NONLINEARITIES, THE GENERATORS PERSPECTIVE IN ADC STATISTICAL TESTING TECHNIQUES

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Abstract – *In this paper we present an experimental method that is suitable to ascertain the degree of nonlinearity present in a given signal as well as its adequacy to be used as a stimulus signal in a statistical method, such as the histogram test of ADCs. We show one possible implementation of the method as well as its theoretical substrate and some preliminary experimental results. Finally we show the importance of determining the distribution of the stimulus signal when using noise.*

Keywords – Nonlinearity measurement, probability density function distortion, histogram method.

1. INTRODUCTION

In virtually all measurements of ADC performance a comparison is established between the properties of a signal at the input and output of the device under test (DUT). The comparison is commonly performed in the time domain (Sine fitting), the frequency domain (Fourier transform) and the statistical domain (Histogram). The relative nature behind these measurements reveals one feebleness: the correct knowledge of the reference, the input signal.

The formulation of this problem is anything but new. However, care must taken since it's all too easy to forget it and assume, as far as the measurement is concerned, the stimulus signal to be ideal.

As an example let us consider the following: in the measurement of ADCs has been common practice to accept generators that have a signal to noise ratio (SNR) higher than $6.02 \cdot n + 1.76$ (dB) [1]. In the case of an 8-bit converter this means that a second harmonic 50dB below the fundamental is acceptable since it is not discernible from the quantification noise. This is certainly true for Fourier based measurements but definitely erroneous in a statistical method. Actually a generator with a second harmonic 50dB below the fundamental will give rise, at best, to a metric of effective number of bits, in an ideal 8-bit converter, of only $n_{ef} = 6.9$. This has to do with the nature of the measurement: in an amplitude distribution measurement (histogram) the finite resolution of the converter simply quantifies this distribution – it does not distort its form, which is exactly what nonlinearities of the converter or the generator do. Statistical measurement techniques impose higher constraints on the generator.

The authors have proposed the use of white gaussian noise as a stimulus signal for the histogram method [2, 3]. Signals that are stochastic in nature present a more daring challenge when trying to determine its distortion.

2. IMPLEMENTATION OF A PDF MEASURING SYSTEM

Several problems have been raised in the preceding paragraphs but common to all is the need to measure the amplitude distribution of the stimulus signal. To this end the authors have adapted an old measurement technique [4] which is illustrated in fig. 1.

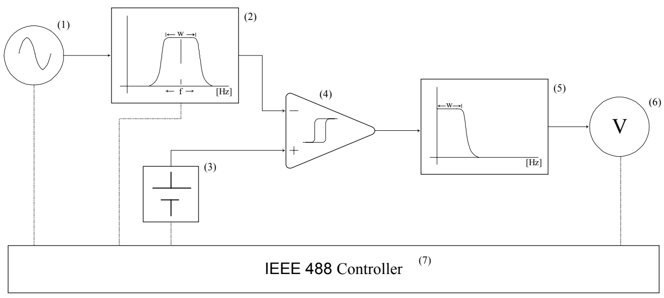

Figure 1 Experimental setup used in the measurement of the distribution function of a waveform. (1) Waveform generator: white gaussian noise, sine or triangular wave. (2) Pass-band filter with center frequency f and width W. (3) DC reference voltage generator. (4) High-Speed comparator. (5) Low-pass filter with a very low cut-off frequency - 1Hz. (6) – Very high resolution voltmeter. (7) – IEEE 488.2 controller using MatLab

Here what is being measured is the distribution function of the stimulus signal amplitude,

$$F_{V_s}(V_{dc}) = P\left\{-\infty < V_s \le V_{dc}\right\} \tag{1}$$

where V_s is the stimulus signal and V_{dc} is the DC reference voltage. The probability expressed in (1) is measured by the precision voltmeter through,

$$F_{V_s} = \frac{V_M - V_{c-}}{V_{c+} - V_{c-}}, \tag{2}$$

being V_M the reading of the voltmeter, V_{c+} the highest output voltage of the comparator and V_{c-} the lowest. At this point we have a high resolution measure of the amplitude distribution of the stimulus signal and one to which can be associated an unequivocal tolerance-confidence pair.

In this setup the main precision limitations stream from the DC voltage source and the precision voltmeter. The comparator can be chosen to possess a very low differential mismatch and a narrow and symmetric histeresis window. The pass-band following the generator fulfills two needs: the restriction of the bandwidth when proceeding with a frequency dependent measurement - white gaussian noise stimulus signal, and the suppression of high frequency noise that can disturb the comparator in terms of its slew rate. The low-pass filter found at the output of the comparator has the simple purpose of providing the amplitude average. Therefore, it must have a very low cutoff frequency (≈ 1Hz) and should be complemented through averaging at the voltmeter end. The comparator is the dynamic culprit of the setup mainly due to input gap mismatch, propagation delays and finite slew rate.

3. MEASURING NON-LINEARITIES

The question now arises as to how we can measure the degree of non-linearity of a given stimulus signal. To that end we must start by choosing the description of the non-linearities. We have chosen to describe them generally through a Taylor series expansion to the third order, that is,

$$y = g(x) = x + b \cdot x^2 + c \cdot x^3 , \tag{3}$$

being y the effective output of the generator and x the ideal waveform (desirable). The choice of a Taylor series expansion for the description of the nonlinear device $g(\cdot)$ seems the most natural due to the absence of time representation that comes out of (1). This lack of time dependency severely hampers the use of the Volterra series expansion.

From the experimental setup in fig. 1 it's easy to see that what is measured is the distribution of y. Therefore we must establish the relation between the distribution of y and of x. It is a well known result [5] that this amounts to

$$f_y(y) = \sum_i \left. \frac{f_x(x)}{\left| \frac{\partial g(x)}{\partial x} \right|} \right|_{x=x_i} , x_i = \underbrace{g_i^{-1}(y)}_{root\ \mathrm{i}} \wedge f_x = \frac{\partial F_x(x)}{\partial x}, \tag{4}$$

being f_y the probability density function (*pdf*) of y. Integrating (4) we get

$$F_y(y) = \sum_i \int_{\min(x)}^{x_i} f_x(x) \partial x = \sum_i F_x(x_i), \tag{5}$$

where i is the number of roots of (3), in this case 3 and $F_x(x)$ is the distribution of the ideal waveform x. Luckily, as these roots are quite involved, it can be proven that only the first one is relevant, for any of the three stimulus signals we are going deal with, Gaussian Noise, Sine and Triangular Waves. This means that (5) resumes to

$$F_y(y) = F_x(x_1(y)). \tag{6}$$

Once the experimental values of (6) are obtained through (2) and the analytical description of (6) is determined, it is relatively easy to obtain the nonlinear coefficients b and c expressed in (3). To that end we have used a numerical search based on the Gauss-Newton method. The risk of oscillation and divergence that might plague this method can be minimized through adequate experimental setup. We have yet to find trouble using it.

3.1 Gaussian Noise

For the Gaussian stimulus signal we have,

$$f_x(x) = \frac{e^{-\frac{(x-\mu)^2}{2\sigma_n^2}}}{\sqrt{2\pi\sigma_n^2}} , \quad F_x(x) = \frac{1}{2}\left[erf\left(\frac{x-\mu}{\sqrt{2\sigma_n^2}} \right) + 1 \right], \tag{7}$$

where $erf(\cdot)$ is the error function, σ_n^2 is the power of the noise and μ its mean, which can be taken to be zero without compromising generality. Inserting (7) in (6) and with a bit of manipulation, we obtain

$$h_1(V_{dc}, \sigma_n, b, c) = \frac{g_1^{-1}(V_{dc})}{\sqrt{2\sigma_n^2}} = erf^{-1}(2 \cdot P_e(V_{dc}) - 1), \tag{8}$$

being $P_e(V_{dc})$ the experimental counterpart of $F_y(V_{dc})$. We have chosen to make a numerical regression analysis of h_1 instead of g_1^{-1} since it allows us to also determine the effective noise power at the output of the generator. Analytically solving g_1^{-1} we obtain the following development for the first identity of (8),

$$h_1(y, \sigma_n, b, c) = \frac{-2b + 2^{2/3} cD}{6c\sqrt{2\sigma_n^2}} , \tag{9}$$

defining,

$$A = \frac{-2b^3 + 9bc + 27c^2 y}{c^3},$$

$$B = 3\sqrt{3}\sqrt{\frac{-b^2 - 4b^3 y + 18bcy + c\left(4 + 27cy^2\right)}{c^4}},$$

$$C = 9\left(-2c^2 - 8b^3 y + 27bcy + 3c\left(2 + 9cy^2\right)\right),$$

$$D = (A-B)^{1/3} + (A+B)^{1/3}, \quad E = \frac{3(b+6cy)}{c^2} - A .$$

In the Gauss-Newton method we have the following identity, at iteration a,

$$[Y_s] - [Y]^{(a)} = [J]^{(a)} \cdot [\beta]^{(a)}, \tag{10}$$

where

$$[\gamma]=\begin{bmatrix}\sigma_n\\ b\\ c\end{bmatrix},[\beta]^{(a)}=\left([\gamma]-[\hat{\gamma}]^{(a)}\right)\Rightarrow$$

$$\Rightarrow[\hat{\gamma}]^{(a+1)}=[\hat{\gamma}]^{(a)}+[\beta]^{(a)},$$

$$[Y_s]=erf^{-1}\left(2[P_e]-1\right),[Y]^{(a)}=\left[h_1\left([\hat{\gamma}]^{(a)}\right)\right]=[h_1]^{(a)},$$

$$[J]^{(a)}=\left[\frac{\partial h_1}{\partial\sigma_n},\ \frac{\partial h_1}{\partial b},\ \frac{\partial h_1}{\partial c}\right]^{(a)}$$

Formalizing for the gaussian noise we reach the following identities for the Jacobian,

$$\frac{\partial h_1}{\partial\sigma_n}=\frac{2b-2^{2/3}cD}{6c\sqrt{2\sigma_n^2}},$$

$$\frac{\partial h_1}{\partial b}=\frac{1}{6c\sqrt{2\sigma_n^2}}\left\{\begin{array}{l}-2+\dfrac{2^{2/3}}{3c^3}\left[\dfrac{3c\left(3c-2b^2\right)-\dfrac{27\left(9cy-b-6b^2y\right)}{B}}{\left(A-B\right)^{2/3}}+\right.\\ \left.+\dfrac{3c\left(3c-2b^2\right)+\dfrac{27\left(9cy-b-6b^2y\right)}{B}}{\left(A+B\right)^{2/3}}\right]\end{array}\right\},$$

$$\frac{\partial h_1}{\partial c}=\frac{1}{6c^2\sqrt{2\sigma_n^2}}\left\{\begin{array}{l}2b-2^{2/3}cD+2^{2/3}c\left[D+\dfrac{E+\dfrac{C}{c^4B}}{\left(A-B\right)^{2/3}}+\right.\\ \left.+\dfrac{E-\dfrac{C}{c^4B}}{\left(A+B\right)^{2/3}}\right]\end{array}\right\},$$

3.2 Sine Wave

In the case of the sine wave we have some well-known results for the amplitude distribution,

$$f_x(x)=\frac{1}{\pi\sqrt{A^2-(x-O)^2}},\tag{11}$$

$$F_x(x)=\frac{ArcSin\left(\dfrac{x-O}{A}\right)}{\pi}+\frac{1}{2},\tag{12}$$

where $A/\sqrt{2}$ is root mean square amplitude and O the offset. Inserting (12) in (6) we obtain the expression which will be used, through regression, to determine both the linear (A, O) and nonlinear (b, c) parameters,

$$h_1\left(V_{dc},A,O,b,c\right)=P_e\left(V_{dc}\right)=$$
$$=\frac{1}{\pi}ArcSin\left(\frac{g_1^{-1}\left(V_{dc}\right)-O}{A}\right).\tag{13}$$

If we solve analytically g_1^{-1} and insert it in (13) we obtain the following development,

$$h_1\left(y,A,O,b,c\right)=\frac{1}{\pi}ArcSin\left[\frac{1}{A}\left(\frac{2^{2/3}cD-2b}{6c}-O\right)\right]+\frac{1}{2},\tag{14}$$

where

$$A=9bc-2b^3+27c^2y,$$

$$B=3c^3\sqrt{\frac{6\left(2+9by\right)c-3b^2\left(1+4by\right)+81c^2y^2}{c^4}},$$

$$C=2^{2/3}c\left[\left(\frac{A-B}{c^3}\right)^{1/3}+\left(\frac{A+B}{c^3}\right)^{1/3}\right]-2b-6cO,$$

$$D=\left(\frac{A-B}{c^3}\right)^{1/3}+\left(\frac{A+B}{c^3}\right)^{1/3},\ E=\pi A\sqrt{1-\frac{1}{A^2}\left(F-O\right)^2},$$

$$F=\frac{1}{6c}\left(2^{2/3}cD-2b\right),\ G=\frac{6b^3}{c^4}-\frac{18b}{c^3}-\frac{27y}{c^2},$$

$$H=\frac{27\left(\dfrac{4+18by+54cy^2}{c}-\dfrac{4B^2}{27c^4}\right)}{2B}.$$

The Jacobian, needed for the Gauss-Newton Method, can now promptly be found,

$$[J]=\left[\frac{\partial h_1}{\partial A},\ \frac{\partial h_1}{\partial O},\ \frac{\partial h_1}{\partial b},\ \frac{\partial h_1}{\partial c}\right],\tag{15}$$

being,

$$\frac{\partial h_1}{\partial A}=-\frac{C}{6\pi A^2c\sqrt{1-\dfrac{C^2}{36A^2c^2}}},\ \frac{\partial h_1}{\partial O}=-\frac{1}{E}$$

$$\frac{\partial h_1}{\partial b}=\frac{1}{6cE}\left\{\begin{array}{l}-2+2^{2/3}c\left[\dfrac{\dfrac{9}{c^2}-\dfrac{6b^2}{c^3}-\dfrac{27\left(18cy-2b-12b^2y\right)}{2cB}}{3\left(\dfrac{A-B}{c^3}\right)^{2/3}}+\right.\\ \left.+\dfrac{\dfrac{9}{c^2}-\dfrac{6b^2}{c^3}+\dfrac{27\left(18cy-2b-12b^2y\right)}{2cB}}{3\left(\dfrac{A+B}{c^3}\right)^{2/3}}\right]\end{array}\right\}$$

$$\frac{\partial h_1}{\partial c}=\frac{1}{E}\left\{\begin{array}{l}\frac{1}{6c}\left[2^{2/3}D+2^{2/3}c\left(\frac{G-H}{3\left(\frac{A-B}{c^3}\right)^{2/3}}+\frac{G+H}{3\left(\frac{A+B}{c^3}\right)^{2/3}}\right)\right]-\\-\frac{F}{c}\end{array}\right\}$$

3.3 Triangular Wave

The triangular wave has a very simple amplitude distribution, identical to uniform noise, given by,

$$f_x(x)=\frac{1}{2A},\ F_x(x)=\frac{x+A-O}{2A}, \tag{16}$$

where A is the amplitude and O is the offset. Similarly to what we have done in the sinusoidal case, here we will also opt for the regression of the distribution, obtained by inserting (16) in (6),

$$h_1(V_{dc},A,O,b,c)=P_e(V_{dc})=\frac{g_1^{-1}(V_{dc})+A-O}{2A}. \tag{17}$$

If we solve analytically g_1^{-1} and insert it in (17) we obtain the following development,

$$h_1(y,A,O,b,c)=-\frac{\left[\left(E-\frac{G}{54}\right)^{1/3}+\left(E+\frac{G}{54}\right)^{1/3}-\frac{b}{3c}-F\right]}{2A}, \tag{18}$$

where

$$A=9bc-2b^3+27c^2y,$$

$$B=3c^3\sqrt{\frac{6(2+9by)c-3b^2(1+4by)+81c^2y^2}{c^4}},$$

$$C=-A+O+b(A-O)^2+c(O-A)^3,$$

$$D=\left(\frac{A-B}{c^3}\right)^{1/3}+\left(\frac{A+B}{c^3}\right)^{1/3},$$

$$E=\frac{b+3cC}{6c^2}-\frac{b^3}{27c^3},\ F=\frac{1}{6c}\left(2^{2/3}cD-2b\right),$$

$$G=\sqrt{\frac{\left[2b^3-9c(b+3cC)\right]^2-4\left(b^2-3c\right)^3}{c^6}},$$

$$k=-b+O, M=-1-2bk-3ck^2,$$

$$H=\frac{M\left[2b^3-9c(b+3ck)\right]}{2c^4G},$$

$$p_1=\frac{9}{c^2}-\frac{6b^2}{c^3},\ p_2=\frac{27\left(-2b-12b^2y+18cy\right)}{2cB},$$

$$p_{10}=\frac{1+3ck^2}{6c^2}-\frac{b^2}{9c^3},$$

$$p_{20}=\frac{2\left[6b^3-9c\left(1+3k^2c\right)\right]\left[2b^3-9c(b+3cC)\right]}{108c^6G}-$$
$$-\frac{24b\left(b^2-3c\right)^2}{108c^6G},$$

$$p_{11}=\frac{b^3}{9c^4}+\frac{3ck^3+3C}{6c^2}-\frac{b+3cC}{3c^3},$$

$$p_{21}=\frac{\frac{36\left(b^2-3c\right)^2}{c^6}-\frac{6G^2}{c}}{108G}-$$
$$-\frac{\frac{18}{c^6}\left[c\left(3ck^3+3C\right)+(b+3cC)\right]\left[2b^3-9c(b+3cC)\right]}{108G}.$$

The Jacobian of h_1 is expressed through (15) where the partial derivatives are now,

$$\frac{\partial h_1}{\partial A}=-\frac{1}{A}\left[\frac{\frac{M}{2c}+H}{3\left(E-\frac{G}{54}\right)^{2/3}}+\frac{\frac{M}{2c}-H}{3\left(E+\frac{G}{54}\right)^{2/3}}\right]+$$
$$+\frac{1}{2A^2}\left[\left(E-\frac{G}{54}\right)^{1/3}+\left(E+\frac{G}{54}\right)^{1/3}-\frac{b}{3c}-F\right],$$

$$\frac{\partial h_1}{\partial O}=-\frac{1}{2A}\left[\frac{H-\frac{M}{2c}}{3\left(E+\frac{G}{54}\right)^{2/3}}+\frac{H+\frac{M}{2c}}{3\left(E-\frac{G}{54}\right)^{2/3}}\right],$$

$$\frac{\partial h_1}{\partial b}=-\frac{1}{2A}\left\{\begin{array}{l}\frac{p_{10}-p_{20}}{3\left(E-\frac{G}{54}\right)^{2/3}}+\frac{p_{10}+p_{20}}{3\left(E+\frac{G}{54}\right)^{2/3}}-\frac{1}{3c}\\-\frac{1}{6c}\left[2^{2/3}c\left(\frac{p_1-p_2}{3\left(\frac{A-B}{c^3}\right)^{2/3}}+\frac{p_1+p_2}{3\left(\frac{A+B}{c^3}\right)^{2/3}}\right)-2\right]\end{array}\right\}$$

$$\frac{\partial h_1}{\partial c}=-\frac{1}{2A}\left\{\begin{array}{l}\frac{b}{3c^2}+\frac{F}{c}+\frac{p_{11}-p_{21}}{3\left(E-\frac{G}{54}\right)^{2/3}}+\frac{p_{11}+p_{21}}{3\left(E+\frac{G}{54}\right)^{2/3}}-\\-\frac{2^{2/3}}{6c}\left[D+c\left[\frac{G-H}{3\left(\frac{A-B}{c^3}\right)^{2/3}}+\frac{G+H}{3\left(\frac{A+B}{c^3}\right)^{2/3}}\right]\right]\end{array}\right\}$$

4. MEASURING NON-LINEARITIES

We have implemented the scheme shown in fig. 1 using a Fluke 5700A as the DC voltage reference, a HP 34401A as the precision voltmeter (with the 3Hz filter turned on) and a 0.8Hz low-pass filter at the output of the comparator. We are currently designing a high performance comparator for the purpose of measuring nonlinearities at very high frequencies but we also wanted to see how this scheme would fare in a low budget setup. For this reason we have used a simple LM311N as the comparator. The results can be seen in fig. 2 to 4 and table I. The LM311N has indeed many limitations and as is it imposes a resolution limit at around -70dBc and a frequency limit of about 50kHz. However, using a very low distortion signal generator, the Stanford Research DS390 as a calibrating source, we built a lookup correction table for the sinusoidal stimulus with which we were able to measure distortions up to -90dBc.

Table I – Experimental results.

Wave	Generator	A [V]	O [mV]	b $\times 10^{-3}$	c $\times 10^{-3}$	Distortion [dBc]
Sine (20kHz)	Wavetek M39	3.00 (3.00)	-4.43 (0)	0.214	0.044	-63.8
Sine (20kHz)	Siemens FG 5-3	3.21 (3.00)	-25.8 (0)	3.95	2.01	-38
Noise	SR – DS360	1.87 (1.73)	(0)	15.6	-	-30.6
Triang.	Wavetek M39	2.01 (2.00)	5.75 (0)	0.40	0.038	-63.2

If we look at table I we might notice that the distortion associated with the noise output of the SR-DS360 generator is quite high. This has not so much to do with the output stage of the generator as with the not perfectly gaussian distribution of the pseudo-random sequence that the generator outputs. In this case the non-linear coefficients b and c do not have their expected meanings. They do alert though that no meaningful characterization of an ADC can be made using this noise as a stimulus signal [3]. The last column in table I shows the distortion that a sinusoidal stimulus would sustain after passing through an amplifier with the nonlinear behaviour expressed in (3). The distortions expressed for the Wavetek generator are very close to the manufacturers specifications at –65dBc for frequencies below 20kHz. The Siemens FG 5-3 is an old and low cost generator.

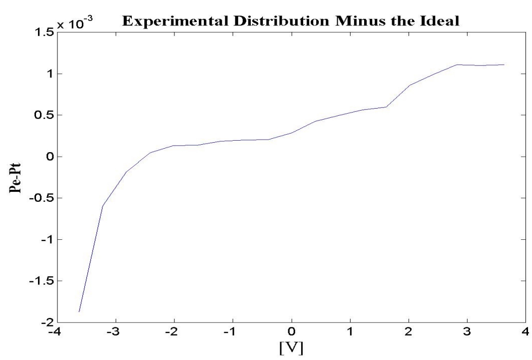

Figure 2. Results from the Wavetek M39 – Sine Output

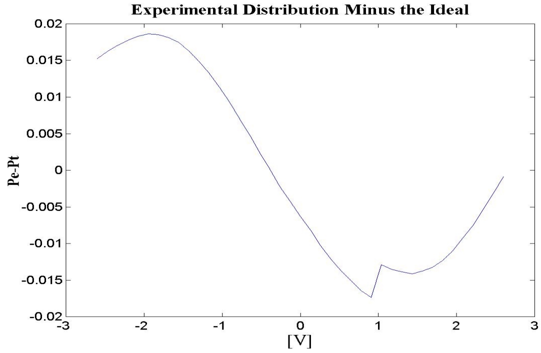

Figure 3 Results from the SR-DS390 – Noise Output

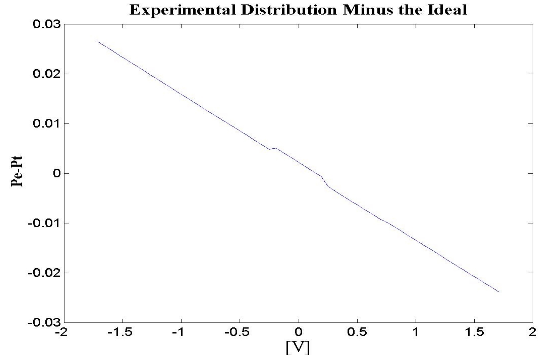

Figure 4 Results from theWavetek M39 – Triangular Output

5. CONCLUSIONS

The method here presented provides a simple and cost effective means to measure the distortion of stimulus signals. It also gives a means to infer about the necessary adequacy of the distribution of the stimulus signal in the case of the histogram method.

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