CHARACTERIZATION OF DIGITIZER TIMEBASE JITTER BY MEANS OF THE ALLAN VARIANCE

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Abstract - *The use of the Allan variance for the characterization of the jitter timebase error in waveform digitizers is proposed. With this aim, the Allan variance is shown to be a sound basis for defining and measuring a suitable figure of merit, diagnosing the jitter noise type, and including a jitter error block into a previously proposed digitizer model. Experimental results highlighting the effectiveness of the Allan variance in the characterization of digitizer jitter error are discussed.*

Keywords – Waveform Digitizers, Jitter, Allan Variance, Modeling, Testing, Diagnostics.

1. INTRODUCTION

At highest sampling frequencies, an error-free reconstruction of the digitized signals depends strongly on the phenomena of short-time instability inside the digitizing architecture [1]. Such phenomena give rise to an uncertainty on the sampling instant denoted mainly as timing jitter (or simply jitter), or as aperture uncertainty, or even as timing-phase noise [2]. In particular, the IEEE 1057-94 Standard defines the jitter as "the standard deviation of the sample instant in time", and measures it as the short-term stability during the longest single-record acquisition [2]. However, in the test procedure (i) all the digitizer errors are assumed to arise from jitter phenomena, and (ii) the jitter is characterized only in the longest record at the maximum sampling rate.

Several alternative techniques have been proposed to test different jitter components in statistical and in time domains both for analog-to-digital converter (ADC) chips and for waveform digitizers [3]-[13]. Ongoing work is devoted mainly: for ADC chips, to separate effects of different kinds of errors, especially in the test bench, from the jitter; and, for waveform digitizers, to define new testing techniques. In any case, the definition of a more suitable figure of merit and a corresponding testing technique still needs for more attention.

The intrinsically random nature of the jitter makes usual its characterization by statistical techniques [2]-[3]. In particular, phenomena related to random variations of the frequency over the time are usually modeled by using power spectrum density (PSD) laws proportional to a power of the sampling rate [14]. In some cases, such as just the jitter noise, the variance and the standard deviation defined in classical statistics diverge [14]-[15]. Therefore, random processes like jitter are not correctly characterized by the classic variance intrinsically. Conversely, in last years, such phenomena have been modeled successfully in terms of Allan variance, and specifically by its "modified" version [14]-[19].

In this paper, the usefulness of the Allan variance in the metrological characterization of jitter errors in waveform digitizers is shown [19]. In particular, in Section 2, the jitter characterization problem is restated by defining a more appropriate figure of merit based on the Allan variance and a related testing procedure. In Section 3, the diagnostic worth of the proposed figure of merit in determining the jitter noise type, and, thus, the consequent jitter error in different acquisition conditions is shown. In Section 4, the modification of an actual digitizer model [20] by including also jitter noise effects is proposed. Finally, in Section 5, experimental results highlighting the effectiveness of the Allan variance in the practical characterization of digitizer jitter error are reported.

2. THE ALLAN VARIANCE FOR DIGITIZER JITTER CHARACTERIZATION

In the following, after a recall on the jitter characterization in the *IEEE 1057 Standard*, a *figure of merit* based on the Allan variance and a related *testing procedure* are proposed.

Apart from more advanced techniques [3], the *IEEE 1057-94 Standard* assesses the jitter as the short-term stability during the longest single-record acquisition of the digitizer under test at a given sampling rate [2]. A corresponding measurement procedure is provided (section 4.9.2) [2], under the hypotheses that the jitter is the predominating error source, and the jitter of the test setup is negligible:

(1) A sample of phase deviation ϕ_i is estimated in each point of the record by: (i) applying a sine-fit algorithm to the samples; (ii) computing the residual in each sample time; and (iii) calculating the time aperture errors in each sample time by multiplying the amplitude residuals for the related time derivatives;

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Table I - Allan variance for some types of noise.

α	Noise type	Allan variance
2	White-noise PM	$a_2\tau^{-3b}$
1	Flicker-noise PM	$a_1\tau^{-2}$
0	Flicker-noise PM	$a_0\tau^{-2}$
-1	Flicker-noise FM	$a_{-1}\tau^{0}$
-2	Random-walk FM	$a_{-2}\tau$

(2) By repeating the previous step in independent acquisitions, the standard deviation of the jitter probability density function is estimated in each point of the longest digitizer record.

(3) An upper bound of the jitter is measured as the RMS of the sequence of the standard deviations achieved inside the record (i.e., more formally, as the standard deviation of the statistical sampling distribution of the achieved standard deviations).

The step (2) involves the acquisition of several records to estimate adequately the sampling distribution of the standard deviations inside the record. Moreover, this jitter estimation is valid only for the operating sampling rate $f_0=1/\tau_0$.

A more comprehensive *figure of merit* can be introduced by exploiting the concept of variance of a time-varying random process. Jitter in the digitizer timebase implies randomly time-varying sample times. In case of an ideal signal acquisition, this causes the instantaneous phase deviation ϕ of the measured signal to become a random process. Consequently, also the estimated instantaneous frequency $\Delta f = (\phi_i - \phi_{i-1})/\tau_0$ becomes a random process. This process can be characterized by the second-order moment of the related statistical distribution. In particular, the Allan variance showed to be useful for characterizing some kinds of time-varying processes, and particularly the jitter noise [14]-[19]. Thus, the Allan variance of the random process of the measured instantaneous frequency Δf is defined as [14]

$$\sigma_f^2(\tau_0)=1/2<\Delta f^2>, \quad (1)$$

where the symbol < > stands for the mathematical expectation.

In this way, the Allan variance expresses concisely the spread of the instantaneous frequency point by point along the record at the sampling rate f_0. A simple relation between $\sigma_f^2(\tau_0)$ and τ_0 can be derived [15], thus the Allan variance can be numerically evaluated versus time for different sampling rates without further experiments. A procedure for this is proposed in the following Section.

Moreover, the Allan variance is also convergent in some cases where the traditional variance diverges [14]–[15]. An exhaustive discussion about the link between the standard and the Allan variances is reported in [18].

A corresponding jitter *testing procedure*, based on the Allan variance determination is proposed:

1. A sequence ϕ_i of the phase deviations in the longest digitizer record taken at the maximum sampling rate f_0 is computed. This can be achieved easily by estimating the frequency, period by period of the sampled signal, through a zero-crossing technique [18].
2. The finite difference Δf between two successive phase deviation samples, i.e. a discrete estimate of the instantaneous frequency, is obtained as $\Delta f= (\phi_i - \phi_{i-1})/\tau_0$. More efficient alternative techniques for the instantaneous frequency estimation have been proposed [18].
3. Then, the jitter error is estimated as the positive square root of the (1).

3. THE ALLAN VARIANCE FOR DIGITIZER JITTER DIAGNOSIS

The Allan variance is useful also for detecting and identifying the jitter noise type for the diagnosis of the digitizer errors, and, moreover, for defining concisely the jitter in all the digitizer sampling conditions.

In problems dealing with the PSD of random processes $S(\omega)$ proportional to a power of the pulsation such as ω^α, the Allan variance was shown to be proportional to a power law of the sampling period: $\sigma_f(\tau)\propto\tau^\mu$, where μ is usually a constant value for constant values of α [14]-[15]. In particular, assuming that $S(\omega)\propto\omega^\alpha$, α and μ are related as: $\mu=-\alpha-1$, $\forall\alpha\in$ [-3,1]; or $\mu \cong -2$, $\forall\ \alpha\geq1$. The relation between α and μ is shown in Table 1 for some common types of noise. In case of PSD with α = 2, the apex *b* indicates the need of using the modified Allan variance to solve the ambiguity in the definition when μ = -2. In fact, this ambiguity does not allow a flicker noise phase modulation (PM) with α = 1 to be distinguished from a white noise PM with α = 2. This problem is solved by using the modified Allan variance with $\mu=-\alpha-1$, $\forall\alpha\in$[1,3].

In any case, a logarithmic diagram of $\sigma_f(\tau)$ allows the type of jitter noise of the digitizer under test to be identified easily by estimating the slope of the diagram.

Consequently, the jitter diagnostic process is:

1. Acquisition of a suitable statistically significant number of periods and samples of a reference sine wave input signal at the sampling rate f_0;
2. Determination of the phase deviation sequence;
3. Calculus of the instantaneous frequency sequence;
4. Estimation of the Allan variance for various values of $\tau=n\tau_0$;
5. Evaluation of μ in the logarithmic law;
6. Characterization of the jitter type by the knowledge of the ω^α law.

The biggest advantage of this process is the possibility of expressing the trend of the jitter error as a function of the sampling rate by only one record acquisition. Let a reference sine wave be acquired for the digitizer longest record at the maximum digitizer sampling rate f_0. The Allan variance is computed according to the above proposed procedure. A change in the sampling step τ can be performed easily by successively computing the (1) on successive couples of instantaneous frequency samples spaced by $\tau=n\tau_0$ in the original sample sequence. In this way, a new estimate of

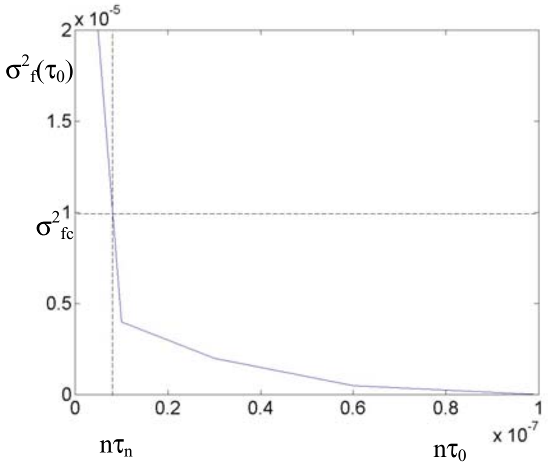

Fig.1 - Allan variance trend with the sampling period.

instantaneous frequency is obtained for each time interval $n\tau_0$. This allows information on the maximum time jitter to be gathered also at sampling rate different from the effective one by means of a unique experimental acquisition. An example of the Allan variance trend with the sampling period $n\tau_0$ is shown in Figure 1.

Moreover, this allows also an easy definition of the maximum sampling rate compatible with a given limit of the jitter error (Fig. 1): given the jitter limit expressed in terms of maximum Allan variance, the corresponding achievable maximum sampling rate is immediately determined by the law $\sigma^2_f(n\tau_0)$.

Another important application is related to the capability of detecting very-slow jitter on quite small time windows. As an example, the phenomenon of Fig.2 can not be detected by an analysis based on the classical variance carried out by acquiring data during the time window T_w. Conversely, the above diagnostic process allows the problem to be solved easily.

4. THE ALLAN VARIANCE FOR DIGITIZER JITTER MODELLING

The above analysis also allows a digitizer error model taking into account jitter errors to be set up. With this aim, a previously proposed actual digitizer model [20] was suitably modified such as depicted in Fig. 3. The new model differs from the former one for the input block taking into account an improved characterization of the jitter noise by means of the Allan variance. The model is the cascade of three sections.

The first section includes only an input block ω^α modeling the jitter effects (time jitter block). This model block is identified according to the procedure described in section 3. However, the jitter was measured through the zero crossing technique [17] in order to have a phase deviation estimate independent from other sources of noise. The previous version of the model [20] embodied here only a simple delay block. Conversely, the inclusion of the jitter

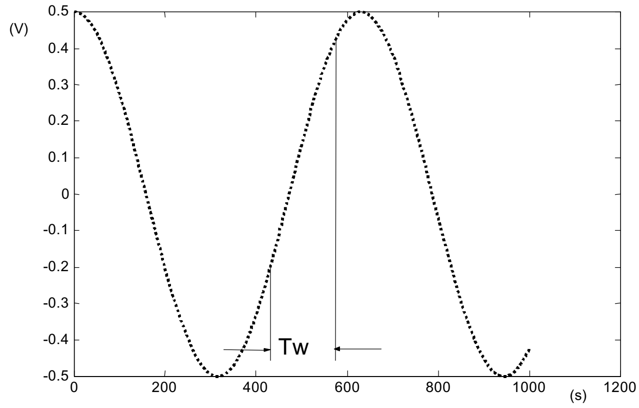

Fig.2 - The Allan variance capability of detecting very-slow jitter on small time windows.

error allows a more complete characterization of the digitizer error sources.

The second section contains three parallel branches: (i) the upper branch models the saturation effects by the odd function "hyperbolic tangent" *a*tanh* (amplitude compression block); (ii) the middle branch models the effects of the nonlinear transfer function by the even function "hyperbolic cosine" $d*[1-cosh(v_I/c)]$ (distortion block); and (iii) the lower branch models the gain error effects by the constant *k* (gain block).

Finally, the third section takes into account the effects of all the noise sources producing a reduction of the signal-to-noise ratio evidenced in the frequency domain by a noise floor growth apart the jitter noise effects. This residual noise is expressed by subtracting the jitter contribution assessed in the time jitter block to the overall noise.

5. EXPERIMENTAL RESULTS

A series of experimental tests on actual digitizers has been carried out in order to validate the proposed approach. A calibrated signal generator Stanford Research DS360 was used to characterize a Tektronix Waveform Analyzer VX4240. The validation scheme is shown in Fig. 4. The parameters of the actual digitizer model were identified by imposing the simulated ratio of the harmonic magnitudes equal to the experimental one through a sine wave acquisition [20]. The modeling error is evaluated by calculating the amplitude differences between the actual and modeled

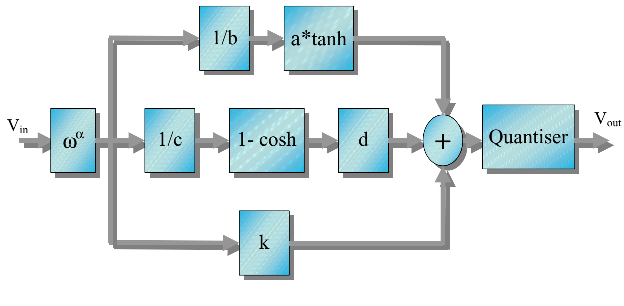

Fig.3 - New ADC model including jitter effects.

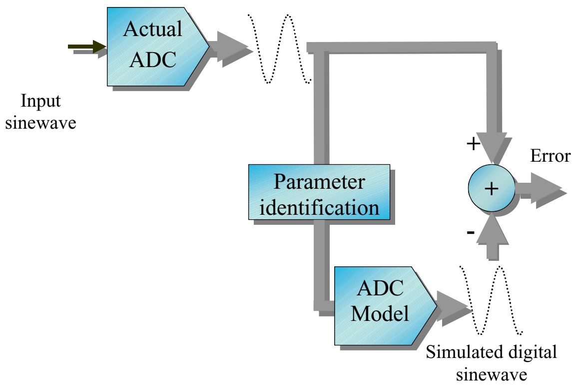

Fig.4 - Validation of the proposed approach.

digitizer outputs.

An example for a test sine wave of amplitude 10 V, and frequency 100 kHz acquired at a sampling rate of 10 MS/s for a total of 10,000 samples is considered. In Fig. 5, the experimental behavior of the Allan variance versus τ, computed according to the diagnostic procedure proposed in Section 3, is reported in logarithmic scale. The resulting value of α indicates a random noise process whose PSD is located between α=-1 and α=0.

The identified values of the other model parameters are: a=0.001; b=0.10; c=0.60; d=0.0025; and k=1.00. The validation error of the new model is reported in Fig. 6. For the sake of the clarity, in Fig.7 a particular of the error trend is zoomed on a different time window. Figs.6 and 7 highlight the effectiveness of the proposed model in taking into account digitizer jitter errors.

6. CONCLUSIONS

A new approach to the metrological characterization, the diagnosis, and the modeling of digitizer jitter based on the Allan variance has been presented. The jitter figure of merit based on the Allan variance expresses concisely the spread of the instantaneous frequency over the record at different sampling frequencies.

Moreover, it allows the diagnosis of the jitter error type, the characterization of the jitter at varying the sampling rate by only one test, as well as the update of a previously

Fig.6 - Errors of the new and old models.

proposed error model. Preliminary experimental results showed the effectiveness of the proposed approach in characterizing the digitizer error jitter.

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REFERENCES

[1] ITU, *Definitions and terminology for synchronization networks*, ITU-T Recommendation G.810 (08/96).

[2] IEEE, *IEEE Standard for digitizing waveform recorders*- IEEE Standard 1057, Dec. 1994.

[3] P. Arpaia, F. Cennamo, P. Daponte, "Metrological characterization of analog-to-digital converters – A state of the art", *Proc. of IEE ADDA99*, Glasgow (UK), July 1999, pp.134-144.

[4] L. Michaeli, "Fast dynamic methods of the systematic error autocorrection", *Proc. of 5th IMEKO TC-4 Int. Symp.*, Vienna, April 1992, pp.247-249.

[5] F. Cennamo, P. Daponte, M. D'Apuzzo, M. Savastano, "Digitizing signal analyzer calibration", *Proc. of IMEKO XII World Congress*, Beegjing (China), September, 1991.

[6] W. Gans, "Dynamic calibrations of waveform recorders and oscilloscopes using pulse standard", *IEEE Trans. on Instrum. and*

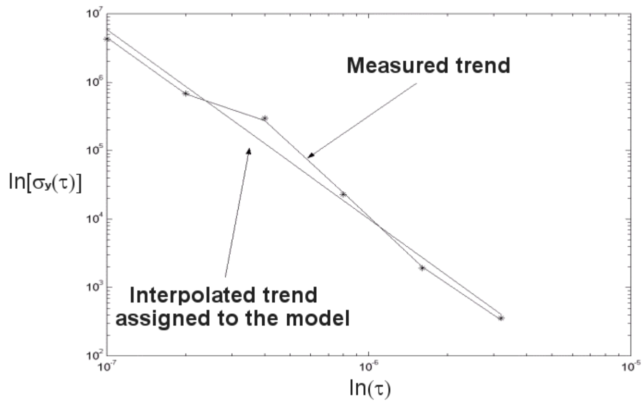

Fig.5 - Logarithmic graph of the Allan variance.

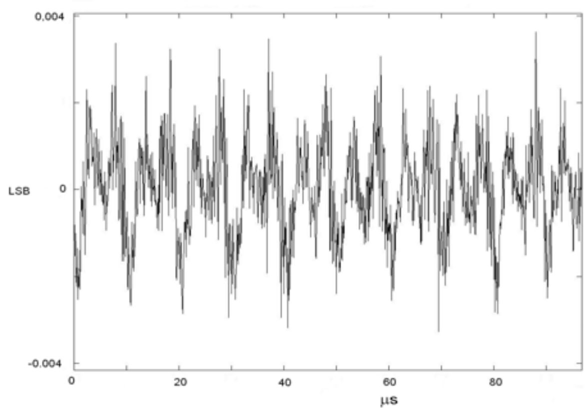

Fig.7 - Particular of the error of the new model.

Meas., vol.39, No.6, December 1990, pp. 952-957.

[7] M. Wagdy, S. Awad, "Effect of sampling jitter on some sine wave measurements", *IEEE Trans. on Instrum. and Meas.*, vol.39, No.1, February 1990, pp.86-89.

[8] Y. Jenq, "Digital spectra of nonuniformly sampled signals: theories and applications - measuring clock/aperure jitter on A/D systems", *IEEE Trans. on Instrum. and Meas.*, vol.39, No.6, December 1990, pp. 969-971.

[9] M. Shinagawa, Y. Akazawa, T. Wakimoto, "Jitter analysis of high-speed sampling systems", *IEEE Trans. on Circ. and Sys.*, vol.25, 1990, pp.220-224.

[10] D. Dallet, S. Le Masson, M. Benkais, P. Marchegay, "Statistical analysis for ADC transfer characteristic parameters and jitter determination", *Proc. of 1st IMEKO TC-4 Workshop on ADC Modeling and Testing*, Smolenice (SK), 7-9 May, 1996, pp. 146-151.

[11] G. Chiorboli, M. Fontanili, C. Morandi, "A new method for estimating the aperture uncertainty of A/D converters", *IEEE Trans. on Instrum. and Meas.*, vol.47, No.1, 1998, pp. 61-64.

[12] G.N. Stenbakken, J.P. Deyst, "Comparison of time base nonlinearity measurement techniques", *IEEE Trans. on Instrum. and Meas.,* Vol. 47, No. 1 , Feb., 1998, pp. 34 –39.

[13] D.M. Hummels, F.H. Irons, W. Ahmed, "Measurement of random sample time jitter for ADCs", *Proc. of ISCAS '95,* Vol. 1, 1995, pp. 708-711.

[14] D. W. Allan, "Should the classical variance be used as a basic measure in standards metrology?", *IEEE Trans. on Instrum. and Meas.*, vol. 36, No.2, June 1987.

[15] C. A. Greenhall , "The third difference approach to modified Allan variance", *IEEE Trans. on Instrum. Meas.*, vol. 46, No. 3, June 1997, pp.696-703.

[16] S. Hongwei, "Study on caracterization of frequency stability in time domain", *Proc. of 48th IEEE Freq. Control Symp.*, 1994, pp. 544 -547.

[17] L. D. Cosart, L. Peregrino, "Time domain analysis and its practical application to the measurement of phase noise and jitter", *IEEE Trans. on Instrum. and Meas.*, vol.46, No.4, August 1997, pp.1016-1019.

[18] A. Carlosena, C. Macua, M. Zivanovic, "Istrument for the measurement of the instantaneous frequency", *IEEE Trans. on Instrum. and Meas.*, vol.40, No.4, August 2000, pp.783-788.

[19] P. Arpaia, P. Daponte, F. Maccaro, S. Rapuano, "Characterisation and Modelling of the ADC Jitter", *Proc. of 5th IMEKO TC-4 Int. Workshop on ADC Modelling and Testing*, Vienna (A), Sept. 2000.

[20] A. Baccigalupi, P. Daponte, M. D'Apuzzo, "An improved error model of data acquisition systems", *IEEE Trans. on Instrum. and Meas.*, vol.43, No.2, April 1994, pp.220-225.