MEASURING THE PHASE ANGLE BETWEEN TWO LOW FREQUENCY SINUSOIDS

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Abstract - *We propose a method for measuring the phase angle between two low-frequency sinusoids. In this method, we integrate one signal between a zero crossing of this signal and a zero crossing of the other signal. Two different integrals are performed, where each integral is a function of the amplitude and frequency of the integrated signal and the phase angle between the two signals. The phase angle is obtained through simple algebraic manipulations.*

Keywords – Phase measurement, power factor measurement.

1. INTRODUCTION

The *phase* of a periodic, varying phenomenon, e.g., an electrical signal, is any distinguishable instantaneous state of the phenomenon, referred to a fixed reference or another periodic varying phenomenon. The *phase time* (frequently abbreviated simply to "*phase"* in colloquial usage) can be specified or expressed by *time of occurrence* relative to a specified reference. The *phase* of a periodic phenomenon can also be expressed or specified by angular measure, with one period usually encompassing 360^o (2π radians). The *phase angle* of a periodic wave is the number of suitable units of angular measure between a point on the wave and a reference point. The reference point may be a point on another periodic wave. The *phase angle* between two periodic waves can be specified or expressed by the *time of occurrence* of the zero crossings of one wave compared to the zero crossings of the other wave. This can simply be expressed by the *time delay* between the zero crossings of the two waves as shown in Fig. 1. The *phase angle* can also be expressed by angular measure as mentioned above.

Low frequency signals are encountered in a variety of applications in our daily life. A popular example is the power system signal, whose nominal frequency is usually 50 or 60 Hz. An important characteristic of low frequency signals is that it is possible to sample and process the samples of such signals at high sampling rates as compared with the frequency of the signals. For example, sampling a 50-Hz signal at 10,000 samples per second produces 200 samples per cycle, with a time interval of 0.1 ms between every two samples. This time interval between successive samples is large enough to perform a number of operations on the sample using a microprocessor or a digital signal processor.

The problem of extracting the phase of a signal has many important practical applications. In a power system, the phase angle between the voltage and current signals is a measure of the "power factor" of the power system. Both the voltage and current signals of a power system are represented as sinusoidal signals. The power factor has an important significance in power systems. Defined as the ratio between the real average power to the apparent power, the power factor (PF) gives a measure of the efficiency of consuming power. The best utilization for power is when PF=1, and no power is utilized at all (case of reactive load) when PF=0.

The phase of a signal is traditionally extracted with the help of a phase-locked-loop (PLL) [1], which tracks both the frequency and phase of a signal. The PLL usually includes a phase detector (PD) and a voltage-controlled-oscillator (VCO). The PD is fed with two signals and generates a voltage that is proportional to the phase difference between the two inputs. Usually the PD uses the zero crossings of the signals to measure the phase angle. This is not practical for low frequency signals, and other more efficient techniques are desirable.

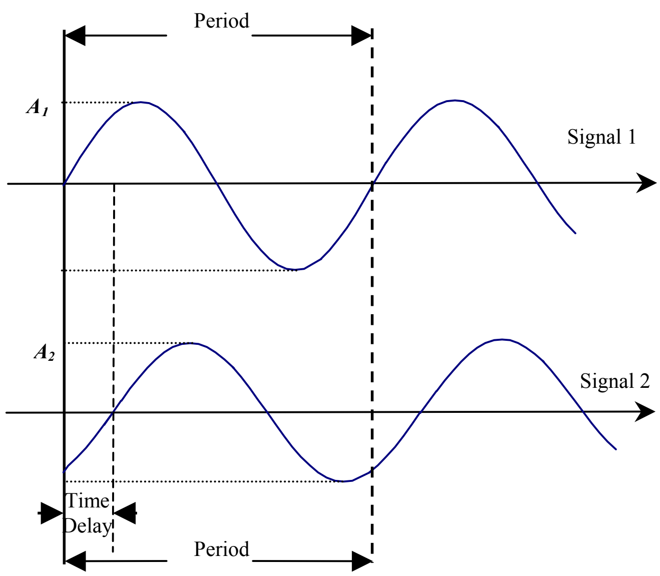

Figure 1. Typical Phase Angle Measurement

The phase angle between two signals can also be measured by using a counter-timer approach. Two repetitive signals having the same period are fed to two channels of the counter-timer, and the device calculates the period of the waveforms and the time delay from one channel relative to the other channel. An attractive phase extraction method has been reported [2], which extracts the phase of a sinusoidal signal in the presence of frequency offset and phase jitter. This method uses a novel "phase filter" for this purpose, rather than a phase-locked-loop. This phase filter processes the phase of an audio-frequency signal transmitted over a telephone line by using simple arithmetic operations. The phase filter was specifically designed to meet the application [2]. Finally, the phase angle can also be measured using the Discrete Fourier Transform (DFT). It was found [3] that the initial sampling position and the resulting snapshot have a significant effect on measurement accuracy, particularly when using a smaller number of samples for the DFT. It was shown in [3] that increasing the number of samples could reduce the overall effect of the initial sample position.

The problem considered in this manuscript is that of estimating the phase angle between two low-frequency sinusoids, where some other methods are not suitable for this case. The principle of the measurement method is based upon integrating one signal between a zero crossing of this signal and a zero crossing of the other signal. Two different integrals are performed, where each integral is a function of the amplitude and frequency of the integrated signal and the phase angle between the two signals. The two signals do not necessarily have the same amplitudes, but they should have the same frequency (or period). With simple algebraic manipulations, the phase angle is readily evaluated as a function of the two integrals. This method has several important practical applications, such as determining the phase factor of a power system as mentioned above.

2. THEORY OF MEASUREMENT

Consider the following two signals for the purpose of measurement:

$$s_1(t) = A_1 \sin(\omega t) \quad (1)$$

$$s_2(t) = A_2 \sin(\omega t + \theta) \quad (2)$$

where A_1 and A_2 are the amplitudes of $s_1(t)$ and $s_2(t)$ respectively, $\omega = 2\pi f$ is the radian frequency, f is the frequency in cycles/s, and θ is the phase difference between the signals $s_1(t)$ and $s_2(t)$.

Let us first integrate the signal $s_2(t)$ starting at a zero-crossing of this signal (i.e., $s_2(t) = 0$ or $\omega t = -\theta$), and ending at a zero-crossing of the signal $s_1(t)$ (i.e., $s_1(t) = 0$ or $\omega t = 0$). It should be noted that the zero-crossings of $s_1(t)$ occur at $\omega t = 0, \pi, 2\pi, 3\pi, \ldots$, and the zero-crossings of $s_2(t)$ occur at $\omega t = -\theta, \pi-\theta, 2\pi-\theta, 3\pi-\theta, \ldots$. Without any loss of generality, we assume that the above integration will be performed in the interval $\omega t = -\theta$ to $\omega t = 0$, which lies in the positive half of the sinusoidal waveform. In order to visualize the process, the signals are depicted in Figure 2, together with the integrals that are calculated. Therefore,

$$I_1 = \int_{-\theta/\omega}^{0} A_2 \sin(\omega t + \theta)\, dt = (A_2/\omega)(1 - \cos\theta) \quad (3)$$

We next integrate the same signal $s_2(t)$ from the zero-crossing of $s_1(t)$ where the first integral ended, until the next zero-crossing of $s_2(t)$, which occurs at $\omega t = \pi - \theta$.

$$I_2 = \int_{0}^{(\pi-\theta)/\omega} A_2 \sin(\omega t + \theta)\, dt = (A_2/\omega)(1 + \cos\theta) \quad (4)$$

Note that I_1 and I_2 were obtained by integrating the signal $s_2(t)$ between two adjacent zero-crossings of this signal. In

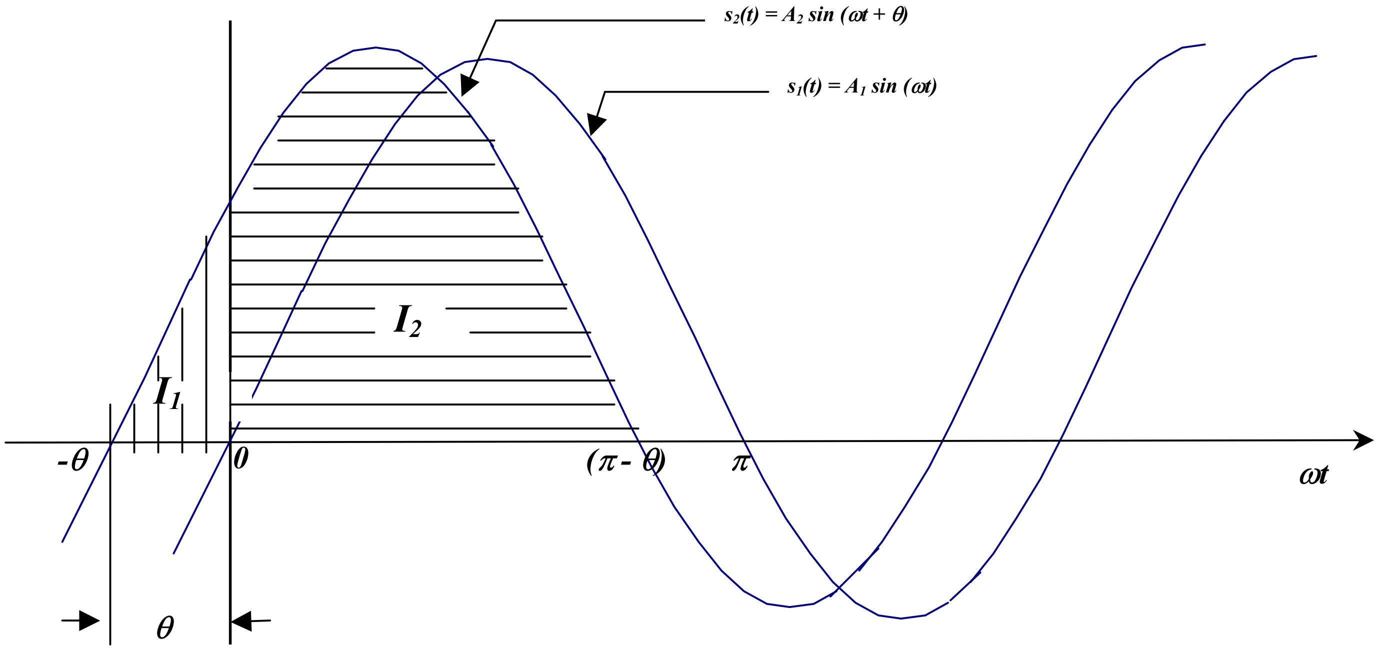

Figure 2. The two signals $s_1(t)$ and $s_2(t)$ that have a phase difference of θ, which is considered for measurement.

other words, both I_1 and I_2 were evaluated using only one-half cycle of the signal $s_2(t)$. The sum of I_1 and I_2 must therefore be equal to the area under one-half cycle of the sinusoidal signal $s_2(t)$, as shown in Fig. 2. The zero-crossing of $s_1(t)$, which is a measure of the phase angle between $s_1(t)$ and $s_2(t)$, puts the border (i.e., draws the line) which separates the area representing the integral I_1 from the area that represents the integral I_2. From equations ***(3)*** and ***(4)*** we obtain:

$$I_2 + I_1 = 2A_2/\omega \qquad (5)$$

$$I_2 - I_1 = (2A_2/\omega)\ \cos\theta \qquad (6)$$

Dividing Equation ***(6)*** over Equation ***(5)***, we obtain a relation between θ and the values of I_1 and I_2 which is independent of A_2 and ω:

$$\cos\theta = (I_2 - I_1) / (I_2 + I_1) \qquad (7)$$

Hence the phase angle between the two signals $s_1(t)$ and $s_2(t)$ can be evaluated as:

$$\theta = \cos^{-1}\{(I_2 - I_1) / (I_2 + I_1)\} \qquad (8)$$

where I_1 and I_2 are given in Equations ***(3)*** and ***(4)*** respectively.

3. DIGITAL REALIZATION OF THE MEASUREMENT METHOD

In order to implement the proposed measurement method using a digital processor, it is necessary to realize the above relations using numeric techniques. The analog signals $s_1(t)$ and $s_2(t)$ should be sampled and processed digitally. The method presented above requires a high sampling rate as compared with the frequency f of the signals. This can be easily achieved for low frequency signals. The measurement accuracy improves as the sampling rate increases, as will be explained next. Hence, it is preferred to increase the sampling rate as much as possible. Two factors put a limit on how high the sampling rate can be, the speed of the analog-to-digital converter, and the time needed to process each sample. Thus it is needed to realize the measurement method with low computational requirements in order to achieve maximum efficiency and obtain the best results.

The integrals I_1 and I_2 can be computed from the samples of $s_2(t)$ using one of the numerical integration techniques, such as the trapezoid rule, Simpson's rule, or others [4]. For sinusoidal signals, good accuracy can be obtained using Simpson's one-third rule. After we calculate the value of $\cos\theta$ from the values of the integrals, we can determine θ using a look-up table, or any other numerical method. If it is decided to use a look-up table, the size of the table and the number of bits needed to represent each table entry (i.e. number of digits to represent θ) can be selected to provide the necessary accuracy. On the other hand, one can use an iterative method to determine the value of θ when $\cos\theta$ is known, by using the formula:

$$\cos\theta = 1 - (\theta^2/2!) + (\theta^4/4!) - (\theta^6/6!) + \ \ldots \qquad (9)$$

In a power system, the phase factor equals $\cos\theta$, and hence equation ***(7)*** gives also the value of the phase factor when the two signals in Equations ***(1)*** and ***(2)*** represent the voltage and current signals. The flowchart in Fig. 3 shows the steps

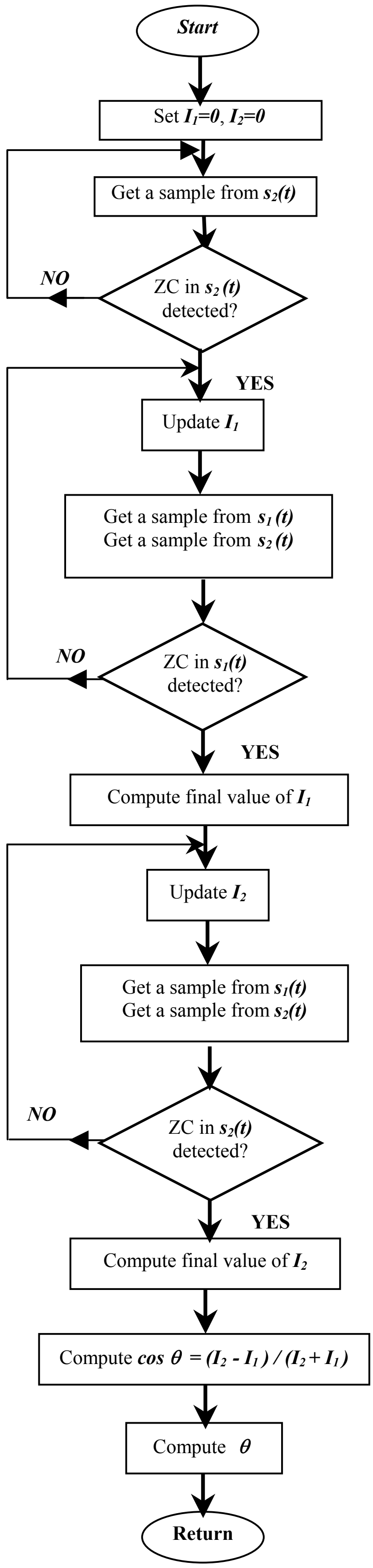

Fig. 3. Flowchart for computing the value of the phase angle between $s_1(t)$ and $s_2(t)$. "ZC" denotes "Zero Crossing"

that can be implemented to perform the required measurement.

4. PRACTICAL CONSIDERATIONS

The accuracy of the measurement method depends upon the accuracy of evaluating the integrals I_1 and I_2. The values of the integrals can be estimated more accurately as more samples of the signal are considered for the measurement. Since the measurement duration is limited to one-half cycle of the sinewave, then the time interval between every two successive samples becomes a critical factor for the accuracy of the measurement. It was mentioned earlier that this method requires high sampling rates compared to the frequency of the signal, and hence the method is best applicable to low frequency signals.

Recall that the integral is updated every sample by adding an area that is approximately equal to the sample value multiplied by the sampling interval. As we obtain more samples in the measurement interval, the number of strips added to obtain the values of the integrals is increased, and the measurement accuracy is improved. The maximum error that could possibly happen is the area of one strip, which approximately equals the value of the sample multiplied by the sampling interval. The effect of sample spacing (i.e., the time interval between every two successive samples) can be reduced as follows.

The sampler usually operates independent of the zero-crossings of either $s_1(t)$ or $s_2(t)$. Therefore, the time instants at which samples are taken will most likely not coincide with the occurrence of zero-crossings of either signal. Since the measurement process depends very much upon the time of occurrence of zero crossings, it becomes necessary to adjust the estimates of the integrals to take the above into consideration.

Consider the two samples of $s_2(t)$ that are obtained just before and just after a zero-crossing of $s_1(t)$. For convenience, let these two samples be denoted by y_1 and y_3 respectively, as shown in Fig. 4. The sample y_1 is the last sample in the area corresponding to I_1 and y_3 is the first sample in the area corresponding to I_2. If we determine the values of the areas ***abcd*** and ***cdef*** with good accuracy, then the values of the integrals will be calculated with better accuracy, and consequently the measurement accuracy will be improved. But this depends upon determining the values of y_2, δ_1, and δ_2 in Fig. 4. Using simple trigonometric identities between the two triangles ***adg*** and ***aeh,*** and noticing that ***ag/ah = dg/eh***, we obtain:

$$\delta_1 / (\delta_1 + \delta_2) = (y_2 - y_1) / (y_3 - y_1) \qquad (10)$$

Note that δ_1 is the time interval between the sample y_1 and the zero-crossing of $s_1(t)$, and δ_2 is the time interval between the same zero-crossing and the sample y_3. These time intervals should be measured in order to improve the accuracy of the measurement method. It is also to be noted that $(\delta_1 + \delta_2)$ is equal to the sampling interval. From Eq. (10), we compute the value of y_2 as follows:

$$y_2 = y_1 + (y_3 - y_1).\ \delta_1 / (\delta_1 + \delta_2) \qquad (11)$$

The value of y_2 can now be used to compute the values of the areas ***abcd*** and ***cdef,*** which will in turn produce better estimates of the integrals I_1 and I_2 respectively.

The area representing the integral I_2 ends at the zero-crossing of $s_2(t)$. In Fig. 4, this area ends at point ***l***. Let y_4 and y_5 be the two samples obtained just before and just after this zero-crossing. The time interval δ_3 between the sample y_4 and the zero-crossing can be either measured or computed as follows:

$$\delta_3 = (\delta_3 + \delta_4).|y_4|/(|y_4| + |y_5|) \qquad (12)$$

Note that $(\delta_3 + \delta_4)$ is equal to the sampling interval. The area of the triangle ***jkl*** can now be computed, and the value of the integral I_2 can be estimated with better accuracy. The value of the integral I_1 can be similarly adjusted by computing the area of the triangle adjacent to the zero crossing at the beginning of this integral.

5. CONCLUSION

Low frequency signals are encountered in a variety of applications in our daily life. The phase of a signal is traditionally extracted with the help of a phase-locked-loop. Such methods depend upon the zero-crossings of the signal, and hence they are not well suited for low frequency signals. In this manuscript, we propose a method for estimating the phase angle between two low-frequency sinusoids, where some other methods are not quite suitable for this case. The principle of the measurement method is based upon integrating one signal between a zero crossing of this signal and a zero crossing of the other signal. Two different integrals are performed, where each integral is a function of the amplitude and frequency of the integrated signal and the phase angle between the two signals. The two signals do not necessarily have the same amplitudes, but they should have the same frequency (or period). With simple algebraic manipulations, the phase angle is readily evaluated as a function of the two integrals. This method has several important practical applications, such as determining the phase factor of a power system as mentioned above.

This paper presents the theory needed for performing the measurement. The phase angle between two low-frequency signals can be measured from one cycle of the waveform, which is around 20 ms for power signals. Since it is based upon zero crossing, this method is sensitive to distortions in the signal, and hence the signal should be smoothed before measurement. Also, better accuracy can be obtained if we base the measurement on few cycles, thus making the measurement interval in the range 100-200 ms. These issues need further investigation.

REFERENCES

[1] R. E. Best, *Phase Locked Loops: Theory, Design, and Applications, Third Edition.* McGraw-Hill Inc. 1996.

[2] A. I. Abu-El-Haija, "Fast Measurement of Telephone Channel Phase Characteristics - Part 1: Method Description," IEE Proceedings F, Communications, Radar and Signal Processing, Vol. 134, No. 5, pp. 481-486, Aug. 1987.

[3] D. M. Wild, "Performance of a DFT in Measuring Phase Angle," B.Sc. Project, University of Saskatchewan, Canada, 1998.

[4] G. Evans, *Practical Numerical Integration.* John Wiley and Sons, 1993.

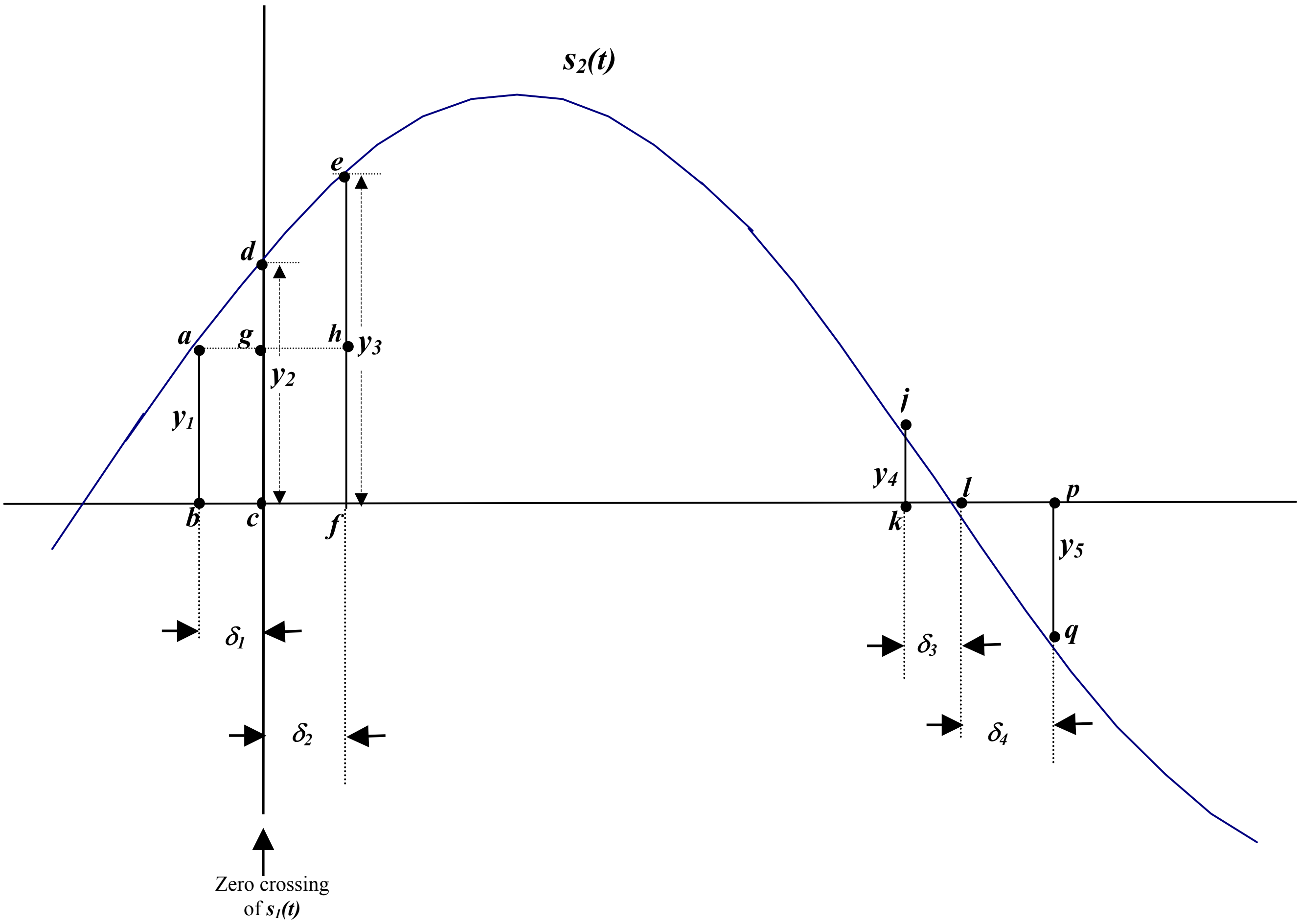

Figure 4. Illustration of how to compute good estimates of the integrals near the zero-crossings of $s_1(t)$ and $s_2(t)$