ANALOGICAL PRE-ESTIMATOR FOR ENHANCED MAXIMUM LIKELIHOOD ESTIMATOR (MLE) APPLIED TO ULTRASOUND SIGNAL ACCURATE DETECTION

Maurizio Caciotta

Department of Electronic Engineering, Rome, via della Vasca Navale, 84 – 00146 Italy
Phone (0039) 06 55177029 Fax (0039) 5579078 e-mail: caciotta@misure.ele.uniroma3.it

Abstract – *The convenience to analyze ultrasound signals with a digital trasform is compared with the application of an analogic module extractor. The accuracy the analogical apparatus can reach, is compatible with the optimal input that needs an improved maximum likelihood analysis. The computing heaviness is relieved by the possibility to pre-estimate the echoes parameters and by the elimination, considering time occurrence, of reflected echoes. The parallel adapting of the recursive algorithm to a hardware parallel structure is discussed to make possible the real time application.*

Keywords – Module extractor, useful echoes, parallel systems, high accuracy spatial measurements

1. INTRODUCTION

Echographic systems are measurement apparata widespread essentially in medical diagnosis. A probe utters mechanical ultrasonic waves, which echoes, collected by the same probe in tissues having different mechanic characteristics. From such characteristic one wants pull up information about precise position with respect to the probe. The action principle is the echo treatment that gets a series of problems about the pull up and the analysis of the signal. The uttered mechanical waves have a proper f_0 frequency and the echo will be formed by an oscillations train with frequency between ($f_0 \pm \Delta f_0$) limits. The frequency displacements are caused by Doppler effect induced by fluids crossed by the mechanical stress. The position information is stored in the time modulation function of the received signal. It is usual that ultrasonic waves cross throughout many slabs of different mechanic properties, rising "multiple echoes" due to many reflection between internal layers that can confuse useful information that is instead centered in the "first reflections" of discontinuity walls.

2. THE PROBLEM ANALYSIS

Problems are complex and must be scaled. First one must choice treatment system of the received signal. Echoes will be received in different time depending on signal lingering in the investigated structure, with the possibility that relative wave trains have different frequency with respect to supplied one. The wavelet transforms seem the most qualified to successfully obtain an efficient analysis, above all if they are bi-orthonormal. The further conditioning that the shape of functions modulating the echoes wave train must be gaussian suggests the Gabor transform. [1]. These kind of transform has the characteristic to give the best possible merit figure in the time-frequency domain that, moreover, in the window of our interest gives bi-orthonormal functions [2,3,4,5] that allow the best efficiency of representation. The f_0 frequency utilized in echographic systems is between 1 and 10 MHz and the maximum bound due to the Doppler effect is of about 10%. The sampling frequency for a correct collected waves shape analysis, can be of the order of 100 MHz with high sensitivity (12-16 bits) that causes an heavy management problem of data. Taking into account the low frequency dispersion and considering the information is centered on modulation function of wave train, not being essential the frequency analysis, it is more interesting to individuate, in the plane time-frequency a way driven by the carrier frequency of the information. The substitution of a complete digitized analysis of the signal, with an analogical one, able to follow the way of carrier in the time-frequency plane, results very interesting. The analogical structure that can be considered is the Phase Locked Loop (P.L.L.) from which is possible to extract in real time the modulation function. The accuracy valuation of this operation is fundamental to establish the implementation possibility of the system that can be considered as a pre-analysis apparatus. The pre-analysis section after allows a digital sampling with frequency much more lower (10 MHz) and moreover in real time and not in time equivalent scale. All received echoes will be identified, applying a recursive algorithm, able, for each one, to individuate position, amplitude and dispersion. The identification will be influenced by the analogical pre-analysis section, but it will allow, by an algorithm based on time recurrences, to obtain the position of useful echoes. Parameters of such echoes will be the initial input of a re-built section based on a Enhanced Maximum Likelihood Estimator. The computing allows great accuracy, but it is very heavy and it requires a parallel computational hardware.

3. ANALOGICAL PRE-ANALYSIS

Analogical synthesis of the Gabor transform is an application of a system based on P.L.L. able to operate with quickness and precision. The classic scheme of such an apparatus is shown in Fig. 1

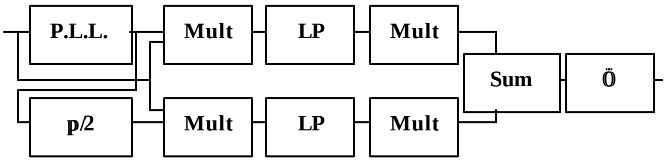

Fig.1: Basic scheme of a module extractor

3.1 The Phase Locked Loop

The scheme of a Phase Locked Loop is shown in Fig. 2.

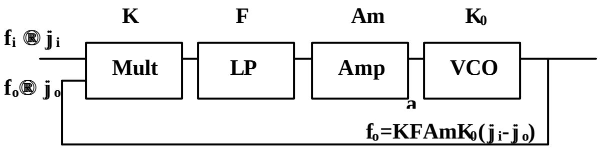

Fig. 2: P.L.L. with multiplier as a phase detector

The phase detector realized by a multiplier causes a frequency modulation of f_0, in the point (a) of Fig.2, with an amplitude α. The choice of a little value of K with an high value of FA_mK_0 allow to write the relation [6]:

$$\omega \cong \frac{\omega_i K s^2}{s\left[\left(1+\frac{s}{FA_mK_0}\right)Ks-\alpha\right]} \tag{1}$$

between input and output frequency. Considering $\omega_i = \Delta\omega_i/s$ the (1) furnishes a frequency transient

$$\omega(t) \cong \frac{2\Delta\omega_i e^{-\frac{1}{2}KFA_mK_0 t}}{\sqrt{1+\frac{4\alpha}{K^2FA_mK_0}}} \sin\left(-\frac{KFA_mK_0}{2}\sqrt{1+\frac{4\alpha}{K^2FA_mK_0}}\,t\right) \tag{2}$$

The maximum value of modulation index is obtained when sinusoidal function became $\pi/2$ that furnishes

$$m_f = 2\left(\frac{\Delta\omega_i}{\omega}\right)\left(1-\frac{2\alpha}{K^2FA_mK_0}\right)e^{-\frac{\pi}{2}} \tag{3}$$

This produces an amplitude mismatching of the order of

$$\frac{m_f}{2}+\frac{9}{10}e^{-\frac{\pi}{2}}m_f = .072\left(1-\frac{2\alpha}{K^2FA_mK_0}\right) \tag{4}$$

3.2 The $\pi/2$ shifter

The shifter must maintain constant the angle between input and output changing the frequency.

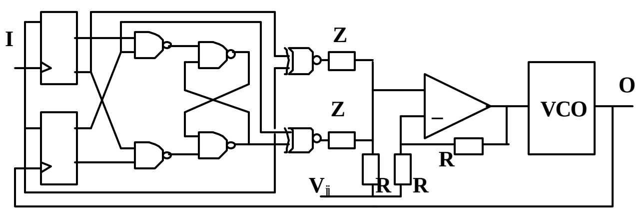

Fig. 3: Phase shifter

To avoid links to harmonics and sub-harmonics, it is utilized a sequential comparator of phase-frequency kind. To avoid influence of frequency on phase difference, the filter must have a pole on the origin and phase difference will can be controlled by a voltage acting on loop filter. The voltage can shift the neutral value that represent the potential that assures constant output signal. To assure stability a zero must compare in the transfer function [7].

3.3 Extracting the module of the sinusoidal input signal

To extract the module of the sinusoidal input avoiding influences of frequency variation, has been utilized an AD 830 device of the Analog Device [8]. The band extension is 500MHz and is present a crosstalk effect with two zeros that correspond to the ω_1 at 32GHz and ω_2 at 35 GHz. The transfer function of such a component is

$$R(j\omega) = K(j\omega)X(j\omega)T(j\omega)+H(j\omega)X(j\omega)+J(j\omega)T(j\omega) \tag{5}$$

with X(t)=Asin ωt e T(t)=Bsin(ωt+φ) in the time domain and

$$K=\frac{L_1}{1+j\frac{\omega}{\omega_B}} \quad H=\frac{L_2}{1-j\frac{\omega_1}{\omega}} \quad J=\frac{L_2}{1-j\frac{\omega_2}{\omega}} \tag{6}$$

with L_1=4mA/V2 and L_2=4mA/V dimensional constants. The LP filter will have a transfer function

$$LP=\frac{1}{1+j\frac{\omega}{\omega_x}} \tag{7}$$

The output signal to the sum circuit of Fig. 1, considering the L equal numerical value of L_1 and L_2, will have a direct component V_{dc}

$$V_{dc}=\frac{L^3A^2B^2}{4}\left\{1+\frac{1}{\left[1+\left(\frac{2\omega}{\omega_B}\right)^2\right]\left[1+\left(\frac{2\omega}{\omega_x}\right)^2\right]}\right\} \tag{8}$$

and a noise component V_N that is the sum of four sinusoidal contributions from the first through the fourth harmonic. Neglecting the phase shift of such components, the upper bound of the noise error can be computed as the sum of their modules, but the action of the zeros of the multipliers allows to consider only the second and fourth harmonic that have such expressions

$$|V_{2\omega}|=\frac{L^3A^2B^2}{2}\frac{\sin\varphi+\cos\varphi}{\left[1+\left(\frac{2\omega}{\omega_B}\right)^2\right]\sqrt{\left[1+\left(\frac{2\omega}{\omega_x}\right)^2\right]}}$$

$$|V_{4\omega}|=\frac{L^3A^2B^2}{8}\frac{1}{\left[1+\left(\frac{2\omega}{\omega_B}\right)^2\right]\left[1+\left(\frac{2\omega}{\omega_B}\right)^2\right]\sqrt{\left[1+\left(\frac{4\omega}{\omega_B}\right)^2\right]}} \tag{9}$$

Choosing $\varphi=\pi/4$, $|V_{2\omega}|=0$ and $|V_N|=|V_{4\omega}|$. An amplitude uncertainty can be written as

$$V_{\Delta A}=\frac{L^3A^4}{4}\frac{\Delta A}{A} \tag{10}$$

as well as the phase uncertainty is

$$V_{\Delta A}=\frac{L^3A^4}{4}\Delta\varphi \tag{11}$$

The next uncertainty must be considered concerns the wavefront of the gaussian for which it is possible consider its slope as well as the co-sinusoidal one. This slope causes a delay due essentially to the LP filter, related to the maximum value that can be encountered. If N is the number of periods in a wave train, the maximum value of the slope will be, at the output of the sum circuit

$$V_{slope} = \frac{L^3 A^4}{4} \frac{1}{\pi^2 N^2} \left(\frac{\omega}{\omega_B} \right)^2 \tag{12}$$

The second relation of the (9) is inversely dependent by the ω_x and it is possible calculate the value, ω_x^* with respect to the working frequency, that minimize the sum of (9) and (12). It is easy to see that

$$\left(\frac{\omega}{\omega_x^*} \right)^2 \cong \frac{\pi N - 1}{4} \tag{13}$$

This influence also the value of α in relations (1),(2),(3),(4), being

$$\alpha = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_X^*} \right)^2}} = \frac{2}{\sqrt{3 + \pi N}} \tag{14}$$

The uncertainty before the squarer has the comprehensive expression

$$\begin{aligned} |V_N| = {} & \frac{L^3 A^4}{8} \left[\frac{\Delta A}{A} + \Delta \varphi + \frac{1}{\pi N} \left(5 + \frac{1}{\pi N} \right) \right] + \\ & + .072 \frac{L^3 A^4}{8} \left(1 - \frac{4}{KFA_m K_0 \sqrt{3 + \pi N}} \right) \end{aligned} \tag{15}$$

The global effect of the squarer is in halving the deterministic component of noise, maintaining equal the stochastic component by the presence of a multiplier in the feedback of the squarer. The relative uncertainty can be considered

$$E_{relative} \approx \frac{1}{8} \left(\frac{\Delta A}{A} + \Delta \varphi + \frac{10}{\pi N} \right) \tag{16}$$

This uncertainty can be easily maintained lower than 2% also operating on N.

4. ECHOES PARAMETERS PRE-ESTIMATION

To determine the position of echoes in signal, one considers a local best-fit on a gaussian approximated by a parabola

$$f_{echo}(t) = A e^{-\frac{(t-t_0)^2}{2\sigma^2}} \approx A \left[1 - \frac{(t-t_0)^2}{2\sigma^2} \right] \quad -\sqrt{2}\sigma \le t - t_0 \le \sqrt{2}\sigma \tag{17}$$

choosing t_0 as time reference and A the actual value of the signal, it results

$$\sigma^2 = \frac{I_2}{K_1^2 A^2} \quad K_1^2 = \frac{128}{9} \quad I_2 = \int_{-\sqrt{2}\sigma}^{\sqrt{2}\sigma} f_{echo}(t) dt \tag{18}$$

The compatibility between integral limits and calculated value imposes an iterative computing. For each value of t_0 it is possible determine σ, but it must be chosen the value that firstly minimize the

$$\Delta(\sigma, A) = \int_{-\sqrt{2}\sigma}^{\sqrt{2}\sigma} \left[f_{echo}(t) - A \left(1 - \frac{t^2}{2\sigma^2} \right) \right]^2 dt \tag{19}$$

The first echo is subtracted from the signal and the procedure goes on.

5. USEFUL ECHOES IDENTIFICATION

The flow chart of the treatment of echoes position identification is shown in Fig.4. One starts considering that the first two echoes are useful ones. This is normally true and, considering all paths that can be considered in a system, it is possible recognize all the multiple ones due to multiple reflection of two walls.

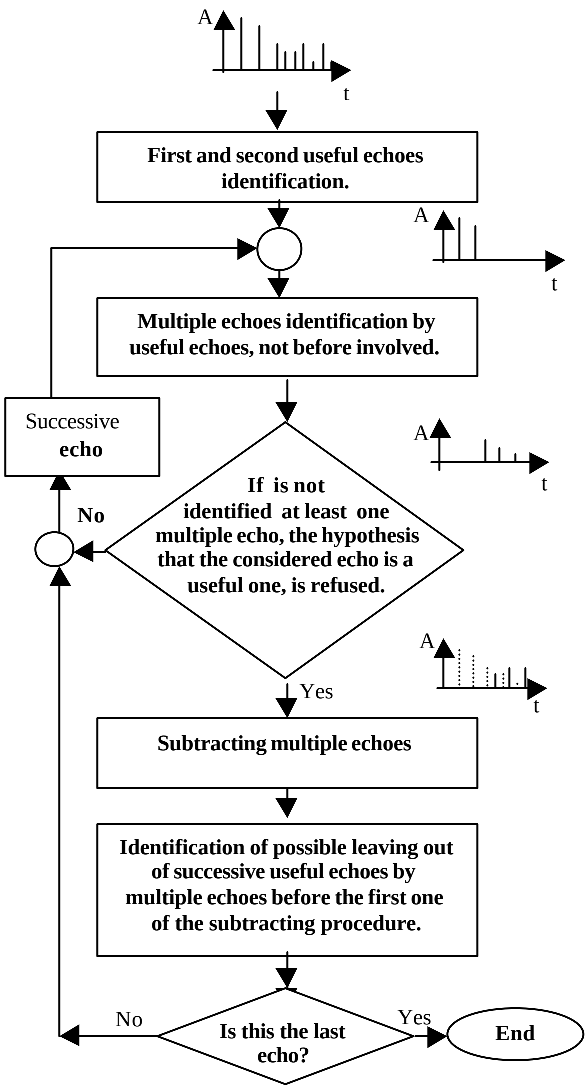

Fig. 4: algorithm of useful echoes identification

The paths determined by r reflections between p walls can be can be identified by this expression

$$n_{r,p} = \prod_{j=1}^{r} 10^{r-j} \left[\sum_{i^{(j)} = \left[i^{(j-1)} - 1 \right](1-k)}^{k = \frac{1}{2}\left[1-(-1)^j\right]} (1) \right] \tag{20}$$

This relation built all possible paths throughout the material, associating to each one a digit number linked to the walls involved. As an example, $n_{3,2}$=212 represents three reflections: the first at the wall 2, the second at the wall 1 and the third again at the wall 2. The order of the (20) is a mathematical structure that is not related with the time occurrence of path on probe, that is resolved by the algorithm.

6. THE IMPROVED MAXIMUM LIKELIHOOD ESTIMATOR

The accurate position of each useful reflected echo, can be obtained by a Maximum Likelihood Estimator that considers the analogic pre-estimation (many percents) of echoes parameters. The vector of parameters α, given any observed value vector r, is an argument of the V (α,r) function. The lower bound of such parameters is given by Rao Cramer limit, developed from the mean square error. The improvement of M.L.E. algorithm, is in the loosening interdependence of actual errors δ using the inversion of error Fischer matrix $[E]^{-1}$.

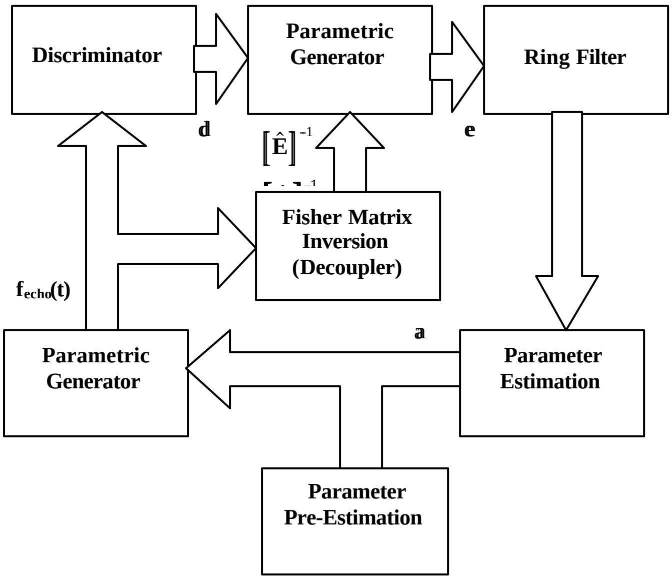

Fig. 5: Improved Maximum Likelihood Estimator

Applying such a matrix to error vector, it is possible to obtain ε parameters errors independent each other, that allow to improve the parameters pre-estimation α. The M.L.E. algorithm became recursive, pounding independent errors, making the Rao Cramer limit better approached. In Fig. 5 it is possible to see the recursive improved M.L.E. structure, where the left side represents the classical M.L.E. and the right one the improving of the algorithm. The algorithm recursion will stop when vector up date will be lower than half last significant bit of A/D converter.

7. COMPUTING ISSUES

The improved Maximum Likelihood Estimator algorithm is recursive and able to give increasing performance lowering the noise. The theoretical limitation in position measurements is linked to the computing numerical truncation, that means about 5μm. Experimental tests [8] evidence that are normally necessary ten recursions to optimize single echo parameters. It is necessary consider that algorithm efficiency is linked to noise overlapped to the signal. This is the reason that obliges, for each echo, to subtract from total input signal, all others rebuilt ones, depending on estimated parameters. The remaining noise of each echo is related to the goodness of parameters estimations of the other ones. This imposes that the procedure must be repeated, until, between two successive iterations, the improvement is negligible. The computing heaviness can be bettered avoiding the procedure repetition for echoes that have already achieved the best estimation. In every case for signals with great noise and only for two echoes it is possible to consume many seconds. Taking into account that one considers the problem of a single row of an echographic image, it appears with evidence that all the procedure cannot be applied in real time.

7.1 Parallel Algorithm

The only real time application possibility to exploit the high spatial accuracy, it is a parallel structure application shown in Fig. 6.

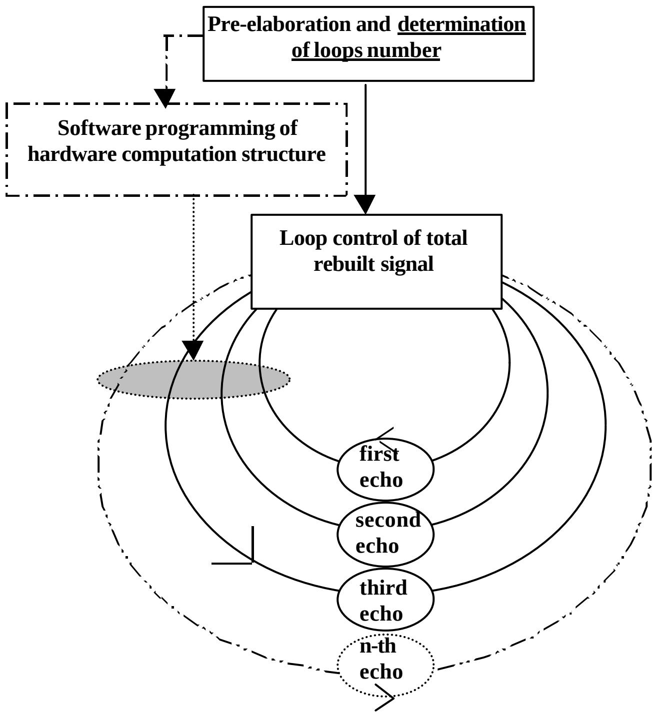

Fig 6: Parallel algorithm structure

7.1.1 Hardware Reconfiguration Software

In the picture appears an adjunctive software that must manage the physical computing parallel structure that, for each concentric loop, computes parameters of a single echo. The software characteristics change depending if one prefers a synchronous or an asynchronous working. In the first case the software can be absent because all the computing nodes are always active. In the other case can be more convenient a different management, adapting the hardware parallel structure, to the actual number of echoes under test.

7.2 Parallel Hardware

Using a parallel system of 17 Transtesh's transputers, assembled into a compatible PC-IBM computer, the computational burden is shared, restoring the on-line application capability. In Fig. 7 is shown a toroidal physical link, between computing nodes.

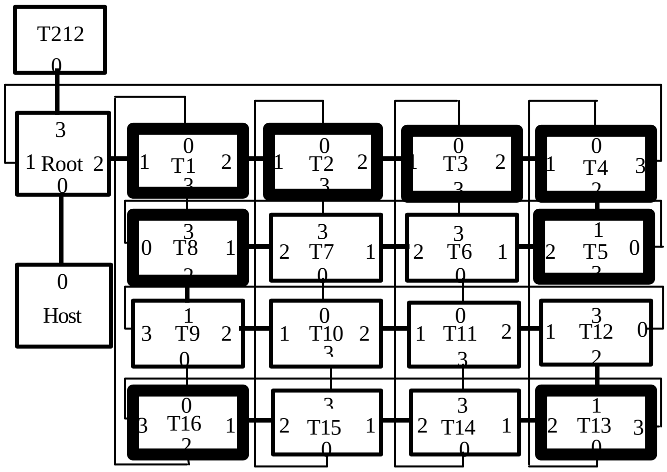

Fig 7: Toroidal parallel structure.

In the picture are also shown the computing nodes occupied by an example of eight echoes. Such an example evidences that the computing structure is not effectively concentric and data of more far echoes transit throughout more other computing nodes. Normally the more far, in time, is the echo, more far from supervisor node is the computing node. This is detrimental because such echoes require more loop applications and the data communication time is further extended. The physical hardware managing software, to balance the total computing time that incorporates the data time transit, must get complicate.

8. CONCLUSIONS

The problem of the analogic pre-analysis apparatus able to extract the module of an ultrasound signal independently from its frequency has been faced considering or the advantage with respect to the more adapt Gabor bi-orthonormal transform or the possible performances with respect to the module accuracy capability. The analysis shows the possibility of 2% in module accuracy allowing a less high sampling frequency that discharge the system from many not useful data acquisition. A recursive algorithm to individuate echoes in signal assigning an estimated standard deviation has been presented. The useful echoes extraction recursive algorithm from signal has been presented. All to obtain optimized data input to an Enhanced Maximum Likelihood Estimator that makes possible to estimate spatial position with great accuracy. The parallel algorithm that can be utilized has been discussed also facing the problem of hardware re-configuration by the specified software that must manage the optimal computer resources exploitation.

REFERENCES

[1] J.G. Goswami, A.K. Chan: *"Fundamentals of wavelet. Theory, Algorithms and Applications"*, Wiley, 1999

[2] J. Wexler, S. Raz : "Discrete Gabor Expansion", *Signal Processing*, vol 21, 1990, pp 207-220

[3] M.J. Bastiaans: "A Sampling Theorem for a Complex Spectrogram and Gabor's Expansion of a Signal in Gaussian Elementary Signals", *Opt. Eng.* Vol 20, 1981, pp 594-598

[4] A.J.E.M. Janssen: "Gabor Representation of Generalized Functions", *J. Math. Appl.* Vol 83, 1981, pp 377-394

[5] W.F. Egan: "*Phase Look Basic*", Wiley, 1998

[6] J.G. Graeme, G.E. Tobey: "*Operational Amplifiers Design and Applications*", Burr-Brown Research Corp., 1971

[7] Analog Devices: "*AD 830 Data Sheet*", 2000

[8] M. Caciotta, F. Leccese: "Multiple Echoes putting down in Ultrasound", *Proc. of IEEE Instrumentation and Measurement Technology Conference*, Hungary, 2001, in press