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# A proposed method for decreasing the losses of the sample number during a weight filtration

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***Abstract-*** Weight filtration is one of the methods for weakening the effect of noise exerted on discrete signals. The wider is the weight window, the higher is the filtration quality. However, weight filtration produces an undesirable side effect, namely a certain number of samples relative to the input is lost. This may be attributed to the concept of weight filtration itself. The number of lost samples is the greater, the wider is the weight window. So, the desire to get a well-filtered signal comes into conflict with the number of samples being lost by virtue of filtration. In the paper an approach is suggested that enables the sample losses occurring during the weight filtration to be reduced.

## I. Introduction

Weight filtration is one of the methods aimed at decreasing the effect of noise affecting discrete signals, inclusive of signals to be measured. Unlike digital FIR (*Finite Impulse Response*) and IIR (*Infinite Impulse Response*) filters, weight filtration is simpler to accomplish, because no additional signal transformation in frequency domain, e.g., Laplace or z-transform, is required. Weight filtration is performed in time domain through multiplying values of appropriate samples by a weighting function, also called *weight window*, and summing the yielded products along the entire window length. The only condition the weighting function has to meet is

$$\sum_{i=-k}^{+k} w_i = 1 \tag{1}$$

i.e., the sum of its components should be equal to 1 [3].

A weight filter is a low-pass filter with a not sharp cutoff. The filtration quality is the greater, the wider is the weight window [4].

## II. Loss of samples due to weight filtration

In the case that the number of samples $L$ of the recorded measured signal is much greater than the number of samples $N$ ( $L >> N$ ) delivered by the $y_w$ signal, called *filtration input signal* in what follows, the weight filtration may be defined by the following relationship

$$y_{f\,j} = \sum_{i=-k}^{k} w_i \cdot y_{w_{i+j}} \quad \text{for } j = 1,\ldots,N \tag{2}$$

Where:

- $y_{w_{i+j}}$ is the filtration input signal,
- $y_{f_j}$ is the signal after being filtered,
- $w_i$ is the weighting function, $k$ is a parameter determining the window width equal $2 \cdot k + 1$,
- $N$ is the number of samples of the measured signal to be filtered.

The subscript of $y_w$ in (2) is shifted to the initial value $i + j$ because the weighting function $w_i$ is symmetrical about the middle of the window, which is $2 \cdot k + 1$ wide. Hence, to get the value of the filtered point $y_{f_i}$ one has to use $k$ preceding and $k$ posterior samples around the $y_{w_i}$ point, as shown in Figure. 1.

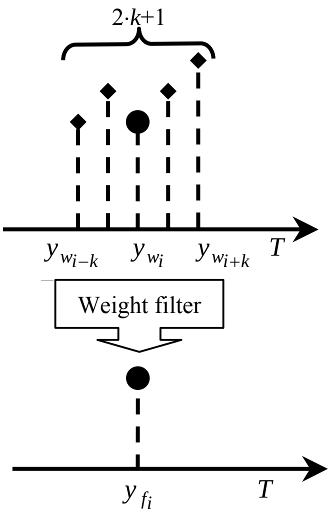


Figure. 1. Weight filtration.

This has the effect of losing $k$ first and $k$ last samples of $y_w$ signal, as depicted in Figure. 2.

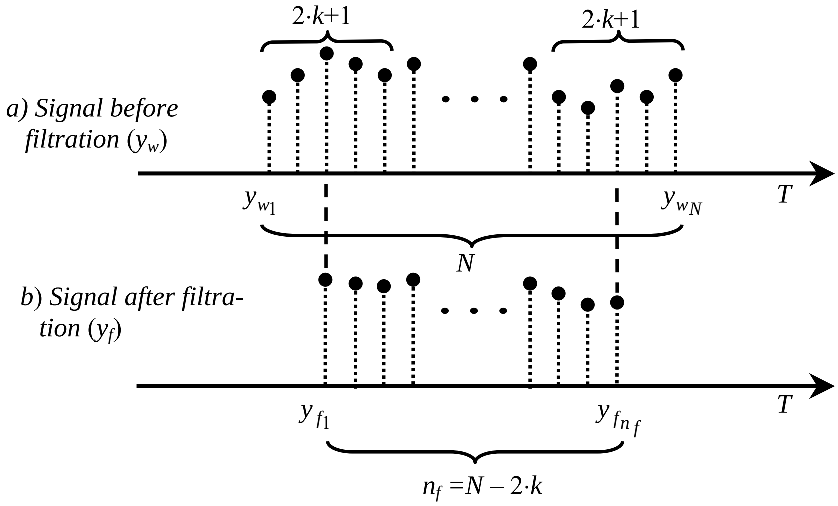


Figure. 2. Effect of weight filtration on the number of samples.

Hence, if the discrete measured signal $y_w$ to be filtered has $N$ samples, and the weight window is $2 \cdot k + 1$ wide, then the following number $n_f$ of samples appears at the output of the weight filter

$$n_f = N - 2 \cdot k \tag{3}$$

In the case that $N = L$, the relationship (2) assumes the form

$$y_{f_j} = \sum_{i=-k}^{k} w_i \cdot y_{w_{i+j}} \quad \text{for } j = k+1, \ldots, N-k \tag{4}$$

Thus, it may be stated that the classic weight filtration is intrinsically characterized by the lost of $2 \cdot k$ samples with respect to $N$ samples the signal features before having been filtered. This can result in a partial lost of information carried by the measured signal, the greater, the wider is the weight window. So, the desire to increase the filtration quality entails widening the weight window, but the number of samples $n$ available after filtration diminishes. This presents no problem if a sufficiently great number of samples relative to the window width is available, hence, the lost of $2 \cdot k$ samples is insignificant here. However, if this is not the case, an increase in the number of samples through higher sampling frequency or condensation of samples through interpolation may present a way-out [5]. Another remedy is to lengthen the time the signal is recorded, or in other words *redundant sampling*. The method consists in taking *k* samples before the proper measured signal begins and *k* samples after it is finished. Such solution is applicable if we have to do with periodic signals, or if a confined fragment of an aperiodic signal is of interest to us. Problems arise when the signal to be sampled is of finite duration, and the entire signal is subjected to filtration. In addition, the above-mentioned way-outs require an extensive *a priori* knowledge about the signal to be sampled in order to choose optimal parameters of sampling and filtration [6,7].

### III. A proposal to resolve the problem of sample losses in weight filtration

As it was mentioned above, the classic approach to weight filtration (Figure. 2) requires that *k* preceding and *k* posterior samples around the point $y_{w_i}$ be known in order to evaluate the sample $y_{f_i}$. Hence, to get the entire filtered signal the number of samples should exceed the number of samples *N* of the input $y_{w_i}$ by $2 \cdot k$. Essentially, the author's proposal consists in increasing the number of samples by $2 \cdot k$ by means of mathematical operations, i.e. by *k* samples before the signal begins and by *k* samples after the signal is finished. This is accomplished by turning *k* samples through an angle $\varphi = 180°$ about the first point $y_{w_1}$ of recorded *N* samples, and by turning *k* samples through the same angle about the last point $y_{w_N}$ of recorded *N* samples of the input measured signal. The concept is illustrated in Figure. 3.

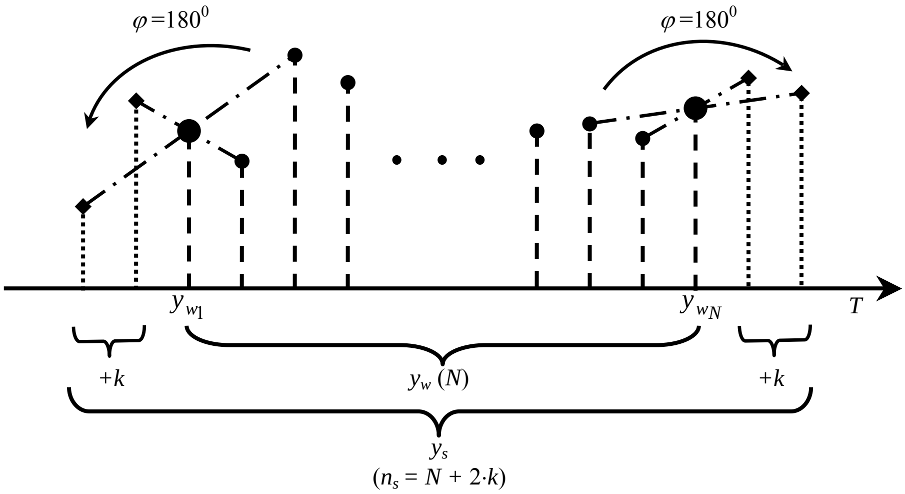


Figure. 3. Concept of a preliminary increasing the number of samples preparatory to weight filtration.

The values of added samples $y_s$ are defined in the following way

– for signal beginning

$$y_{s_{-i}} = \left(y_{w_1} - y_{w_i}\right) + y_{w_1} = 2 \cdot y_{w_1} - y_{w_i} \tag{5}$$

– for signal finishing

$$y_{s_{N+i}} = \left(y_{w_N} - y_{w_{N-i}}\right) + y_{w_N} = 2 \cdot y_{w_N} - y_{w_{N-i}} \tag{6}$$

As a result, the preliminary number of samples $n_s = N + 2 \cdot k$ of an augmented measured input $y_s$ is obtained. The last operation to be done before the signal is filtered is to rearrange sample subscripts in the newly obtained augmented set of samples of the input signal $y_s$. Such augmented signal $y_s$ is subjected to weight filtration, which yields eventually the following number of samples

$$n_f = n_s - 2 \cdot k = \left(N + 2 \cdot k\right) - 2 \cdot k = N \tag{7}$$

In Figure. 4 results of simulations carried out are depicted.

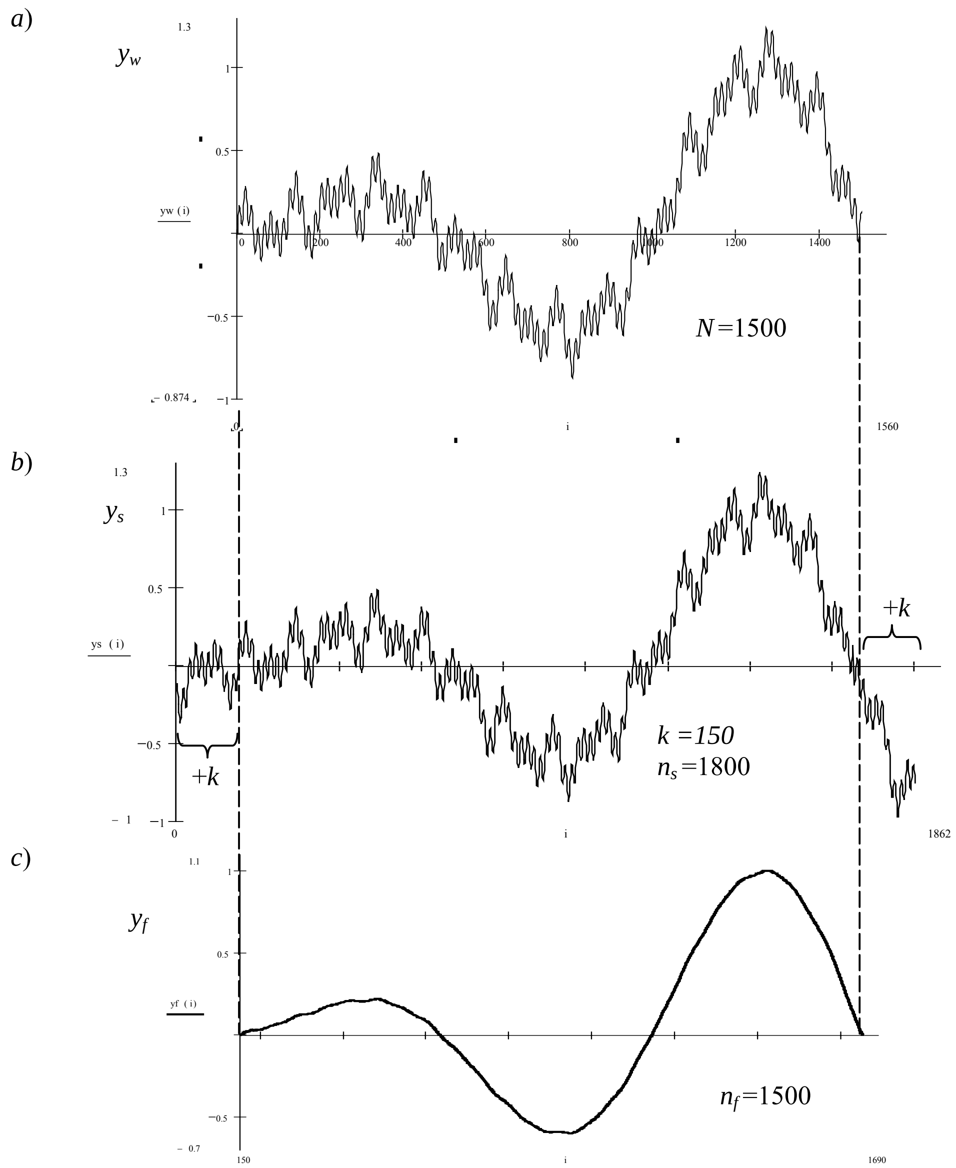

Figure. 4. Results of the simulated method for preliminary increasing the number of samples preparatory to weight filtration according to author's proposal.

In the presented example the simulated measured signal $y_w$ ($N = 1500$ samples) (Figure. 4a) is affected by a uniformly distributed random noise. Figure 4b shows the $y_s$ signal augmented by $2 \cdot k = 300$ samples ($k = 150$ samples at the beginning of the signal and $k = 150$ samples at the end of the signal). Figure 4c shows the signal $y_f$ after the noise has been filtered off by means of the weight filter with a rectangular window given by

$$w_i = \frac{1}{2 \cdot k + 1} \tag{8}$$

where $2 \cdot k + 1$ is the window width. In the above example the window width amounts to $2 \cdot k + 1 = 301$ samples.

### IV. Practical use of the proposed method

This is best illustrated by a concrete example of a study conducted by the author, which was concerned with filtration-approximation [6] of measured signals affected by noise [7,8]. Here filtration provided a basis for determining the position of spline approximation knots of the 3rd order. Analysis of carried out simulations and comparisons between IIR, FIR and weight filtrations suggested that weight filtration with a rectangular window (8) offers best results, a selection of which is exemplified in Figure. 5.

*a*)

*b*)

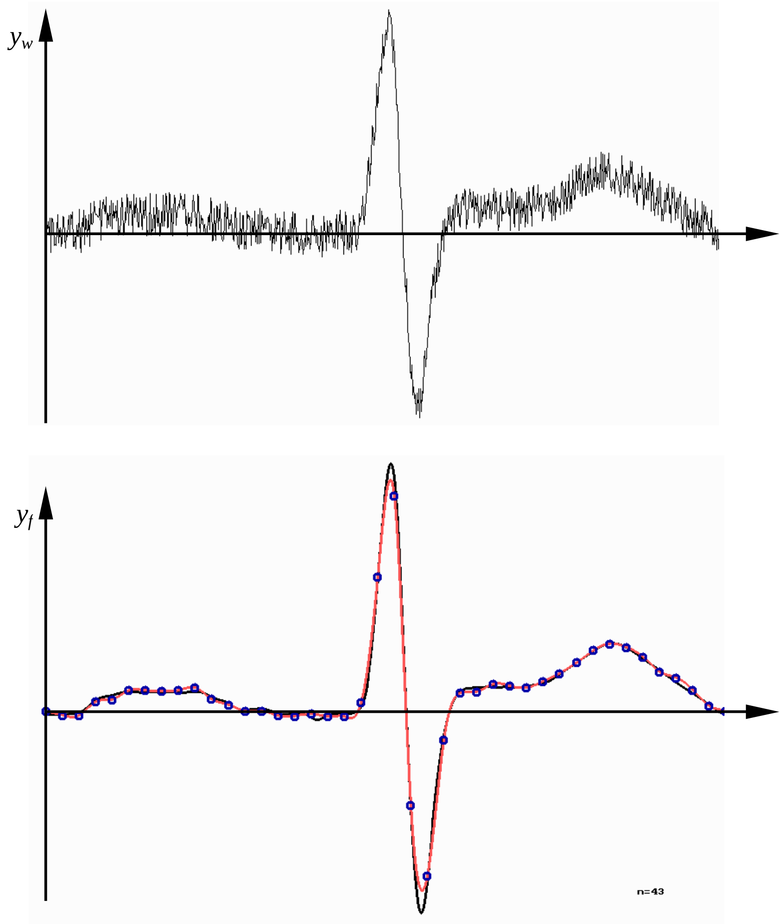


Figure. 5. Using weight filtration with a rectangular window
to position spline approximation knots of the 3rd order [7,8].

Figure 5 shows a measured signal (ECG) affected by a uniformly distributed noise the amplitude of which amounts to 10% of the standardized measurand amplitude. Figure 5b illustrates approximation knots (designated by circles) obtained as a result of weight filtration the signal of Figure. 5a has been subjected to.

## V. Conclusions

Results of tests carried out both on actual signals and by way of simulation [7,8] lend support to the proposed method of increasing the number of samples for weight filtration of noise-affected signals to be measured. the method enables one to make up for samples lost as a result of weight filtration itself, and to avoid a contradiction between filtration quality increasing with the window width, and the number of samples lost by virtue of filtration.

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