EVALUATION OF FREQUENCY DEPENDENCY OF EFFECTIVE NUMBER OF BITS BY MEANS OF STOCHASTIC TESTING SIGNAL

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Abstract – *The article describes a new method of Analog-to-Digital Converters testing that is suitable for testing of high-resolution AD converters (e.g. Σ-Δ or dither-based) or on the contrary ultra high-speed AD converters. The method is based on the histogram test driven by stochastic signal with defined probability density function. By repeating of the test for different settings of band-pass filter that is inserted to the input testing signal path it is possible to obtain an estimation of frequency dependency of effective number of bits. This important information was obtainable by deterministic test only. Practical demonstration confirmed the usability of the method.*

Keywords - ADC, ENOB, testing, stochastic signal

1. INTRODUCTION

Testing of AD converters by means of deterministic testing signal becomes problem when low sampling frequency, high-resolution AD converters (e.g. Σ-Δ or dither-based) or on the contrary ultra high-speed AD converters (e.g. with opto-electronic core) are to be tested. In such cases, stochastic input signals seem to be better applicable for testing [1], [2].

Histogram test can use various input signals and principally allows use noise as the input signal. To overcome difficulties related to generation of large-scale uniformly distributed stochastic signal, a method based on superposition of Gaussian noises with equidistantly spaced DC shifts (Fig.1) has been proposed [3] and theoretical analysis has been provided. It is necessary to summarize it in the following chapter.

2. HISTOGRAM STOCHASTIC TEST

Let us have Gaussian noise with probability density function (p.d.f.) $f_G(\mu,\sigma)$ where μ is the mean value and σ means standard deviation. It is easy to show that

$$\lim_{\Delta\to 0}\left[\left(\sum_{k=-\infty}^{\infty} f_G(\mu + k.\Delta,\sigma)\right) - \frac{1}{\Delta}\right] = 0 \text{, } k \text{ integer} \quad (1)$$

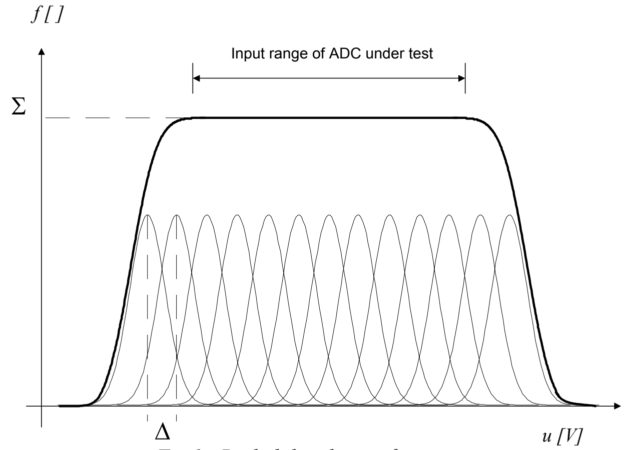

Fig.1 - Probability density function of the proposed stochastic signal

Independently on the value of μ and σ. In other words, the superposition of Gaussian distributed noises with the equidistantly spaced DC values by step Δ is for suitably small values of Δ an excellent approximation of uniformly distributed signal. Measurement is provided for each Gaussian noise separately but histograms are cumulated. Theoretical analysis of such signal has been provided in [3]. The sensitivity of resulting probability density function to the variances in DC positions of Gaussian sub-signals, to variations of their power (r.m.s. value) and the border error was described there. It is important to know that from that analysis follows that testing signal that can be used to test high-resolution ADCs can be built in this way. Moreover, it is useful to note that in the case that particular Gaussian stochastic signals are generated by DAC, the used DAC should provide better accuracy than only the relevant portion of tested ADC. It means it is not necessary to compare the linearity of the full scales of tested ADC and testing DAC.
The following formula should be used to calculate the necessary amount of samples to achieve a required accuracy:

$$k = m \cdot \frac{a^2}{\varepsilon^2} \quad (2)$$

where k is number of required samples, m is number of code words of the tested ADC, $a = 1,96$ for 5% confidence level of DNL evaluation and ε is the statistical error of DNL evaluation (5% in our case).

3. ENOB CALCULATION

ENOB (Effective Number of Bits) calculation follows the usual way that is described in [3] or in IEEE-STD-1241. In the concrete, DNL values are estimated from the measured histogram and INL values are then calculated using DNL:

$$DNL_i = \frac{Oi - O}{O} \qquad INLj = -\sum_{i=1}^{m-2} DNL_i \tag{3,4}$$

where O_i is the value of the i-th code of cumulative histogram and O is its ideal value that can be achieved by the following way (the common way of indexing of code words 0,1,…,m-1 is expected) :

$$O = \frac{\sum_{i=1}^{m-2} O_i}{m-2} \tag{5}$$

Assuming statistical independence of INL values and quantisation error, the standard deviation of ADC output can be calculated as

$$\sigma_c = \sqrt{\frac{1}{12} + \frac{1}{m-2}\sum_{i=1}^{m-2} INL_i^2} \tag{6}$$

Then, ENOB can be calculated as

$$ENOB = \log_2 \frac{m}{\sigma_c \sqrt{12}} \tag{7}$$

The practical applicability of this method has been verified by comparison of results (Tab.1) obtained by this method and by standard histogram test using deterministic (ramp) testing signal for an internal ADC of digitizing oscilloscope [4].

4. FREQUENCY DEPENDENCY EVALUATION

One can see that both methods provide comparable results. The only problem that remains is the nature of measured Effective Number of Bits (ENOB). While deterministic version provides ENOB for almost static case (the frequency of the input ramp has to be very low), the stochastic version as described above provides “wide-band” results (depending on the spectral characteristic of the noise used) and, therefore, ENOB is usually lower. Moreover, by repeating the deterministic version of histogram test for sine-wave input signals with different frequencies, one can obtain (after recalculation according to probability density function of sinusoidal signal) the frequency dependency of ENOB. This is very useful piece of information that the stochastic version of histogram test could not provide yet.

Tab.1 - Comparison between standard and stochastic variants of histogram test

	STOCHASTIC TEST SIGNAL (as described above)	DETERMINISTIC TEST SIGNAL (ramp)
DNL max	0,60	0,63
DNL min	-0,68	-0,66
INL max	0,42	0,47
INL min	-1,70	-1,52
SNR [dB]	39,9	40,3
ENOB	6,33	6,39

However, it is possible to obtain a similar dependency – integral ENOB for particular (narrow) frequency bands. It is necessary to include a tunable band-pass filter to an input signal path (Fig.2) and repeat the test procedure for different settings of middle frequency of this filter. In this case, the variance of (wide-band) noise source has to be high enough, or, in different words, the step of DC calibrator has to be small enough.

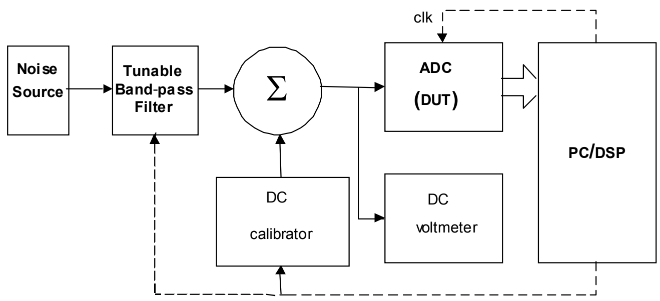

Fig.2 - Measurement of frequency dependency of effective number of bits using stochastic histogram test

Generally, there are four parameters/effects that has to be considered to evaluate suitable ration between DC step and RMS value of noise used. These are:
-sampling frequency of the tested ADC
-antialiasing filter of the tested ADC, if any
-bandwidth of the used tunable band-pass filter
-spectral aspects of the used noise generator

As for sampling frequency, there is no direct relation between noise bandwidth and sampling frequency. The equivalent noise bandwidth can be many times higher than one half of the used sampling frequency (it means it is not necessary to fulfill sampling theorem). It is easy to show that all the noise energy is mirrored/shifted to the useful frequency range of the ADC.

On the other hand, antialiasing filter (if any) influences directly the power of the noise delivered to input of the tested ADC. Therefore, it should be set to the highest possible value (if tunable). Moreover, its cut-off frequency f_{AL} has to be considered as the bandwidth-limiting factor for the equivalent noise-bandwidth F_N.

Bandwidth of the used tunable band-pass filter F_{BP} has to be smaller enough in comparison with sampling frequency to produce enough detailed result (ENOB frequency dependency).

Spectral content of the noise source can be described using the following approximation. We assume that in the useful range (which is usually determined by antialiasing cut-off frequency f_{AL}) the noise has white character, that means that the energy is uniformly distributed over all the frequency range $(0,f_{AL})$. If there is no such filter, the whole equivalent bandwidth F_N has to be considered.

To design a proper equivalent RMS value of the generated noise for this test, the following relation has been derived:

$$\frac{U_{NRMS}^2}{U_{ENRMS}^2} = \frac{\min(f_{AL}, F_N)}{F_{BP}} \tag{8}$$

where U_{NRMS} is the RMS value of the generated noise, distributed in the frequency range F_N. U_{ENRMS} is the required equivalent noise RMS value (σ) for noise generator setting (1).

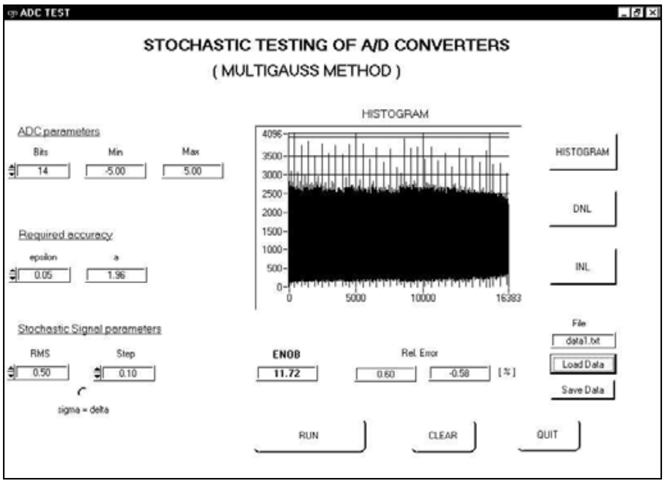

Fig.3 User interface of the developed testing system

5. RESULTS

To verify above mentioned procedures, the ADC plug-in card AD14 (nominally 14 bits, 250 kSa/s) has been tested. Such object can be tested by means of standard test procedures (IEEE 1241) as well because acceptably accurate deterministic testing signal (e.g. harmonic) is available. Fig.3 shows the user interface of the test software that has been developed for stochastic tests of ADC.

The schematic diagram of the system is identical with the one at Fig. 2. A common 12-bit arbitrary generator HP 33120A has been used to generate the normally distributed noise with uniformly distributed power in the area $(0,F_N.)$. It is declared by the vendor and it was confirmed by spectrum analysis that its F_N=15 MHz. Fig. 4 shows measured DNL of the ADC for the following settings: σ = 0,5V, Δ = 0,1V, number of samples 25.10^6.

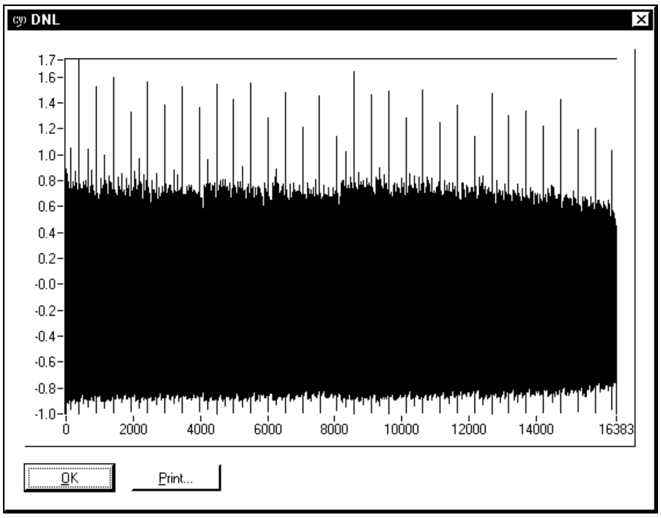

Fig. 4 -Differential non-linearity of ADC on plug-in card AD14 without filtering ("wide-band ENOB")

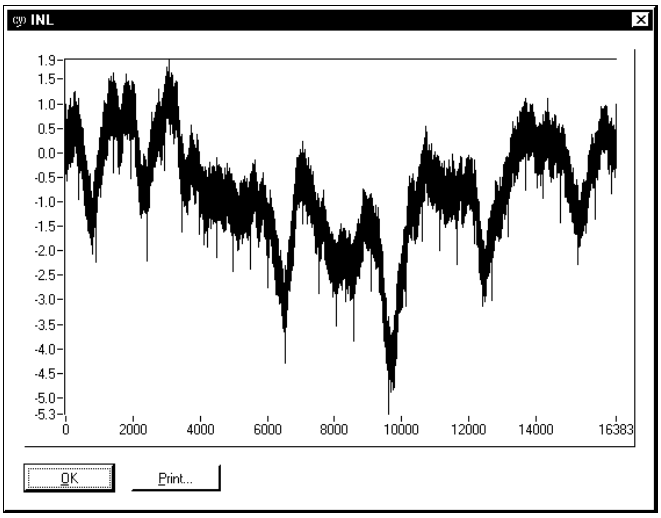

Fig.5 - Integrated non-linearity of ADC on plug-in card AD14 without filtering ("wide-band ENOB")

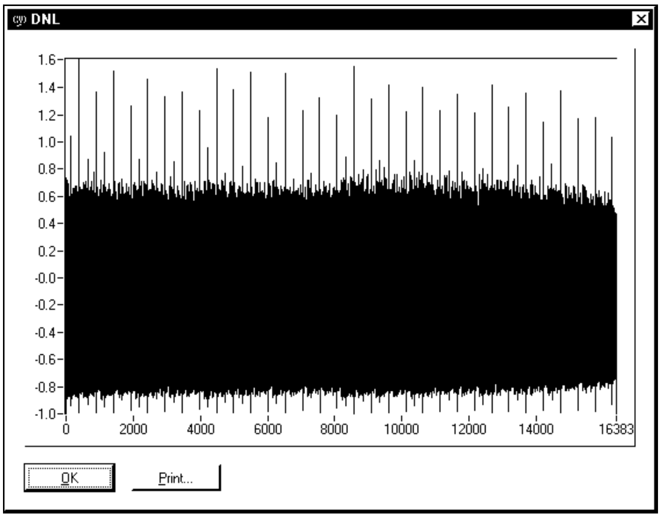

Fig. 6 - Differential non-linearity of of ADC on plug-in card AD14 with BP filter, F_{BP}=10 kHz, center frequency 100 kHz

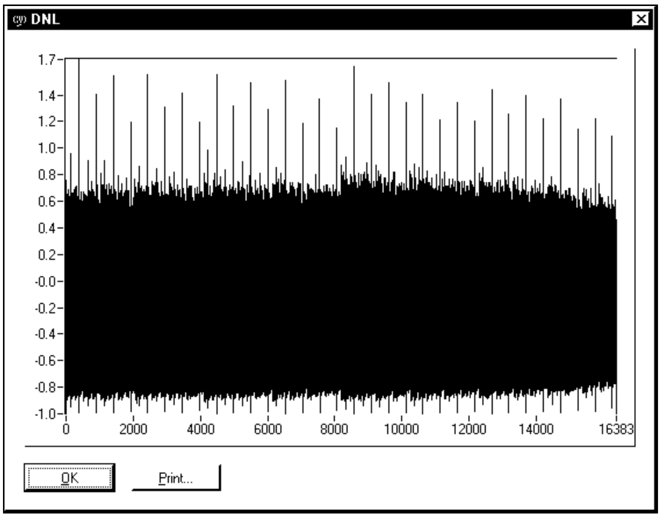

Fig. 8 - Differential non-linearity of of ADC on plug-in card AD14 with BP filter, F_{BP}=10 kHz, center frequency 5 kHz

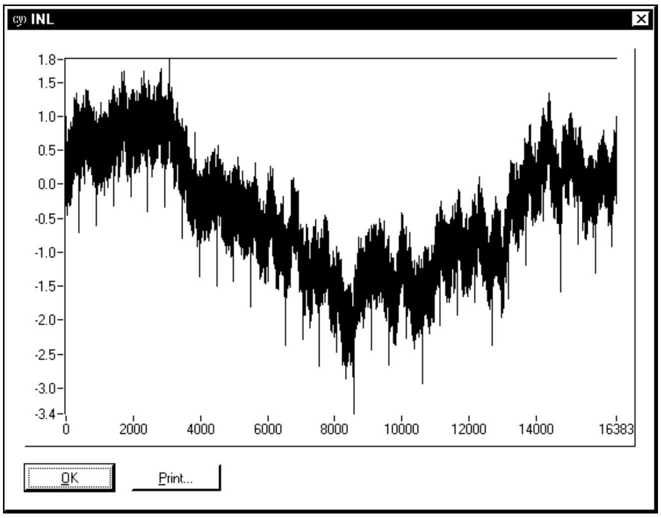

Fig. 7 - Integral non-linearity of of ADC on plug-in card AD14 with BP filter, F_{BP}=10 kHz, center frequency 100 kHz

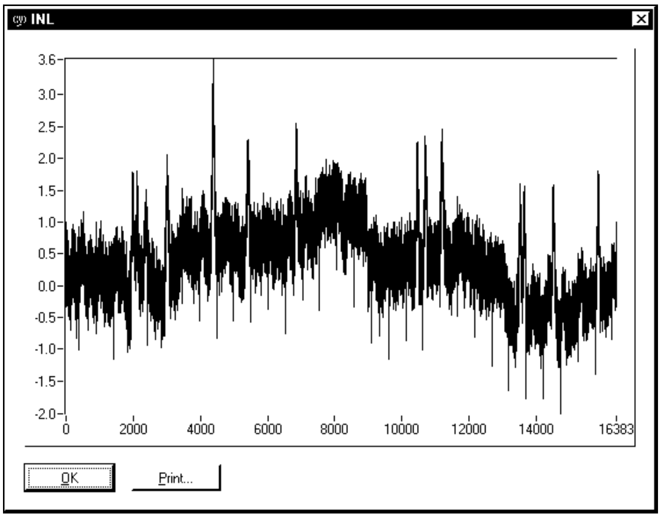

Fig. 9 - Differential non-linearity of of ADC on plug-in card AD14 with BP filter, F_{BP}=10 kHz, center frequency 5 kHz

Fig. 5 shows the calculated INL for the same settings σ=0,5V, Δ=0,1V, number of samples 25.10^6. The calculated ENOB for this case is 11,72 bit (+ 0,6% -0,58%), according to (7).

To introduce band-pass filter as depicted at Fig.2, it was necessary to increase the RMS value of the generated wide-band noise according to (8). In our case, since the F_{BP}=10kHz, the settings have been changed to the values σ=0,5V, Δ=0,01V and the test was repeated for various positions of the centre frequency of the band-pass filter. To show the non-valid results when (8) is not taken into account, the original generator settings (σ = 0,5V, Δ = 0,1V) have been used with filter connected. The cumulated probability density function is not uniform in this case (Fig. 10).

Tab.2 - Measured Effective Number of Bits for different settings of band-pass filter

	no BP filter	10kHzBP @5kHz	10kHzBP @100kHz
ENOB (bits)	**11,72**	**12,56**	**12,18**

CONCLUSIONS

Stochastic testing of ADC enables to test some kinds of ADCs that can not be tested satisfactorily by deterministic methods yet. Even the estimation of ENOB dependency on input signal frequency can be obtained. Due to high amount of samples required, testing of low sampling frequency is problematic.

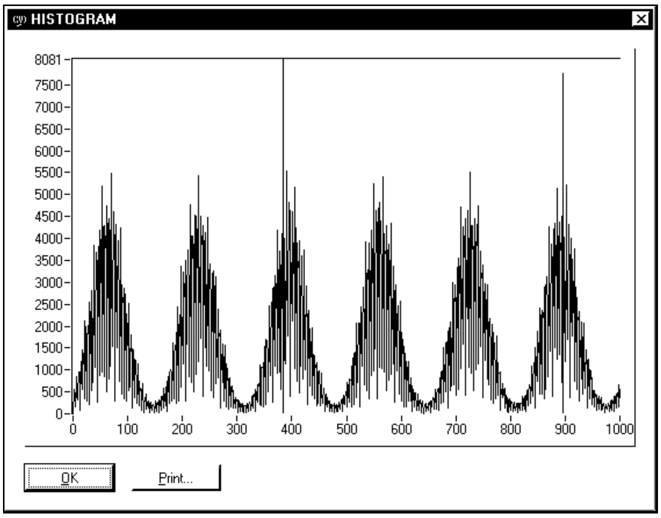

Fig.10 - The wrong setting of noise generator after filter introduction. Cumulated probability density function is not uniform in this case.

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