USING THE INTERPOLATIONS METHOD FOR NOISE SHAPING AD CONVERTERS

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Abstract:

Digital post processing of ADC output data requires among others a fast computational algorithms aimed on the enhancement of ENOB restricted by nonlinearities of the ADC. The paper deals with advantages and disadvantages of implementation Bayesian interpolation theorem to reduce large scale errors in output signal in spite of dithering with high peak-peak voltage. Interpolation process based on Bayesan model smoothes the distortion in the output signal caused by the integral nonlinearity in the ADC transfer characteristic. It suppresses also the random errors generated by allegory errors affecting the analog to digital conversion process.

Keywords: Interpolation, Noise Shaping, ADC Correction,

Theoretical background

The probabilistic model of ADC is presented by noisy channel on the base of Bayesian model. It includes effects of both random behavior in the alternational band around the ADC transition level T(k) and its shift caused by integral nonlinearities *INL(k).*

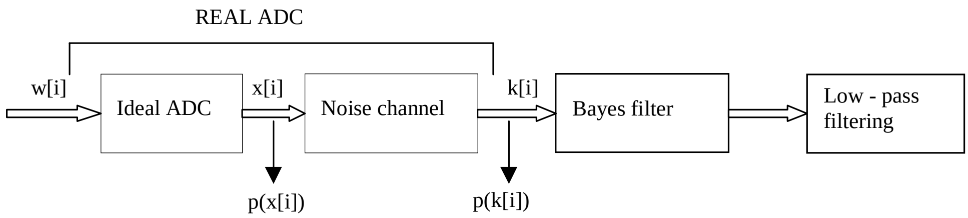

Fig 1. Probabilistic model of ADC

The first stage is represented by ideal ADC providing rounding operation. In noise channel are changed of the codes from the ideal ADC output to the real one. This process of code alternation could be described as the data transfer across a noisy channel. After noise channel the data flux could be filtered by the inverse Bayesian filter. This filter return all damaged data in their right position. Finally we can use low pass filtering.

Lets apply composition: oversampling ADC with dithered signal (and noise) added to the input, inverse Bayesan filter. The oversampling allow to apply the second approach for low band noise reduction the noise shaping. After noise shaping block the final low pass filtering algorithm has been implemented. Input ADC performs conversion by the conversion frequency much more higher than Niquist frequency. Digital output data flux is distorted by nonlinearities of input ADC. [1]

Dithering signal reduce only such a value of nonlinearities which are overlapped by dithering signal amplitude swing. A large scale errors require large dithering amplitude. Interpolation of oversampled data flux taking in account the integral nonlinearities *INL(k)* improves virtual effect reducing nonlinear distortion of ADC.

The proposed Bayesian interpolation allow to reduce the large scale errors using dithering signal with small peak-peak value. In such case the occurrence of dithered data with certain code bin enhance proportionally to peak-peak value reduction. In ideal ADC if input analog value is equal to *T[k]* transient code levels of two codes *k* and *k+1* probabilities of appearance this two codes is 50%.

In real ADC was many event "random" type. Effect *INL* make that probabilities of this two codes is not equal. Moreover the *INL* value higher than 1 causes that ideal value *x[i]* will be transferred into code bin $k[i] \neq x[i]$. If the corresponding value *DNL[k]* of some code bin *k* is –1, such code cannot fall. In order to describe alternation of ideal code bins *x* into their real counterparts *k* (output of noise transfer channel) the transfer characteristic was described in terms of conditioned probabilities *p(k[i]/x[i]).* This probability describes the effect of real INL on occurrence of the codes *k[i]* caused by input sample with value *x[i]* *(p(k[i]/x[i])).* Course of *p(k[i]/x[i])* is equal to probabilities profile of channel with code bin *k*. The probability density function *p(k[i]/x[i])* is being calculated from known values of transient code levels *T[k]* by algorithm shown on Fig.3. The values *T[k]* are determined by suitable testing procedure of ADC.

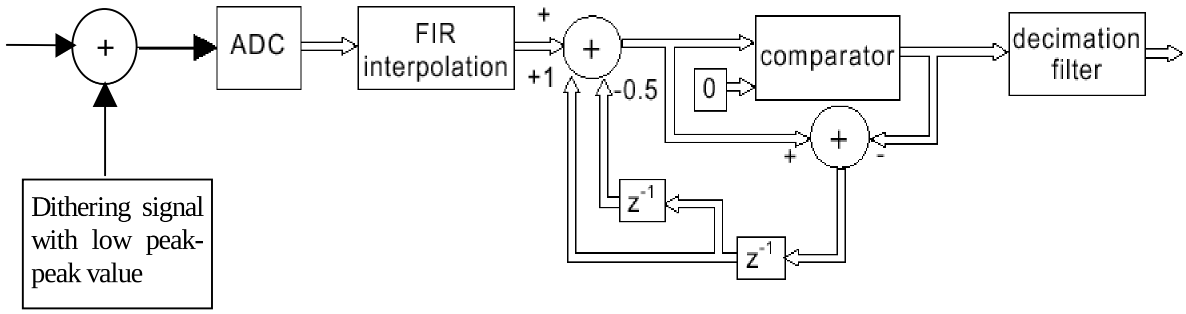

Fig. 2. Principe of digital correction based on a preliminary Bayesian interpolation of samples (and a successive second order noise shaping).

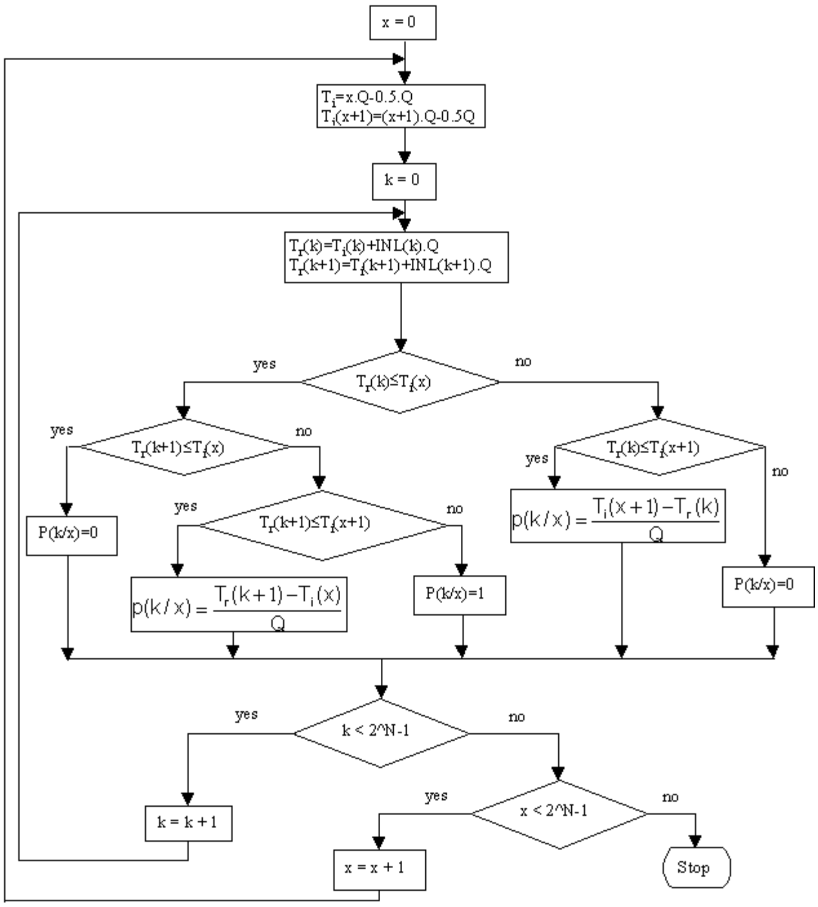

Fig. 3. Probabilistic model of distorted known nonlinearities

Bayes:

The fundamental theorem of inverse probability - Bayes' Theorem, is given below for the discrete case:

$$p(x[i]/k[i]) = \frac{p(k[i]/x[i]).p(x[i])}{\sum_{x=0}^{2^N-1} p(x[i]).p(k[i]/x[i])}$$

The inverse conditioned probabilities $p(x[i]/k[i])$ determines the probability of ideally quantised value $x[i]$ for the sample $k[i]$ acquired from the real ADC output. The best estimated input voltage $x_e[i]$ for output code $k[i]$ is expressed by Bayesian relation:

$$x_e[i] = \sum_{x[i]=0}^{2^{N-1}} x[i].p(x[i]/k[i]) \doteq$$

$$\doteq \sum_{x[i]=0}^{2^{N-1}} x[i] \frac{p(k[i]/x[i]).p(x[i])}{\sum_{x[i]=0}^{2^N-1} p(x[i]).p(k[i]/x[i])}$$

where *p(x[i])* is probability density function of input signal.

The noise invaluably dithering signal for the oversampled data flux could be removed by the average value $\overline{x[i]}$, calculated by a moving window of length L=2L_s+1. The optimal reconstructed sample *x[i]* which represented the windowed output of filter with length L_s is:

$$\overline{x[i]} = \frac{1}{2L_s+1}\sum_{j=-L_s}^{L_s} x_e[i+j] =$$

$$= \frac{1}{2L_s+1}\sum_{j=-L_s}^{L_s}\sum_{x[i]=0}^{2^{N-1}} x[i]\frac{p(k[i+j]/x[i+j]).p(x[i+j])}{\sum_{x=0}^{2^N-1} p(x[i+j]).p(k[i+j]/x[i+j])} =$$

$$= \frac{1}{2L_s+1}\sum_{j=-L_s}^{L_s}\sum_{x[i]=0}^{2^{N-1}} x[i]\frac{p(k[i+j]/x[i+j]).p(x[i+j])}{p(k[i+j])}$$

The denominator of this formula is *p(k[i])* and could be assessed for the stationary signals by continuous histogram acquisition. The conditioned probability *p(k[i]/x[i])* is a function of real integral nonlinearity shape. The computing algorithm for its determination is shown on Fig. 3. It presents by the probability figures the fact that *INL(k)* errors causes for the ideally digitalised sample *x[i]* the distorted real output sample. The only parameter in this formula which is unknown is *p(x[i])*. For the ADCs with *INL(k)* with polynomial shape the *DNL(k)* are ≈ 1 (almost equal) and the probability *p(x[i])* could be approximated by *p(k[i])*.

$$\overline{x[i]} \doteq \frac{1}{2L_s+1}\sum_{j=-L_s}^{L_s}\sum_{x[i]=0}^{2^{N-1}} x[i].p(x[i+j]/k[i+j])$$

The calculated Bayesian model was assessed using half period of triangular signal In this case is probability *p(x[i])* constant. The conditioned probability *p(x[i]/k[i])* was calculated for simulation model of real ADC with known *INL* course.

In order to involve the converting block in the stage where the set of samples with certain probability during the window (2L_s+1) occurs the dithering voltage with the necessary low peak – peak value Q is implemented.

Simulation results:

Efficiency of Bayesian filtration depends on precision how the conditioned probabilities *p(x[i]/k[i])* are calculated. Consistent increase number of samples in window in first decrease level of spectral elements in high frequency band and smooth increase this spectral elements in low frequencies.

In order to assess the efficiency of proposed algorithms the proper figure of merit gain was proposed. This is represented effective number of bits defined by formula:

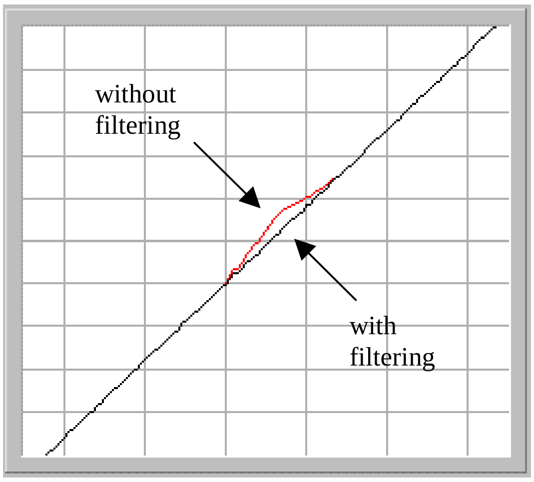

Fig.4. Output after Bayes filtration using main Bayes theorem to triangular signal

$$G\_ENOB = \frac{1}{2}\log_2\frac{e_r^2}{e_{out}^2}$$

where e_r is the real quantisation noise power at the output of ADC without dithering and Bayesian filtering. e_{out} is the quantisation noise power after dithering, Bayesian estimation and FIR filtering by moving windows of L_s.

The continuous character of INL generates distorted frequency components of higher order related with signal frequency. Reduction of signal distortion

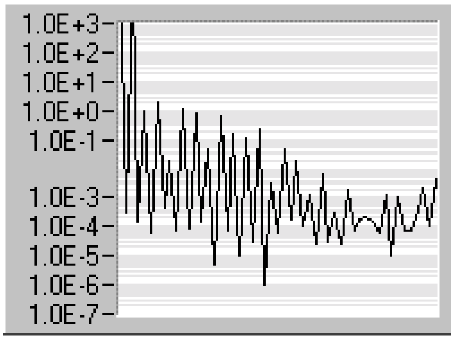

Fig 5 Spectrum of real ADC without error reduction by Bayesian filter on low frequencies (N=10bit, sampl=2^10, fvz=2^7)

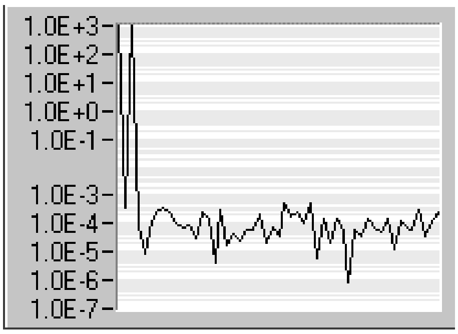

Fig.6 Spectrum of the output signal from Bayesian filter on low frequencies without low pass filtering (N=10bit, w=2, sampl=2^10, fvz=2^7)

observable in time domain (Fig.4) has been displayed in frequency domain by reduction of high order frequency components in the neighbouring of basic frequency component. Fig. 5, Fig. 6.

Consistent increase of length L_s in the output of FIR filter causes decrease spectral elements not only on high frequencies but also in low frequencies. In this case Bayes estimation increase G_ENOB. In simulation was ascertained that Bayesian filter has effect if number of samples in window is minimal 1.4% from total samples number.

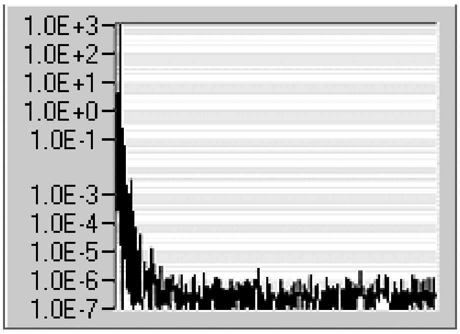

Fig.8 Spectrum of signal from the output of Bayesian filter smoothed with short window FIR - full spectrum (N=9bit, w=64, sampl=2^10, fvz=2^7)

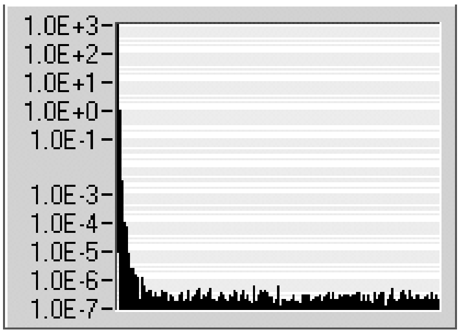

Fig 9 Spectrum of signal from the output of Bayesian filter smoothed with large window FIR - full spectrum (N=10bit, w=100, sampl=2^12, fvz=2^7)

In all this results it's a deal positive effect of dithering. Effect of dithering increase with higher dither frequency (higher than sample frequency), but just for higher resampling FA.

Finally it's better combine Bayesian filtration with use dithering but just with large width window. Fig. 8, Fig.9

Conclusions

A suitable Bayesian filter which was proposed suppress the distorted spectral components overriding the reduced quantisation noise. Bayesian filter allow to implement dithering signal with small peak - peak value for suppression of large scale errors. The fixed digital value below quantization level can be reduced by dithering with low peak – peak value of the input signal. The proposed Bayesian processing approach offers another advantage The distortion is significantly reduced with smoothing procedure based on FIR with short window. This enable faster calculations for obtaining sufficiently corrected output spectrum. Disadvantage of interpolation algorithms based on mentioned stimulation is the generation of new spectral components by harmonic distortion processed signal. There are some constrains which obstruct for this time full function of Bayesian filter. E.g. calculation of probabilities p(x) for sinusoid signal. The faster algorithm and symmetry of the error features could accelerate the digital processing time.

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