CALIBRATION OF IMPEDANCES BY THE SUBSTITUTION METHOD: NUMERICAL UNCERTAINTY EVALUATION

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Abstract – *Several theoretical and practical difficulties affect the analytical evaluation of uncertainty of measurements dealing with complex variables. Computer-intensive methods based on bootstrap and Monte Carlo techniques are here employed to investigate the uncertainty evaluation of a basic measurement procedure, the calibration of an impedance standard with the substitution method, using an RLC automatic bridge as a comparator.*
A procedure giving confidence intervals for the impedance parameter is shown in principle; results of calculations made with an implementation on actual measurement data are shown.

Keywords – Uncertainty; Impedance calibration; Numerical methods.

1. THE SUBSTITUTION METHOD

Top-level automatic RLC bridges currently available have relative accuracies in the order of 10^{-3}-10^{-4}, not sufficient to perform calibrations of artifact impedance standards. These instruments, however, provide measurement outputs having very high resolution, in the order of 10^{-6} or better [1,2]; repeatability and short-term stability are accordingly good.

This suggest the possibility of performing impedance standard calibrations with the so-called *substitution method*, using an automatic RLC bridge as a comparator. The method relies on a two-step measurement procedure: 1. measure the impedance standard to be calibrated (DUT) and 2. measure a reference impedance standard having a known value (S). Both measurements have to be conducted with the same instrument within a short time delay, and the two standards should have the same nominal value. The readings of the instrument are combined with the known value of S to give the value of DUT.

The method has been applied in the past to the dissemination of NIST inductance unit with good results [3] and an automation of the method is under development at the Istituto Elettrotecnico Nazionale Galileo Ferraris (IEN) [4].

Going into details of the substitution procedure, we will express the impedance as a vector-valued, complex quantity $\boldsymbol{Z}$, and choose the equivalent-series representation $\boldsymbol{Z} = R + \mathrm{j}X$ where R and X are the real-valued quantities resistance and reactance. We will call:

- Z_x the impedance standard to be calibrated, having a nominal complex impedance $\boldsymbol{Z}_x^N$;
- $\boldsymbol{Z}_x$ the complex impedance of Z_x to be estimated;
- Z_s the reference standard, having a nominal value $\boldsymbol{Z}_s^N$ equal to $\boldsymbol{Z}_x^N$: $\boldsymbol{Z}^N = \boldsymbol{Z}_s^N = \boldsymbol{Z}_x^N$;
- $\boldsymbol{Z}_s$ the estimate of Z_s, e.g. the value coming from the calibration certificate.

With this notation the substitution method can be described as follows:

1. connect the RLC bridge to Z_x and take the reading $\boldsymbol{Z}_x^{RLC}$;
2. connect the bridge to Z_s and take the reading $\boldsymbol{Z}_s^{RLC}$;

The two steps are repeated m times, giving a measurement sample M of m ordered pairs of complex numbers $\mathsf{M} = \{(\boldsymbol{Z}_x^{RLC}, \boldsymbol{Z}_s^{RLC})_i\}$, with $i = 1...m$.

Since $\boldsymbol{Z}_s^N = \boldsymbol{Z}_x^N = \boldsymbol{Z}^N$, we have $\boldsymbol{Z}_x \approx \boldsymbol{Z}_s$ and $\boldsymbol{Z}_x^{RLC} \approx \boldsymbol{Z}_s^{RLC}$, and the (unknown) deviation of $\boldsymbol{Z}_x^{RLC}$ from $\boldsymbol{Z}_x$ can be corrected with the (known) deviation of $\boldsymbol{Z}_s^{RLC}$ from $\boldsymbol{Z}_s$.

The substitution procedure permits to correct *either* the offset [3] *or* the gain $\boldsymbol{G}$ error of the RLC bridge; since most automatic bridges permit to perform offset-nulling before the actual measurement, it seems preferable to correct the gain error by calculating $\boldsymbol{Z}_x$, for every measurement pair in M, as

$$\boldsymbol{Z}_x = \boldsymbol{G}\,\boldsymbol{Z}_s; \qquad (1.1)$$

where

$$\boldsymbol{G} = \boldsymbol{Z}_x^{RLC} / \boldsymbol{Z}_s^{RLC}.$$

2. DIFFICULTIES IN UNCERTAINTY EVALUATION

Equation (1.1) is the model from which $\boldsymbol{Z}_x$ is estimated with its uncertainty. The simplicity of (1.1) is only apparent, because of the compact notation possible in the complex domain.

An uncertainty evaluation compliant with [5] implies:

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2a) expansion of equation (1.1), by separating real and imaginary components of each parameter. The model splits in a system of two strongly nonlinear equations, with 6 input and 2 output quantities;

2b) calculation of sensitivity coefficients; although the properties of analytical functions give a significant help, the calculations are painstaking;

2c) construction of confidence intervals. It is worth noting that, for the complex (multivariate) case, this construction is not covered [6] by the present edition of [5].

3. COMPUTER-INTENSIVE METHODS FOR THE EVALUATION OF UNCERTAINTY

The analytical and conceptual difficulties in the treatment of model (1.1) can be overcome with the use of two powerful tools of computer-intensive statistical analysis: *Monte Carlo randomization*, and *bootstrap resampling*. Although applications to the uncertainty estimation in precision metrology are still rare [7], these methods have been thoroughly analyzed and employed by statisticians since 1950s and 1970s respectively; for an extended treatment and exaustive bibliographies we send back to recent reviews [8,9].

The idea behind both methods is to substitute computing heaviness to analytical difficulties. The model is left formally untouched, and simply recomputed a very large number of times varying the input quantities according to simple rules (this corresponds to step 2b); the corresponding set of outputs is collected, and can be used to generate, for example, distributions, estimators, confidence intervals (step 2c). If the computer program is written in a language capable (directly or through routines) of complex arithmetics, even step 2a can be avoided. Our discussion will be focussed on model (1.1), although the method can be applied to different models without conceptual variations.

Calling $U(\boldsymbol{Z}_s)$ the confidence domain of the estimate $\boldsymbol{Z}_s$, the steps of the procedure are:

3a) generate a Monte Carlo sample $\boldsymbol{Z}_s^k$ for Z_s. This is done by choosing a a pseudo-random complex number with a given distribution $p_s(\boldsymbol{Z}_s)$ in $U(\boldsymbol{Z}_s)$.

3b) obtain a bootstrap sample M^{*k} from the original sample M, choosing m pairs $(\boldsymbol{Z}_x^{RLC}, \boldsymbol{Z}_s^{RLC})_i$ from M, at random, *with replacement* (the same pair can be picked more than one time). The gain $\boldsymbol{G}_i^k$ is computed for each pair.

3c) compute a set Z^k of $\boldsymbol{Z}_{x,i}^k$ of size m directly by means of the model equation (1.1); $\boldsymbol{Z}_{x,i}^k = \boldsymbol{G}_i^k \boldsymbol{Z}_s^k$;

3d) compute $\boldsymbol{Z}_x^k$ with the chosen estimator (e.g., the mean) from the m values $\boldsymbol{Z}_{x,i}$ in Z^k. If other estimators are required (e.g, the correlation coefficient between $\mathrm{Re}(\boldsymbol{Z}_x)$ and $\mathrm{Im}(\boldsymbol{Z}_x)$) compute them also.

The steps 3a)...3d) are iterated a large number of times b, obtaining the set $\mathsf{B} = \{\boldsymbol{Z}_x^k\}$ (and, if required, other sets for other estimators).

From the set B the distribution of $\boldsymbol{Z}_x$ can be constructed. This can be viewed as a numerical approximation from which the relevant statistics can be obtained.

We can see that, in the algorithm above, the Monte Carlo method (3a) manages the Type B uncertainty of $\boldsymbol{Z}_s$.

The boostrap resampling (3b), instead, directly manages Type A uncertainty, making no particular assumptions on the distribution from which $\boldsymbol{Z}_x^{RLC}$ and $\boldsymbol{Z}_s^{RLC}$ are sampled. This is roughly equivalent to avoid any piece of information coming from previous measurements, and is known as a risky procedure in the case of a small sample. Other methods, like *parametric bootstrap* [10] could be used to avoid this problem; for our particular case, however, a sufficiently large (a few tens of elements) sample can easily be obtained. By converse, it is very difficult to make a good characterization of a system capable of measuring resistors, capacitors, and inductors, of different constructions and noise properties, with an impedance ranging over several decades.

3. THE MEASUREMENT SETUP

An automatic measurement station for the calibration of impedance standards with the substitution method has been developed [4], and is now under characterization. The station is based on a Quadtech 7400 automatic RLC bridge, and a coaxial switching system which connects, alternatively, one of the two standards to the bridge with four-terminal pair connections. The bridge and the switch are remotely controlled (via IEEE-488 and RS-232 buses respectively) by a personal computer running a LabWindows/CVI acquisition program. Each measurement cycle has two steps: the first connects Z_X to the bridge, acquires the reading $\boldsymbol{Z}_x^{RLC}$, the second connects Z_S and take the reading $\boldsymbol{Z}_S^{RLC}$. The number of cycles m, the measurement timings and idle delays, the bridge settings and so on are user defined; the results of each cycle, the m couples of complex numbers of the set M, are recorded in a file for off-line processing with the procedure of Par. 4.

4. IMPLEMENTATION OF THE PROCEDURE

The procedure has been implemented as a MATLAB routine (complex arithmetics, random numbers generation, and even bootstrap resampling are factory-defined functions); it is at present called with manual commands, but an integration in the acquisition program is under development.

The routine first asks for the confidence limits of $R_s = \mathrm{Re}(\boldsymbol{Z}_s)$ and $X_s = \mathrm{Im}(\boldsymbol{Z}_s)$; $U(\boldsymbol{Z}_s)$ is constructed as a rectangular domain with these limits as bounds, and $p_s(\boldsymbol{Z}_s)$ is assumed as a uniform distribution in $U(\boldsymbol{Z}_s)$. If additional information on $p_s(\boldsymbol{Z}_s)$ is available, the program can be modified accordingly.

The computation of B is performed following precisely the algorithm described in Chap. 3; the computation for $b = 20000$ takes seconds. The distributions of $R_x = \mathrm{Re}(\boldsymbol{Z}_x)$ and $X_x = \mathrm{Im}(\boldsymbol{Z}_x)$, and the distribution of $\boldsymbol{Z}_x$ are plotted for visual control.

The location of $\boldsymbol{Z}_x$ is computed with the mean of the set B. The confidence region $U_{95}(\boldsymbol{Z}_x)$ is computed as the rectangular domain, centered on the mean($\boldsymbol{Z}_x$), containing $0.95 \cdot b$ elements of B.

5. RESULTS

To test the implementation, we chose for S and DUT two 10 kΩ ac resistors ($\boldsymbol{Z}^N = 10000 + j0\ \Omega$), configured as four terminal-pair standards.

S belong to the Italian national impedance standard, and its impedance is traceable at IEN to dc resistance national standard through ac-dc corrections.

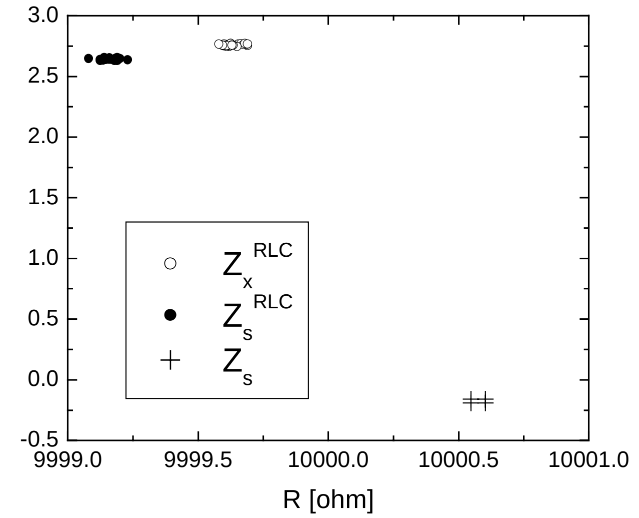

Fig.1 – Values of the standard impedance Z_S^{RLC} and the unknown impedance Z_X^{RLC} obtained with the automatic RLC bridge. The confidence domain for the known value of Z_S is also shown.

A set of $n = 30$ measurements for each standard have been taken in about 10 minutes. The results are shown in Fig. 1.

The procedure has been evaluated with $b = 20000$. Figure 2 shows the histogram corresponding to the set $B = \{\boldsymbol{Z}_x^k\}$, which is taken as an estimator of the true distribution of $\boldsymbol{Z}_x$. Figure 3 shows the histograms Re(B) and Im(B) representing the marginal distributions of R_X and X_X, respectively. It is apparent that the distributions are not Gaussian. From the set B we can easily compute the means for R_X and X_X and their corresponding 95% rectangular-domain confidence limits.

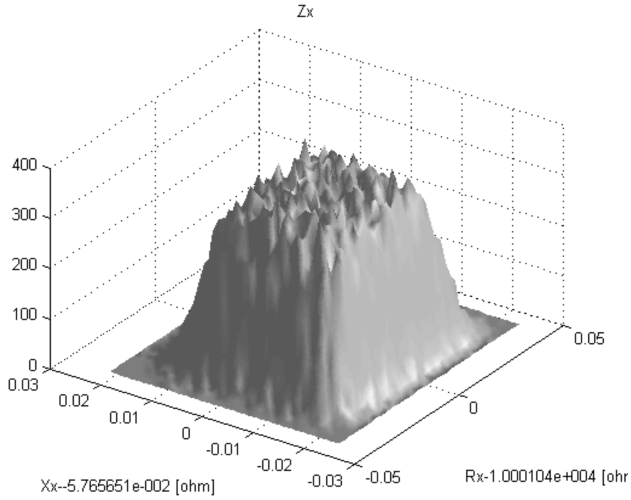

Fig.2 – Two-variate histogram of the set B, the result of the procedure described in the paper. The mean value, i.e. the estimate of Z_X has been subtracted from the data for clarity.

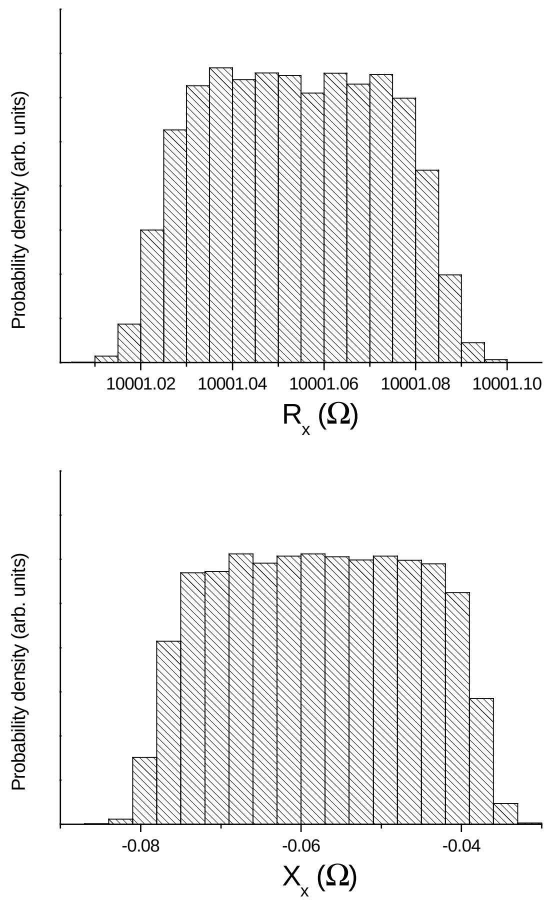

Fig.3 – Histograms of Re(B) (upper part) and Im(B) (lower part), projections of the two-variate histogram of Fig. 2.

6. FURTHER DEVELOPMENTS

The power of numerical uncertainty evaluation is that, after the model and the estimators have been chosen, no further elaboration is necessary and the procedure is "mechanistic". More complicated models can be easily treated by appropriate extensions of the algorithm proposed.

An estimate for the correlation coefficient can be obtained with a confidence interval.

Other, more robust estimators than the mean (median, trimmed mean) can be easily implemented.

The trivial rectangular confidence domain here employed can be refined to confidence ellipses or zones.

Alternative representations of the impedance (different equivalent circuits, representations using quality factor or dissipation factor, ...) can be easily treated.

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