A THEORETICAL APPROACH TO EVALUATE THE VIRTUAL INSTRUMENT MEASUREMENT UNCERTAINTIES

Salvatore Nuccio   Ciro Spataro

Dipartimento di Ingegneria Elettrica - Università degli Studi di Palermo
Viale delle scienze, 90128 - Palermo, ITALY
Phone (+39) 0916566270, e-mail: nuccio@.unipa.it

Abstract – *In the paper an approach to assess the virtual instruments measurement uncertainties is presented. It is a theoretical method, which strictly follows the procedures of the "ISO-Guide to the expression of uncertainty in measurement". By means of this method, the combined standard uncertainty of the measurement result is obtained starting from the standard uncertainties generated by each single source and then trough the standard uncertainty of each acquired sample. To avoid calculating the correlation coefficients, which often is a time consuming and hard task, in the proposed method there are some approximations. The results obtained by means of this approximated theoretical analysis are compared with the ones obtained by means of experimental tests. The good agreement between these results, justifies the approximations of the proposed method.*

Keywords - Virtual Instruments, Uncertainty Estimation; A/D Converters, Digital Signal Processing.

1. INTRODUCTION

In our department we designed and realized some Virtual Instruments (VI) for measurements in the electric power system [1], so we had to evaluate the associated standard uncertainties. But in the Standards and in the scientific literature there is no a systematic approach which deals with this topic. Therefore to characterize these instruments from a metrological point of view, we had to take into account only the procedures described in the "ISO-Guide to the expression of uncertainty in measurement" (GUM) [2].

Starting form the particular considerations we made about those particular instruments [3], we tried to carry out a more systematic treatment, in the hope it could be useful for the characterization of a generic VI.

In fact, in the last years the VIs are becoming more and more widespread, in particular in the industrial environment, and for a correct employment in a quality management system, it is essential to characterise these instruments and to estimate the uncertainties associated with the measurement results [4]. The uncertainty evaluation must be an economically acceptable process, so the evaluation method has to be, even if approximate, easy to apply.

The working principle of a VI is very easy to explain: the physical quantity is transduced in electric signal which is conditioned to be adapted to the successive circuits; the signal is sampled at a frequency at least twice his bandwidth and converted in numerical codes; the acquired samples are processed by the suitable measurement digital signal processing block, usually developed by the user, to obtain the measurement results which are displayed in a virtual panel of the PC monitor.

For a correct VI measurement uncertainty evaluation, according to the GUM, four stages are necessary:

1. Identification of the uncertainty sources which give a contribution to the uncertainty of the measurement result during the transduction of the quantities, the conditioning of the signals and the A/D conversion.
2. Evaluation of the standard uncertainties associated with each source.
3. Composition of these standard uncertainties to obtain the combined standard uncertainty of each acquired sample.
4. Study of how the uncertainties of each acquired sample combine and propagate through the processing algorithms, which, in their turn, are uncertainty sources.

So in the following we deal with the identification of the uncertainty sources and with the evaluation of the standard uncertainties associated with each source (session 2 and 3). In session 4 we analyze how the uncertainties of each acquired sample combine and propagate through the processing algorithms, which, in turn, are uncertainty sources. In session 5 we apply the proposed uncertainty evaluation procedures to a realized virtual instrument and then we verify experimentally the proposed method.

2. THE SOURCES OF UNCERTAINTY IN THE A/D CONVERSION

In this context, we do not consider the errors due to transducers and conditioning accessories. Even if these errors are often predominant compared to the uncertainties generated in the A/D conversion, the transducers and conditioning accessories variety is so wide, that it is necessary to analyze separately each particular situation.

ISBN 972-98115-4-7 © 2001 IT

On the contrary, it is possible to carry out a general treatment in the case of the A/D conversion process.

For as regard this process, the main uncertainty sources are: pre-gain offset and its temperature drift, gain and its temperature drift, long term stability and temperature drift of the onboard calibration reference, integral non-linearity, post-gain offset and its temperature drift, noise, cross-talk, settling time, time jitter, quantization and differential non-linearity [5,6,7].

To characterize the data acquisition boards are often used some parameters which regard the overall behavior of the boards, such as the effective number of bit. But these parameters are not always present in the manufacturers' specifications, so to get them, it is necessary to carry out expensive and laborious tests. They are measured for conventional signals (usually sinusoid) so they have not validity for any other signal and moreover they do not take into account some uncertainty sources such as offset and gain. If well determined, these parameters are useful to characterize the quality of a data acquisition board, whereas they lose their validity when it is necessary to evaluate the VI actual measurement uncertainty. For this purpose, it is our opinion that the starting point must be evaluating the standard uncertainties associated with each uncertainty source.

It can be carried out by means of statistical methods with a Type A evaluation according to the GUM, (but we need a statistically sufficient number of VIs of the same kind), or it is also possible to turn to manufacturers' specifications (Type B evaluation). An uncertainty evaluation starting from the manufacturers' specifications is of course less expensive and less time consuming, since it does not require any kind of test.

For the offset, gain, temperature drift, long term stability and non-linearity errors, the manufacturers declare an interval (a where the error surely lies. According to the GUM, provided that there is no contradictory information, each input quantity deviation has to be considered equally probable to lie anywhere within the interval given by specification, that is modeled by a rectangular probability distribution. The best estimate of the uncertainty is then $u=a/\sqrt{3}$.

The quantization error is usually considered uniformly lying within an interval of 1 LSB, so the best estimate of the standard uncertainty is $1/\sqrt{12}$ LSB.

The standard uncertainty related to noise can be directly obtained from the technical specifications, since it is usually expressed as rms value.

The cross-talk errors are produced by the interference in the multi-channel acquisition. Its related uncertainty is expressed as minimum ratio between the signal rms value and the interference signal rms value.

The settling time is the amount of time required for a signal that is amplified to reach a certain accuracy and stay within the specified range of accuracy. The manufacturer declares this range for the maximum sampling rate and for the full scale step, but the errors on the measured signal depend on the actual sampling rate and on the actual signal step.

Impact of time jitter uncertainties of measuring chain is being transformed on the signal uncertainty as a function of signal derivatives. For the worst case, it is possible to use the following expression [8]:

$$u_{jitter} = 2^{-\left(\log_2\left(\frac{2}{\sqrt{3}\pi f_x t_a}\right)-1\right)} \cdot V_{range} \tag{1}$$

where t_a is the rms aperture jitter and fx is the maximum frequency component of the signal.

All these uncertainties can be considered not correlated, so the standard uncertainty of each acquired sample can be calculated as the root sum square of the standard uncertainties of every considered source.

3. THE SOURCES OF UNCERTAINTY IN THE DIGITAL SIGNAL PROCESSING BLOCK

As regards the digital signal processing (DSP) we have to take into account the bias of the processing algorithms and the uncertainties related with the rounding phenomena. The algorithm bias is caused by the finite implementation of the measurement algorithms and represents the deviation of the actually measured result with respect to the theoretical response which the instrument should give. The estimate of the bias is often a very hard task, since it depends on the input signals as well as the algorithms. The search of the worst case could be useful to find an upper limit to the uncertainty. In many cases, the lack of knowledge of the bias becomes the main uncertainty source.

The rounding phenomenon is caused by the microprocessor finite wordlength. It can occur in every multiplication carried out in a fixed-point representation and in every addition and multiplication carried out in a floating-point representation. The related uncertainties can be modeled as follows [9]:

$$u_{fixed} = \sqrt{\frac{2^{-2 \cdot B_x}}{12}} \tag{2}$$

for each multiplication carried out by a fixed-point processor;

$$u_{floating,multipl} = \sqrt{0.18 \cdot 2^{-2 \cdot B_m}} \tag{3}$$

$$u_{floating,add} = \sqrt{p \cdot 0.18 \cdot 2^{-2 \cdot B_m}} \tag{4}$$

for each multiplication and addition, respectively, carried out by a floating-point processor, where Bx is the fixed wordlength, Bm is the number of bit used to represent the mantissa and p is a factor depending on the probability of rounding occurrence in an addition.

As regards the software for the data acquisition board control and for the user interface, we can say that this do not contribute to the measurement result uncertainty, but obviously it is essential for a correct working of the VI.

4. PROPAGATION OF THE UNCERTAINTIES

The VI measurement result is a function of many acquired samples and this function is described by the measurement DSP algorithm. So to evaluate the measurement result uncertainty we have to know how the uncertainties of each acquired sample propagate through the DSP block.

When a measurand estimate y is determined from N other samples $x_1, x_2, \ldots, x_N$, through a functional relation $y = f(x_1, x_2, \ldots, x_N)$, the combined standard uncertainty estimate $u_c(y)$ of the measurement result is the positive square root of the estimated variance $u^2_c(y)$, obtained from:

$$u_c^2(y) = \sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + 2\sum_{i=1}^{N-1}\sum_{j=1, j\neq i}^{N}\frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j} r(x_i, x_j)u(x_i)u(x_j) \quad (5)$$

where $u(x_i)$ is the estimated standard uncertainty associated with the sample estimate x_i and $r(x_i,x_j)$ is the estimated correlation coefficient associated with the samples x_i and x_j.

The evaluation of the correlation coefficients is often a very hard task, also because they are strictly depending on the input signal. On the other hand to ignore the correlations could cause an heavy underestimate of the uncertainties. So the theoretical approach seems to be economically inapplicable because of difficulties in the exact identification of correlation coefficients.

But if we consider separately each uncertainty source, we can observe that as for the offset and its temperature drift, the gain and its temperature drift and the long term stability, the correlation coefficients are approximately equal to 1, while as regards the other uncertainty source the correlation coefficients can be supposed equal to 0. Moreover, in case of errors due to gain, the relative standard uncertainty $u_r(x)=u(x)/|x|$ has to be considered constant on each input sample. In all other cases it is the absolute standard uncertainty u, to be considered constant on each input sample.

Therefore all the uncertainty sources can be divided approximately in three classes:

I. completely correlated input quantities and $u_I=const$;
II. completely correlated input quantities and $u_{rII}=const$ (u_r = relative uncertainty);
III. not correlated input quantities and $u_{III}=const$.

The same uncertainty source can belong to different classes; for example the thermal drift offset belongs to the first class for an rms value measurement of a signal, but it belongs to the third class for the daily mean temperature calculated from 24 samples measured at each hour.

In this way, it is not necessary to calculate the correlation coefficients and moreover the uncertainty propagation law becomes easier to apply, that is respectively for the three classes:

$$u_{cI}(y) = u_I \sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\right) \quad (6)$$

$$u_{cII}(y) = u_{rII} \sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i} x_i\right) \quad (7)$$

$$u_{cIII}(y) = u_{III}\sqrt{\sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\right)^2} \quad (8)$$

Starting from the above considerations, the idea of an approximated theoretical method has arisen. It consists of the following steps:

- to subdivide the uncertainty sources in the three classes;
- to carry out the root sum square of the uncertainties of each class, obtaining three values of uncertainty for each acquired sample;
- to apply the propagation law separately for each source class, getting three standard uncertainty values u_{cI}, u_{cII}, u_{cIII}..
- to carry out the root sum square of these three values obtaining the combined standard uncertainty of the measurement result.

There are some approximations in this method: the first one is implicit in the propagation law which is based on a first-order Taylor series approximation of y; the second one consists of combining the uncertainties after they are propagated, whereas actually the uncertainties first are combined in each acquired sample and then propagate through the software block; the last approximation is the subdivision of the uncertainty sources in the three classes with supposed correlation coefficient exactly equal to 1 or 0.

5. VALIDATION OF THE PROPOSED APPROACHES

With the aim of verifying the proposed methods, we applied both of them on various DSP basic blocks which are typical of a measurement chain, as RMS, DFT, IDFT, FFT, IFFT, FIR filters, IIR filters, windowing and so on.

As examples in the following, the procedure for the mean of 2000 samples, the RMS of 2000 samples and a finite impulse response (FIR) filter with 11 coefficients, is reported.

The VI is constituted of a Krokn-Hite™ VIII order Butterworth lowpass filter, the National Instruments™ AT-MIO-16E10 data acquisition board (DAQ) (16 single-ended or 8 differential channels, successive approximation 12 bit ADC, 100 kS/s max sampling rate, ± 10 V maximum input signal range) and a PC with an INTEL™ 200 MHz processor; the LabView™ 5.1 is the programming language used to drive the acquisition board, to process the acquired samples and to realize the user interface. The sampling rate is 10 KS/s.

The example test signal is a 2 KHz sinusoid with a 9 V peak value and is generated by the National Instruments™ PCI-MIO-16XE10 board with a 16 bit D/A converter. The sampling is coherent with the generated sinusoid, so in this way, the bias of the three algorithms is equal to 0.

We consider a Type B evaluation of standard uncertainties, based on manufacturer's specifications, assume rectangular distributions and suppose to operate within ± 1 K of the DAQ self-calibration temperature, within ± 10 K of factory calibration temperature, after one year of the factory calibration and to set the gain equal to 0.5.

In Table I the considered uncertainty sources, the manufacturer specification and the standard uncertainty values (evaluated as in session II) are reported.

Table I - Uncertainty sources

Uncertainty source	Manufacturer specification	Class	Standard uncertainty
pregain offset	± 2 µV	I	1.2 µV
post gain offset	± 1000 µV	I	577 µV
pregain offset temperature coefficient	± 15 µV/°C	I	8.7 µV
postgain offset temperature coefficient	± 480 µV/°C	I	277 µV
gain	0,05 %	II	289 ppm
gain temperature coefficient	± 20 ppm/°C	II	12 ppm
temperature coefficients of the onboard calibration reference	± 5 ppm/°C	II	2.9 ppm
long term stability of the onboard calibration reference	± 15 ppm/√(1000 h)	II	25 ppm
INL	± 1 LSB	III	2819 µV
DNL	± 0.5 LSB	III	1410 µV
quantization	± 0.5 LSB	III	1410 µV
noise	0.07 LSB rms	III	342 µV
settling time for full scale step	± 0.1 LSB in 100 µs	III	282 µV
time jitter	± 5 ps	III	140 µV
cross talk	- 80 dB	III	707 µV

Because the number of bits used to represent the mantissa is equal to 52, the uncertainties introduced by microprocessor finite wordlength are negligible compared with the other ones.

To apply the proposed theoretical method we have to carry out, as first step. the root sum square of the uncertainties of each class, obtaining the three values of uncertainty for each acquired sample:

u_I = 640µV u_{rII} = 290 ppm u_{III} = 3555 µV.

The second step consists of applying the uncertainty propagation law.

Let us consider the mean of N samples. In this case

$$y = \frac{\sum_{i=1}^{N} x_i}{N} \quad \text{and} \quad \frac{\partial f}{\partial x_i} = \frac{1}{N} \tag{9,10}$$

so the (6),(7) and (8) become respectively:

$$u_{cI}(y) = u_I \tag{11}$$

$$u_{cII}(y) = u_{rII}\frac{\sum_{i=1}^{N} x_i}{N} \Rightarrow u_{r\,cII}(y) = u_{rII}(x) \tag{12}$$

$$u_{cIII}(y) = \frac{u_{III}}{\sqrt{N}} \tag{13}$$

For the RMS value of N samples:

$$y = \sqrt{\frac{\sum_{i=1}^{N} x_i^2}{N}} \tag{14}$$

$$\frac{\partial f}{\partial x_i} = \frac{x_i}{N\sqrt{\frac{\sum_{i=1}^{N} x_i^2}{N}}} \tag{15}$$

so the (6),(7) and (8) become respectively:

$$u_{cI}(y) = u_I \frac{\sum_{i=1}^{N} x_i}{N} \sqrt{\frac{N}{\sum_{i=1}^{N} x_i^2}} \tag{16}$$

$$u_{cII}(y) = u_{rII}\sqrt{\frac{\sum_{i=1}^{N} x_i^2}{N}} \Rightarrow u_{r\,cII}(y) = u_{rII}(x) \tag{17}$$

$$u_{cIII}(y) = \frac{u_{III}}{\sqrt{N}} \tag{18}$$

As regarding a finite impulse response filter:

$$y = \sum_{i=1}^{N} a_i x_i \quad \text{and} \quad \frac{\partial f}{\partial x_i} = a_i \qquad (19,20)$$

where a_i are the coefficient of the filter.

In this case the (6),(7) and (8) become respectively:

$$u_{cI}(y) = u_I \sum_{i=1}^{N} a_i \qquad (21)$$

$$u_{cII}(y) = u_{rII} \sum_{i=1}^{N} a_i x_i \Rightarrow u_{rcII}(y) = u_{rII}(x) \qquad (22)$$

$$u_{cIII}(y) = u_{III} \sqrt{\sum_{i=1}^{N} a_i^2} \qquad (23)$$

At last, carrying out the root sum square of u_{cI}, u_{cII} and u_{cIII} we get the combined standard uncertainty of the measurement result.

In Table II the so obtained values of the combined uncertainty are reported. The measurands are respectively the mean, the RMS value and the filtered sinusoid peak value.

In the same table we report also the results of the experimental tests, obtained from a set of 10000 measurements. The experimental obtained uncertainties are (as prescribed in the GUM) the root sum square of the uncertainty actually measured and of the uncertainties due to offset, gain, temperature drift and non-linearity because the last ones, having a systematic behavior, cannot be pointed out as uncertainty in a single DAQ test.

Table II - Combined standard uncertainty

Algorithm	**Theoretical uncertainty**	**Experimental uncertainty**
Mean	645 µV	512 µV
RMS	1847 µV	1532 µV
FIR filter	2895 µV	2365 µV

The experimental results are lower than the theoretical obtained ones, also without considering the uncertainties introduced in the signal generation process and in anti-alias filtering. Therefore, these results validate the considered model and the values of the various uncertainty sources of the utilized data acquisition board.

6. CONCLUSIONS

In this paper the problem of characterization, in terms of uncertainty, of the virtual instruments performances has been considered.

As prescribed in the GUM, the combined output uncertainty of the measurement result is obtained starting from each uncertainty source, without taking into account the parameters which regard the overall behavior of an acquisition board, such as the effective number of bits.

To study how the uncertainties of each acquired sample propagate through the DSP block, an approximated theoretical method has been introduced.

The main advantage of this method is that it is no necessary to calculate the correlation coefficients as prescribed in the GUM, avoiding this hard and sometimes impracticable task.

The results obtained by using this approach have been compared with the ones obtained by means of experimental tests, validating the considered model and the values of the various uncertainty sources of the utilized data acquisition board.

We carried out other experimental tests on other DSP blocks, using various signal and other acquisition board and also in these cases the results validate the proposed approach.

Our method could be extended also to the uncertainties generated by the transducers and the signal conditioning accessories. These ones are often preponderant compared to the uncertainties generated in the A/D conversion stage, which in turn are often much greater than the uncertainties generated during the digital signal processing.

REFERENCES

[1] S.Caldara, S.Nuccio, C.Spataro: A virtual instrument for measurement of flicker, *IEEE Trans. on Instrumentation and Measurement*, Vol. 47, N.4, August 1998.

[2] ISO - Guide to the Expression of Uncertainty in Measurement (1995).

[3] S. Nuccio, C. Spataro: Assessment of Virtual Instruments Measurement Uncertainty, Computer Standards & Interfaces, Vol. 23, N. 1, 2001, pp. 39-46.

[4] ISO 9000 Series - Quality Management and Quality Assurance Standards .

[5] IEEE Std 1057, 1994, IEEE Standard for Digitizing Waveform Recorder.

[6] IEEE Std 1241, 1998, IEEE Draft Standard for Terminology and Test Methods for Analog-to-Digital Converters.

[7] DYNAD - SMT4-CT98-2214, Methods and draft standards for the DYNamic characterisation and testing of Analogue to Digital converters.

[8] R. H. Walden: Analog-to-Digital Converter Survey and Analysis, IEEE Journal on selected areas in communication, Vol.17, N. 4, April 1999.

[9] K. Kalliojarvi, J. Astola: Roundoff Errors in Block-floating-point System, IEEE Trans. on Signal Processing, Vol.44, N. 4, 1996, pp. 783-790.