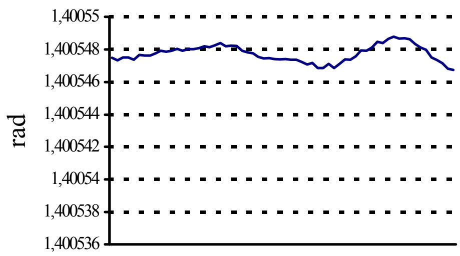

Figure 2 : All values of parameter y obtained from samples in example illustrated at Fig.1, but after neutralization of parabola of errors applying the iterative procedure. The abscissa represents the (time) duration of sampling.

Obviously, there is no "parabolicity" remained in parameters y, and the overall scattering is less than 1 ppm relatively. Slight sinusoid deviations, which can be noticed in the curve of all parameters *y*, are power line interference in samples of the measured signal.

As the final proof of declared accuracy of our method for phase difference measurement, we provide a typical result of 30 consecutive measurements performed in CPEL (Fig.3). Measurement system was very simple, as the method itself does not impose any too hard requirements. The source was HP3245A Universal Source, which applies DDS Synthesis. That enabled us to set the desired phase difference of its two channels and to have it relatively stable and accurate. The sampling device was HP3458A Digital Multimeter. We sampled both channels with the same instrument, commutating its inputs. Such procedure ensured maximal possible invariance of instrument's delays T_o and T_d. Start moment of sampling was synchronized with the same channel for both samplings through the TTL output of the source. This preserved the phase relationships during two separate samplings. In this example (Fig.3), the preset phase difference was zero, i.e., both signals were phase matched, so every particular point at the Figure represents absolute error of phase difference measurement. With no rigorous demands on the laboratory conditions and measurement system, we obtained very good results. Maximal error in 30 measurements was less than 60 μrad, and there were only two extreme values that are to be ascribed to the DDS synthesis, rather than to the measurement method. All other values were within ±40 μrad. When we think about disadvantages of this method, the only significant one would be that

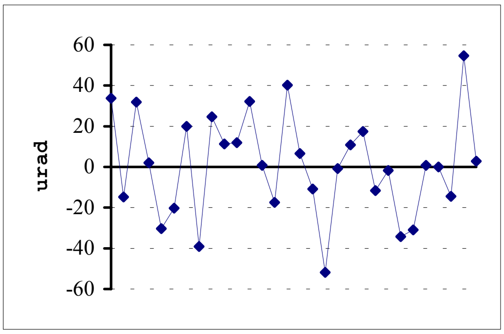

Figure 3 : Absolute error of phase difference measurement, expressed in μrad. On the abscissa are ordinal numbers of measurements (omitted as irrelevant).

sampling lasts about 90 s, so signals must be stable during that time. That makes this method suitable, first of all, for laboratory measurements.

REFERENCES

[1] N. Hlupic , M. Kresic and I. Lenicek : Comparison of two sampling methods for sine wave measurement, XVI IMEKO World Congress 2000, Proceedings Volume IV, p. 131-135

[2] N. Hlupic, : Compensation of integration interval inaccuracy in sampling methods for AC measurements, IMEKO 2001

[3] N. Hlupic : Precise measurement of power applying a sampling method, Ph. Dissertation, University of Zagreb 2001. (in Croatian)

[4] D.Ilic, J. Butorac, M.Kresic : Use of precise digital voltmeters for AC voltage measurements, CPEM 2000