

SOME ASPECTS OF MEASUREMENT ERRORS ANALYSIS IN ELECTROMAGNETIC FLOW METERS FOR OPEN CHANNELS

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Abstract - This paper deals with some problems of measurement error analysis for flow measurement by electromagnetic flow meter. Two main groups of errors are discussed. The first group of errors is caused by various flow parameters which differ from the flow characteristics during calibration. The authors present a new method of errors estimating, called the moving stream method that allows to estimate on error of a flow meter during the design stage much more precisely than classical methods. The second group of errors is caused by a variation of liquid level in a flow channel during flow meters' operation. Authors propose a selecting an appropriate liquid level as a parameter in designing the exciting coil of electromagnetic flow meter as a best way of that error minimization. Some results of numerical simulation are presented.

1. INTRODUCTION

The electromagnetic technique of flow measurements imposes an artificial magnetic field in a conductive fluid flow and then measures the resulting potential difference across the flow (Fig.1).

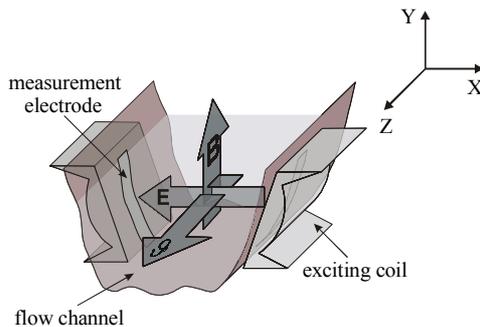


Fig.1. Idea of electromagnetic flow meter

The magnetic flux density vector, \vec{B} , has to be perpendicular to the fluid's velocity vector, \vec{v} . The moving fluid in the flow channel acts as the conductor and the induced voltage between the electrodes, which are placed on both sides of a flow channel, is proportional to the mean velocity of the flow in the area between the electrodes. The measured voltage can be described as follows:

$$U = \int_V \vec{W} \cdot \vec{v} dv \quad (1)$$

where: \vec{W} is the weight vector, ($\vec{W} = \vec{B} \times \vec{J}$, where \vec{J} is the virtual current density vector, \vec{B} is the magnetic flux density vector), U is the voltage across the electrodes, V is the flow volume, \vec{v} is the velocity vector of the liquid.

If the excitation coil of the flow meter generates a field such that

$$\text{rot}(\vec{B} \times \vec{J}) = \text{rot}(\vec{W}) = 0, \quad (2)$$

over the entire domain V [2], then the voltage between the electrodes would be proportional to the mean velocity of unspecified flow, ϑ_m . This kind of flow meters is considered "ideal" [1,4,6].

The main goal of designer is to mould the excited coil in a such way to fulfils the formula (2), but technical limitation allows to meet this requirement only approximately [2,3,6]. Resulting measurement error consists of a two main parts: error which is caused by various flow parameters which differ from the flow characteristics during calibration and error caused by liquid level variation.

From the point of view of the design procedures it is possible to precisely estimate the value of the first error and effectively minimize the value of the second error.

2. ERRORS CAUSED BY VARIOUS FLOW PARAMETERS - METHODOLOGY OF ESTIMATION

Classical methods for estimating the error caused by various flow parameters are based on determining the distribution of the weight vector. [3,6] To determine the distribution of a weight vector, first of all, we must determine the distribution of the virtual current density vector and distribution of the magnetic flux density vector as well.

According to the Bevir method, the distribution of a virtual current density vector in the measurement zone is equivalent to the distribution of a hypothetical unitary current in the area between the electrodes [1]. Because of numerical calculations, the distribution of \vec{J} can be easier found as a solution of the Laplace equation ($\text{div}(\gamma \text{grad} \varphi) = 0$) for

hypothetical electric potential φ with a Dirichlet boundary conditions on measurements electrodes ($\varphi_{e1} = 1, \varphi_{e2} = 0$) [6].

If the analysed problem is linear, than the distribution of \vec{J} can be described, as follows:

$$\vec{J} = -\alpha \gamma \text{grad} \varphi, \quad (3)$$

where α is a coefficient described by formula (4).

The value of the coefficient α depends on the actual shape of the flow channel and the conductivity of the channel bed and banks. Coefficient α can be described by

$$\alpha = \frac{1}{\int_{S_e} \gamma \frac{\partial \varphi}{\partial n} dS} \quad (4),$$

where γ is the conductivity of the liquid, S_e – is the area of 1m length of the electrode, $\int_{S_e} \gamma \frac{\partial \varphi}{\partial n} dS$ is the real value of the electrode current.

For two dimensional calculations the total hypothetical current will be equal 1 A/1m of electrode's length. Once you determine the distribution of \vec{J} then you can begin designing the excitation coil.

The distribution of the magnetic flux density can be calculated (in most cases using finite element analysis and a variable metric method [2]) during the design of the cross-section of the excitation coil in the electromagnetic flowmeter.

The velocity vector, \vec{v} , has only one component, \vec{v}_z , with the appropriate installation of the primary transducer for most natural and artificial flow channels, (Fig.1). Consequently, the three-dimensional problem can be analysed in a two-dimensional co-ordinate system and the formula (2) can be rewritten as follows:

$$J_y B_x - J_x B_y = w \quad (5)$$

In formula (5), the constant w should be chosen to obtain a measurable value of the output signal (1) and yet be sufficiently small to maintain reasonable coil dimensions and supply power.

The coil shape must minimize the following objective function to obtain uniformity in the distribution of w for a given channel configuration:

$$F = \frac{\frac{1}{2} \int_S [w_0 - (J_x B_y - J_y B_x)]^2 ds}{w_0^2 S} \cong \frac{\frac{1}{2} \sum_{i=1}^n [w_0 - (J_{x,i} B_{y,i} - J_{y,i} B_{x,i})]^2 S_i}{w_0^2 S} \quad (6)$$

where: S is the area of a cross-section of analysed channel, w_0 is the desired value of flow signal (for $\int_S v ds = 1$), n is the number of finite elements in which the area S was divided into S_i (Finite Elements Method).

The objective function generally describes the nonhomogeneity of the distribution of \vec{W} . The final value of the objective function can be used to estimate, during the design procedure, the expected error for an electromagnetic flowmeter caused by the variable characteristics of the flow.

The new approach to determining the error of a primary transducer is based on a "moving stream method" derived by the authors. The *moving stream* method allows the designer to check the sensitivity of the transducer's design for various velocity distributions in the flow channel. Therefore, it is possible to estimate the error of a flowmeter for the actual conditions in the field.

The value of objective function (4) is determined mainly by the non-uniform distribution of the vector product $\vec{B} \times \vec{J}$. This non-uniformity reaches maximum values near the electrodes and the channel's walls [2]. When considering the real velocity profile (e.g. $v = 0$ near the walls), it does not seem reasonable to use the final value of the objective function in estimating the error. These results will always be quite different from the actual values of measurement error.

Generating velocity profiles similar to real ones in the channel's cross-section provides the basis for determining the flowmeter's response to variable profiles of velocity. These profiles should provide null boundary conditions ($v = 0$) at the walls of the channel.

The function that describes the velocity profile depends on geometric parameters of the channel and is defined by the following equation:

$$v(x, y) = f(x, y) \cdot h(x, y) \quad (7)$$

where

$$\text{if } \begin{cases} x > p \text{ then } f(x, y) = \cos\left(\frac{\pi/2 \cdot (a(x, y) - a(p, q))}{(1 - a(p, q))}\right) \\ x < p \text{ then } f(x, y) = 0.5 + 0.5 \cdot \sin\left(\frac{a(x, y)}{a(p, q) \cdot \pi/2}\right) \\ y > q \text{ then } h(x, y) = 0.5 + 0.5 \cdot \sin\left(\frac{c(x, y)}{c(p, q) \cdot \pi/2}\right) \\ y < q \text{ then } h(x, y) = \cos\left(\frac{\pi/2 \cdot (c(x, y) - c(p, q))}{(1 - c(p, q))}\right) \end{cases}$$

The variables $a(x, y)$ and $c(x, y)$ are defined as follows:

$$a(x, y) = \frac{x}{w_1 + (d - y)}, \quad c(x, y) = \frac{y}{d}$$

Parameters w_1 , w_2 and d describe the width of the channel bottom, the width of the water's surface, and the depth of the channel respectively. You may determine the point "M" of maximum flow velocity for the given distribution by setting the parameters p and q (coordinates in a two-dimensional model of the flow channel). You can change the simulated velocity profile by changing the coordinates of point "M", but the average speed in a measurement zone always remains constant. If the flow is not caused by the force of gravity, virtually any velocity profile is valid. The range of p and q is limited only by the geometry of the channel. Figure 2 shows

the examples of velocity profiles for half of a rectangular flow channel that measures 1m deep by 2 m wide.

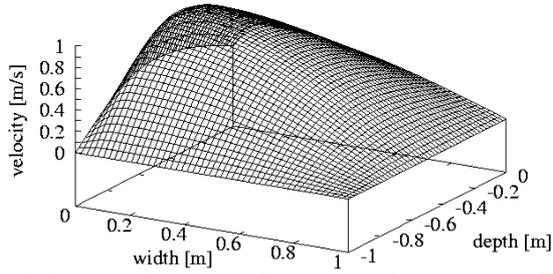


Fig. 2. Example of velocity profiles generated for a rectangular flow channel.

The function defined in (7) can generate a velocity profile very similar to the real one regardless of the driving force, gravity or artificial. For an ideal flow meter, the voltage between the measurement electrodes given by (1) should be constant for all velocity profiles having the same average value.

The estimated error in a flow meter caused by variation in the flow can be defined as the standard deviation of a mean value of electrode potential versus the mean value of electrode potential obtained for a series of different simulated velocity profiles, where the average velocity remained constant:

$$\eta = \sqrt{\frac{1}{(n-1)} \sum_{m=1}^{m=n} (\bar{u} - u_m)^2} \times 100\% \quad (8)$$

where:

n – is the number of simulated velocity profiles,

\bar{u} is the mean value of electrode potential calculated for a series of different velocity profiles,

u_m is the calculated electrode potential for m -th velocity profile.

Several flow channels were analyzed for the expected error by simulating the transducer behavior under different velocity profiles. Table 1 shows results for rectangular (Rect.) and trapezoid (Trap.) channels with various water-to-bed conductivity ratios, γ_w/γ_g (γ - [S/m]).

Tab.1 Results of numerical calculations for various channel cross-sections and conductivity ratios.

Channel	Conductivity ratio [S/m]	Objective function defined by (6) [%]	Error defined by (8) [%]
Rect.	0.1/0.01	5.26	0.278
Rect.	0.01/0.1	11.88	0.387
Trap.	0.1/0.01	4.20	0.199
Trap.	0.01/0.1	14.1	0.247

Sixty different velocity profiles were analyzed for each channel, all having the same mean velocity of 1 m/s. The error was calculated according to formula (8). The moving stream method shows apparent correlation between the error and the final value of the objective function (compare the first and second rows or the third and fourth rows in table 1). We could not estimate the real value of the error by considering the final value of the objective function alone.

3. ERRORS CAUSED BY LIQUID LEVEL VARIATION - METHODOLOGY OF MINIMIZATION

Classical synthesis of a cross-sectional shape of the excitation coil takes into consideration full filling flow channel [2,6]. Full filling of a flow meters' cross-section can be easily achieved in case of closed channels, whereas in open channels such conditions practically never exist. So the initial conditions for design procedure were quite different from reality. During the numerical simulations the changes of a voltage signal across the electrodes as a function of channel's filling were monitored with constant mean velocity of the liquid. Calculations were carried out for a coil designed for fully filled channel. The results indicated signal changes related to the level of the liquid even though theoretically voltage across the electrodes is only a function of the mean velocity. This effect is caused by non-uniform distribution of a weight vector introduced by variable filling.

It can be clearly seen that the acquired shape of the exciting coil is not optimal for applications where liquid level changes are expected and transfer function of such a flow meter is ambiguous. Minimization of errors introduced by changing level of the liquid is conducted mainly by so-called "dry calibration" [3] procedures when the level is constantly monitored and appropriate correction factors are applied. But it turns out, that for extreme level changes and small liquid-to-ground conductivity ratios those coefficients exceed value of one, making this whole procedure quite useless. Authors suggest to choose such level of the liquid during calculations of the shape of the coil, for which the influence of level changes on voltage signal would be minimal.

Assumption of certain level of filling during the designing process affects the results. Hitherto existing results of works, in which classical approach was used, showed, that full filling of channel does not resolve problem univocally. Choices of level of filling based on average one year's filling or random selection also are not best because of necessity of measurement of high states. Diminishing of filling accepted to calculations in approximate manner will not permit fully to verify received results which is caused by non-linear transfer function of a flow meter.

To point out the filling which could be accepted to calculations, over a dozen of coils were projected, each for different filling. Every one of them assured uniform distribution of weight vector only at filling for which it was projected. For every case a different conversion characteristic ($U = f(h)$) was obtained. To measure this characteristic, a series of computer simulation was conducted and voltage across the electrodes was calculated. The flow velocity was set at 1 m/s and filling was changed from 20% to 100% of maximal value. On the basis of those results an optimal coil could be chosen. The results, however, could not be compared directly because of different fillings. The proposed method of comparison was a normalization obtained by multiplying an electrode potential by a following coefficient (9):

$$k = \frac{1}{S_{opt} \cdot w_0} \quad (9)$$

where: S_{opt} - the area of a filled cross section of the flow channel for which the exciting coil was optimized, w_0 - the desired value of coefficient w .

After applying this normalization, a set of n -element data vectors was obtained for each coil. The definition of normalization procedure is so constructed, that for ideal coil a unitary vector is expected. For such an ideal coil the measurement signal depends only on the mean velocity of the flow. With the unity vector as reference, it is possible to find the coil which transfer function (represented by its specific data vector) is closest to the ideal one. The problem of classification can be resolved by choosing an appropriate representation of the signal treated as a vector in space. In that case the representation is a normalized electrode potential. For all data are in a form of vectors, it is natural to use metric spaces as a most convenient tool of classification. Every coil is represented by a n -element sequence of real numbers. Let's consider a set R^n of all such sequences. In this set a metric can be introduced in many ways. For preliminary classification some basic metrics were chosen. Those were the following functionals:

$$d_1(x, y) = \sqrt{\sum_{i=1}^n |x_i - y_i|^2}; \quad d_2(x, y) = \sum_{i=1}^n |x_i - y_i|;$$

$$d_3(x, y) = \sum_{i=1}^n |x_i - y_i|; \quad d_4(x, y) = \frac{1}{n} \sum_{i=1}^n |x_i - y_i|;$$

$$d_5(x, y) = \min \left\{ |x_i - y_i| \right\}; \quad d_6(x, y) = \left(\sum_{i=1}^n |x_i - y_i|^p \right)^{\frac{1}{p}} \quad (5)$$

where: $d_1(x,y) \div d_6(x,y)$ - the value of metric, x_i - sequence or vector of desired values (unity vector), y_i - sequence or vector of measured data ($y_i = k \cdot U_i$).

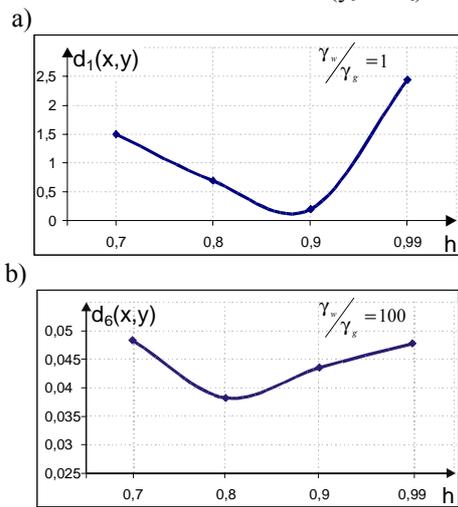


Fig.3. The relations between the metric value and relative water level (h) taken to design process a) for metric d_1 and conductivity ratio $\gamma_w/\gamma_g=1$, b) for metric d_6 and conductivity ratio $\gamma_w/\gamma_g=100$.

The value of metrics is a quantitative value of quality for each representation. For considered metrics, the better

solution is for the smaller values of the functional. This means that given representation approaches optimal expected value. Large values of functional mean that the coil has to be rejected. The relations between the metric value and relative water level taken to design process is shown on Fig. 3 and Fig. 4.

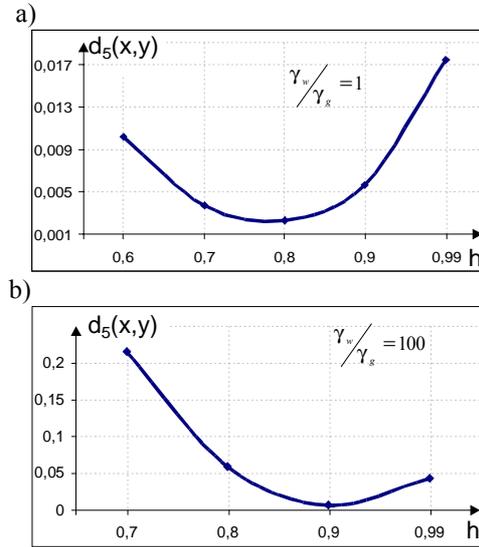


Fig.4. The relations between the metric value and relative water level (h) taken to design process, a) for metric d_5 and conductivity ratio $\gamma_w/\gamma_g=1$, b) for metric d_5 and conductivity ratio $\gamma_w/\gamma_g=100$.

The final decision which representation to choose is not always clear. It is caused by generally large values of functional for fillings lower then 40%. Those values influence the final result and probably only the greater fillings should be considered (Fig.3,4). By analysing the characteristics $d(x,y)$ it can be seen, that there is always an optimal coil, closest to the ideal one. This coil is optimized for a specific filling, which should be taken as a starting point for a optimization process.

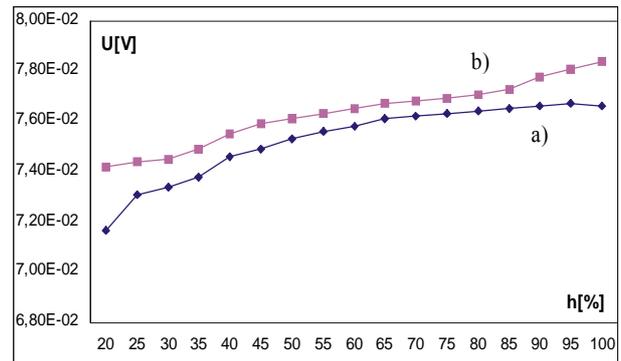


Fig. 5 The relation between flow signal and channels filling for two flow meters, a) - with 100% filling taken to the design procedures, b) - with optimized channel's filling taken to the design procedures, both flow meters were designed for insulated flow channel.

It should be noticed that the filling, for which a functional reaches its minimum depends on many environmental

parameters. The most important of them is ground-to-liquid conductivity ratio. Fig. 5 presents the relation between flow signal and channel's filling for two flow meters, b) - with optimized channel's filling taken to the design procedures a) – with 100% filling taken to the design procedures, both flow meters were designed for insulated flow channel. All calculated metrics, for that flow channel, pointed out the optimal filling at $0.9 h_{max}$.

The testing procedure was carried out for several different flow channels. The obtained final results allows to summarise, that applying the proposed procedure to selecting an appropriate liquid level taken to the design procedures, could effectively minimize (about 20% less) the sensitivity of the flow meter to various liquid level.

As a final testing routine, authors have decided to check the sensitivity of the improved flow meter for various velocity distribution (variable flow parameters). The results of testing are presented in Table 2.

Tab.2 Error generated by variable flow parameters for full and optimal filling for flow channel $2m \times 1m$.

Conductivity ratio [-]	Liquid level [%]	Error defined by (8) [%]
1	80 (optimal)	5,34
	100	10,04
2	80 (optimal)	3,46
	100	10,19
5	90 (optimal)	7,57
	100	10,06
10	90 (optimal)	5,39
	100	9,57
10 000 000	90 (optimal)	6,66
	100	9,43

Designing the excitation coil of a electromagnetic flow meter according to the new procedures of choosing initial values, leads additionally to 50% reduction of error defined by (8). Significant improvement can be obtained especially for ground - to - liquid conductivity ratios close to 1. For insulated channels the error reduction reaches 30%.

4. CONCLUSIONS

From the point of view of the designer and end user of the flow meters is very important to be able to precisely estimate the value of the error caused by various flow parameters as well as effectively minimize the sensitivity of the flow meter to various liquid level in the flow channel. The most important is that all estimations could be done on the design stage. The classical method of error, caused by various flow parameters, estimation, would give an error almost 20 times higher than the value obtained using the moving stream method. (See Table 1.)

This is because the objective function includes a non-uniform distribution of the weight vector in the measurement zone. The moving stream method minimizes this non-uniformity by setting the “real” velocity profile (e.g. $v = 0$ near the walls) existing in the field.

Consequently, the estimated error is much more closer to the actual value found in the field. The reduction of error over the classical method's shows how much the non-uniformity of the distribution of the weight vector can be decreased by applying “real” distributions of the flow velocity.

All simulation carried out for a wide group of a flow channels confirm the dependence between flow signal and liquid level in a flow channel. Primarily that dependence was minimized using “dry calibration procedure”. Presented approach, based on applying a metric space to selecting an appropriate liquid level taken to design procedures, allows to effectively minimize this dependence. The most important is that all calculation and simulation could be done on the design stage. Generally we can say that presented procedure allows to minimize the sensitivity of the flow meter to various liquid level. Additionally we can observe the serious improvement in measurement accuracy. The value of measurement error caused by various flow parameters has been decreasing about 40% in comparison to the flow meter designed for fully filling flow channel. The biggest improvement of measurement accuracy can be obtained when ground-to liquid conductivity ratio is around one.

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