Two Parameter (2D) Measurements in Four Terminal (4T) Impedance Bridges as the New Tool for Signal Conditioning (1)

Part 1. Main Equations and Terminal Parameters

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Abstract. *In two parts of this paper is presented the synthesis of some author's works on unified backgrounds of immitance multivariable (nD) measurements of few jointed variables and on the unconventional signal conditioning in four terminal (4T) bridge circuits used for that. In Part 1 four different circuits for 2D measurements are proposed. Short description of equations and main parameters of the 4R bridge as twoport type **X** of variable parameters and then as circuit 4T unconventionally supplied by two current sources, named double current bridge (2J) are given and shortly discussed. In Part 2 application of both types of 4R bridges for 2D measurements is presented and discussed in details.*

1. Introduction

Four-terminal (4T) bridge networks are presented in equivalent circuits of many objects of variable internal parameters tested on their terminals or by a couple of contacting probes. They are frequently used on inputs of many data acquisition circuits of autonomic immitance measurement instruments and of system transmitters. It is due to two main reasons:

- direct relations are known of bridge terminal parameters to changes of arm immitances or also indirect to other variables detected by immitance sensors
- bridges could realize some simply and easily with required accuracy signal conditioning functions, such as output signal dependence on difference or sum of increments of several parameters of the immitance sensor set from its initial stage.

During a couple of last decades many of new types of single variable (1D) signal conditioning circuits have been developed. They use variously connected differential operational amplifiers [1-6], many methods of switching (e.g. various integrating or successive balancing converters), single and double high accuracy stabilized supply sources or supplying of input analogue circuit and its AD converter from common source (ratio measurement case).. But in measurements with immitance sensors the most popular are still bridges working mainly with the practically open circuit voltage output, very rare with the current one.

In many multivariable measurements (nD) starting from 2D, values and increments of circuit internal parameters have to be checked indirectly from its terminals. Also in several practical situations additionally should be measured some number of external variables which simultaneously and, in most cases, unselectively are influencing these parameters, e.g. few parameters of single immitance sensors or integrated set of them. Methods of successive testing individual transfer impedances or admittances between any two terminals of the network as used in the impedance tomography, is in many situations unsatisfying because of poor accuracy. Especially, if subtraction of near values of few increments is being made, another methods have to be applied. Some proposals have been presented below.

Four circuits of 2D measurements are given on the Fig 1a-d. All have two voltage outputs and unconventionally applied the four arms resistance bridge (4R), differently supplied by current sources. Circuit 1a contains two classic Wheatstone bridges, 1 and 2 cascade-connected, both working as twoports on diagonals and are supplied to one of them. These bridges are in balance for initial values R_{i0} of arm resistances of the bridge 1. First voltage output U'_{DC} depends on increments of its arm resistances from their initial values in balance, second one, ΔU_{AB} of the bridge 2, depends on increment ε_{AB} of input terminal resistance R_{AB} of the bridge 1. Next three circuits of the Fig 1 are supplied unconventionally by ideal current sources connected in parallel to opposite bridge arms or switched between them. For these new circuits author have proposed name to use: **double current bridges** [7, 10] and **acronym 2J**. Their outputs are on both bridge diagonals and are balanced for initial values R_{i0}. Two current sources J_1 and J_3 of the circuit 1b should be equal. In circuits 1c, 1d all measurements are made twice with replacing two unequal sources $J_1 \neq J_3$ (1c), or even only the single one J (1d), and two obtained results are averaged. Proper understanding of the operation of all these circuits makes necessary to present the short description of equations and main parameters of the 4R bridge as twoport type **X** of variable parameters and then as 4T circuit unconventionally supplied by two current sources.

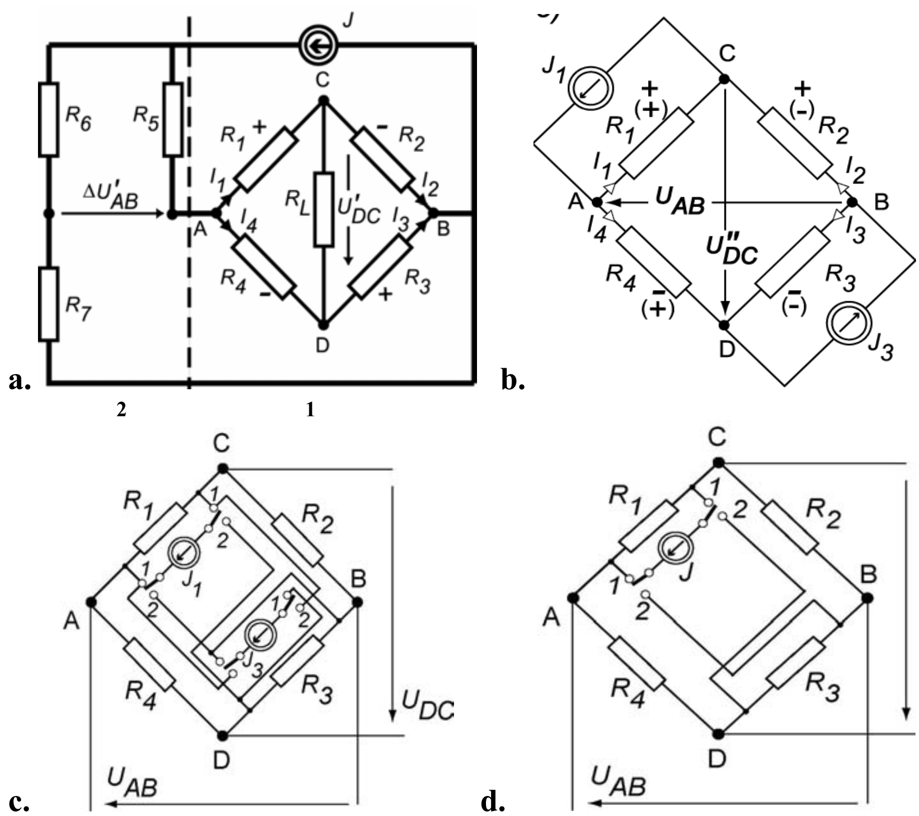

Fig. 1. Current supply circuits of 4R bridges for use in two variable (2D) measurements:
a. Cascade of two classic bridges of basic relations: $R_5 >> R_{AB}$; $R_L >> R_{CD}$ and balance conditions: bridge no 1 $R_{10}R_{30} = R_{20}R_{40}$, bridge no 2 $R_5 R_7 = R_6 R_{AB0}$.
Double current bridges: **b.** 2J – of equal two current supply sources; **c.** 2x2J – of two switched current sources $J_1 \neq J_3$; **d.** 2xJ –of the single switched current source J.

2. Basic Equations for the Four-Arm Resistance Bridge (4R) as Twoport

Circuit of the Fig.1a contains two jointed bridges 1 and 2; both powered classically to one of their diagonals. The input current I_{AB} of the bridge 1 depends on equivalent parameters of the supply branch terminals of the bridge 2. Ideal current source J and internal resistance R_G in parallel or ideal voltage source E and serial R_G could represent them. Current I_{AB} depends also on the input bridge resistance R_{AB}. The last one is a function of actual values of all bridge resistances R_i and of its load R_L, if any. Then for the full description of the bridge as linear circuit (SLS) of variable parameters, relations of voltages and currents of bridge input and output terminal pairs should be used, e.g. impedance type twoport equations expressed in generalized matrix form as:

$$\mathbf{U} = \mathbf{Z} \ \mathbf{I} \tag{1}$$

When all parameters are real, matrix Z = Re (Z) ≡ Z_R and for the 4R bridge it is:

$$\mathbf{Z}_R \equiv \begin{vmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{vmatrix} = \begin{vmatrix} \dfrac{(R_1+R_2)(R_3+R_4)}{\sum R_i} & \dfrac{R_1R_3-R_2R_4}{\sum R_i} \\ \dfrac{R_1R_3-R_2R_4}{\sum R_i} & \dfrac{(R_1+R_4)(R_2+R_3)}{\sum R_i} \end{vmatrix} \tag{1a}$$

where: $r_{11} \equiv R^{\infty}_{AB}$, $r_{22} \equiv R^{\infty}_{CD}$ – input and output resistances of the opposite open-circuited twoport sides.

r_{12}, r_{21} – current to voltage both direction transmittances (transfer resistances) when the opposite side port is open-circuited ($R_G \to \infty$ or $R_L \to \infty$)

$\sum R_i = R_1 + R_2 + R_3 + R_4$ – sum of resistances of bridge arms;

After expansion of matrix formula (1) to algebraic form it is:

$$\begin{aligned} U_{AB} &= R^{\infty}_{AB} I_{AB} - r_{12} I_{DC} \\ U_{DC} &= r_{21} I_{AB} - R^{\infty}_{CD} I_{DC} \end{aligned} \tag{2}$$

where: bridge output current $I_{DC} = -I_{CD}$

Because transmittances (or transfer resistances) of the 4R bridge in both directions are equal, i.e. $r_{21}=r_{12}$, then even in general case only three elements of the $\mathbf{Z_R}$ matrix are different. They depend on four bridge circuit resistances. Values of these elements are always finite, including ones in the initial balance state $r_{21}=0$. The open circuit output voltage U^{∞}_{DC} of the bridge 1 (in practice: $R_L>>R_{CD}$) is given by the equation:

$$U^{\infty}_{DC} = I_{AB}\, r_{21} = \frac{I_{AB}\,(R_1R_3 - R_2R_4)}{\sum R_i} \tag{3}$$

In a general case of the bridge all arm resistances are variable. Farther analysis should be more general and easier if arm resistances are referenced to one of them of balanced bridge, e.g. to R_{10} and their changes - to initial values of resistances in this state. Then:

$$R_i \equiv R_{i0} + \Delta R_i \equiv R_{i0}(1+\varepsilon_i) \equiv r_{io} R_{10}(1+\varepsilon_i) \tag{4}$$

where: R_{i0} – initial (in balance) value of the R_i resistance

r_{i0} – relative values of arms' resistances in balance conditions

$\Delta R_i = r_{i0}\,\varepsilon_i$; ε_i – absolute and relative arm resistance increments.

After transformation of (3) with notations (4) one gets:

$$U^{\infty}_{DC} = I_{AB}\, R_{10}\, \frac{r_{20}\, r_{40}}{\sum r_{i0}} \cdot \frac{(\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4 + \varepsilon_1\varepsilon_3 - \varepsilon_2\varepsilon_4)}{1 + \dfrac{\sum r_{i0}\varepsilon_i}{\sum r_{i0}}} \equiv T'_0 \cdot f'(\varepsilon_i) \tag{5}$$

where: $T'_0 \equiv I_{AB}\, t'_0$ and $f'(\varepsilon_i)$ – initial sensitivity and normalized unbalance function of open-circuitry output voltage of the 4R bridge.

Commonly known bridge balance condition (when $U_{DC}=0$) for initial resistances is

$$R_{10}R_{30} = R_{20}R_{40} \tag{5a}$$

In (5) it is directly expressed how output open voltage U^{∞}_{DC} and transmittance r_{21} are depended on signs and values of the relative resistance increment ε_i of particular bridge arm. Increments of opposite signs in neighboring arms, given on Fig. 1a unbalance the bridge in the same direction. If absolute values of these increments are the same ($\varepsilon_i = \pm\varepsilon$) the output voltage is proportional to the number of variable arms (e.g. multiplied by 2 or by 4).

If also terminal parameters of the bridge as twoport are referenced to their initial values in balance and - for simplified notation- it is put down additionally that: $R_{20} \equiv mR_{10}$, $R_{40} \equiv nR_{10}$ and $R_{30}=mnR_{10}$, then matrix $\mathbf{Z_R}$ from equation (2) could be written as:

$$\mathbf{Z}_R = \begin{vmatrix} R^{\infty}_{AB\,0}\left(1+\varepsilon^{\infty}_{AB}\right) & t'_0\, f'\left(\varepsilon_i\right) \\ t'_0\, f'\left(\varepsilon_i\right) & R^{\infty}_{CD\,0}\left(1+\varepsilon^{\infty}_{CD}\right) \end{vmatrix} \tag{6}$$

where:

$$t'_0 \equiv \frac{R_{10} R_{30}}{\sum R_{i0}} = R_{10} \frac{mn}{(1+m)(1+n)} \tag{6a}$$

- initial open circuit sensitivity of transmittance r_{21}

$$f'\left(\varepsilon_i\right) = \frac{(\varepsilon_1-\varepsilon_2+\varepsilon_3-\varepsilon_4+\varepsilon_1\varepsilon_3-\varepsilon_2\varepsilon_4)}{1+\dfrac{\varepsilon_1+m\varepsilon_2+n\left(m\varepsilon_3-\varepsilon_4\right)}{(1+m)(1+n)}} \equiv \frac{\Delta L(\varepsilon_i)}{1+\varepsilon_{\Sigma R}(\varepsilon_i)} \tag{6b}$$

- normalized unbalance function of r_{21}

$\Delta L\left(\varepsilon_i\right)$; $\varepsilon_{\Sigma R}$ - increments of numerator and denominator of (6b)

$\sum R_{i0}$ - value of ΣR_i of the balanced bridge.

Transmittance r_{21} and it's unbalance function $f'\left(\varepsilon_i\right)$ could theoretically take on values from the range (-∞, +∞). In practice there are some limitations e.g. due to extreme values of increments ε_i, limited dissipated powers of arms and limited voltage of the real current source. Transmittance $r_{21}=0$, if $f'(\varepsilon_i)=0$. It could happen for many different combinations of ε_i. The basic balance state was defined already as such one, when all $\varepsilon_i=0$.

Open-circuits terminal resistances of the 4R bridge are:

$$R^{\infty}_{AB} = R_{AB\,0}\left(1+\varepsilon^{\infty}_{AB}\right) = R_{AB\,0} \frac{\left(1+\varepsilon_{12}\right)\left(1+\varepsilon_{34}\right)}{\left(1+\varepsilon_{\Sigma R}\right)} \tag{7}$$

$$R^{\infty}_{CD} = R_{CD\,0}\left(1+\varepsilon^{\infty}_{DC}\right) = R_{CD\,0} \frac{\left(1+\varepsilon_{14}\right)\left(1+\varepsilon_{23}\right)}{\left(1+\varepsilon_{\Sigma R}\right)} \tag{8}$$

where:

$R_{AB\,0} = R_{10} \dfrac{n\left(1+m\right)}{1+n}$ (7a), $\qquad R_{CD\,0} = R_{10} \dfrac{m\left(1+n\right)}{1+m}$ (8a)

-- input and output bridge resistances in the balance state

$\varepsilon^{\infty}_{AB} = \dfrac{1}{1+\varepsilon_{\Sigma R}\left(\varepsilon_i\right)}\left(\dfrac{n\varepsilon_{12}+\varepsilon_{43}}{1+n}+\varepsilon_{12}\varepsilon_{43}\right)$ (7b), $\qquad \varepsilon^{\infty}_{CD} = \dfrac{1}{1+\varepsilon_{\Sigma R}\left(\varepsilon_i\right)}\left(\dfrac{m\varepsilon_{14}+\varepsilon_{23}}{1+m}+\varepsilon_{14}\varepsilon_{23}\right)$ (8b)

-- relative increments of R_{AB} and of R_{CD} from the balance state

$\varepsilon_{ij} \equiv \dfrac{\Delta R_i + \Delta R_j}{R_{i0}+R_{j0}}$ (9a), $\qquad \varepsilon_{\Sigma R} \equiv \dfrac{\varepsilon_{12}+n\,\varepsilon_{43}}{1+n} = \dfrac{\varepsilon_{14}+m\varepsilon_{23}}{1+m}$ (9b)

-- relative increments of the sum of R_i+R_j resistances of *i, j* arms and of the sum $\sum R_i$.

In author's papers [8-10] are given tables containing elements of matrix Z_R of the resistance 4R bridge operating as the twoport. There are open circuit terminal parameters r_{21}, R^{∞}_{AB}, R^{∞}_{DC} and their components corresponding to the general case and to some particular cases of the bridge, depending on:

- number of variable arms,
- relations between their increments
- relations between initial arm resistances (in balance).

In papers [9, 10] there are also tables of terminal parameters of the 4R bridge working with arbitrary values of equivalent resistances R_G and R_L of branches connected to it. From these tables it is possible to find terminal parameters of any 4R bridge and any kind of its operation as twoport.

3. Principles of Double Current Bridges Operation

The circuit in Fig 1b has output voltages given by equations:

$$U''_{DC} = J_1 \frac{(R_1R_2 - R_3R_4)}{\sum R_i} - \frac{\Delta J(R_1 + R_4)R_3}{\sum R_i} \tag{10a}$$

$$U_{AB} = J_1 \frac{(R_1R_4 - R_2R_3)}{\sum R_i} - \frac{\Delta J(R_1 + R_2)R_3}{\sum R_i} \tag{10b}$$

where: $\Delta J = J_1 - J_3$

Their two balance conditions (when $U''_{DC} = 0$ or $U_{AB} = 0$) depend on supply currents J_1, J_3. When the supply sources are equal then $J_3 = J_1$, $\Delta J = 0$ and equations (10a,b) are simplified to:

$$U''^{\infty}_{DC} = J \frac{R_1R_2 - R_3R_4}{\sum R_i} \equiv J\, t_{DC} \quad (11a) \qquad U^{\infty}_{AB} = J \frac{R_1R_4 - R_2R_3}{\sum R_i} \equiv J\, t_{AB} \quad (11b)$$

where: t_{DC}, t_{AB} – open-circuit voltage to current sensitivities of DC and AB outputs of the bridge 1b.

From (11a,b) initial balance conditions of both outputs are now obtained as follow:

$$R_{10}R_{20} = R_{30}R_{40} \quad (12a) \qquad \text{or} \qquad R_{10}R_{40} = R_{20}R_{30} \quad (12b)$$

Hence, the 2J bridge of two equal current supply sources is in balance when pairs of the impedance products of the neighboring arms to each of terminals of its output diagonal (CD or AB) are equal.
Formulas of balance conditions (12a,b) together with given before (5a) for the classically supplied bride completed the set of all three possible equality of products of pair impedances of four arm bridge.
If the initial resistances R_{10}, R_{20}, R_{40} have the same values in circuit 1a and 1b and they are balanced by adjusting R_{30}, twice in the 2J bridge 1b - separately for each output, then resistances of balance are as follow:

$$R'_{30} = r_{20}\, r_{40}\, R_{10}; \qquad R''_{30} = \frac{r_{20}}{r_{40}} R_{10}; \qquad R'''_{30} = \frac{r_{40}}{r_{20}} R_{10}; \qquad (13a, b, c)$$

In general case these three resistances are different
The author developed also some circuits allowing using 2J bridge with not equal supply sources. Two of them are given in Fig 1c,d. In this method each one of the output voltages should by measured twice with exchange of current sources. From (10a) and (10b) it is clear that second components in both results have the same absolute value, but opposite signs. Mean value of such two results in each output is proportional to the mean value of the supply currents as follow:

$$\overline{U}''_{DC} = \frac{J_1 + J_3}{2} \frac{(R_1R_2 - R_3R_4)}{\sum R_i} \quad (14a) \qquad \overline{U}_{AB} = \frac{J_1 + J_3}{2} \frac{(R_1R_4 - R_3R_2)}{\sum R_i} \quad (14b)$$

It is possible to use single current J source only, switched between opposite arms as it is shown on the Fig.1d. In this case sum of two measured output voltages is proportional to J.
Equations (11a,b) and (14a,b) of the double current bridges have the form similar to the equation (3) of classic bridge, but in these three formulas arm impedances are taking other places dependending on the type of supply and on the output diagonal.
Every one of 1b-d circuits are in balance for both outputs only when $r_{20} = r_{40}$. All circuits of Fig 1a-d are in balance together when $r_{20} = r_{30} = r_{40} = 1$ and all balance resistances R_{i0} are then the same.
With the notations (4) and next ones used for classic bridge and after transformation of (11a,b) or (13a,b) formulas of their voltage to current sensitivities t_{DC}, t_{AB} of unbalance states are obtained:

$$t_{DC} = R_{10} \frac{r_{10}\, r''_{20}}{\sum r''_{i0}} \bullet \frac{(\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4 + \varepsilon_1\varepsilon_2 - \varepsilon_3\varepsilon_4)}{1 + \frac{\sum r''_{i0}\varepsilon_i}{\sum r''_{i0}}} \equiv t''_0 \bullet f''(\varepsilon_i) \tag{15a}$$

$$t_{AB} = R_{10}\frac{r_{10}\, r'''_{40}}{\sum r'''_{i0}} \bullet \frac{(\varepsilon_1 - \varepsilon_2 - \varepsilon_3 + \varepsilon_4 + \varepsilon_1\varepsilon_4 - \varepsilon_2\varepsilon_3)}{1 + \frac{\sum r'''_{i0}\varepsilon_i}{\sum r'''_{i0}}} \equiv t'''_0 \cdot \bullet f'''(\varepsilon_i) \quad (15b)$$

where: $t''_0 = R_{10}\frac{mn}{(n+m)(1+n)}$, $t'''_0 = R_{10}\frac{mn}{(n+m)(1+m)}$ – initial voltage to current open circuit sensitivities

$f''(\varepsilon_i)$, $f'''(\varepsilon_i)$ – normalized unbalance functions of t_{DC}, t_{AB}

Even with the same values r_{20}, r_{40}, of Wheatstone bridge and of circuits of 2J bridge denominators of initial sensitivities t'_0, t''_0, t'''_0, and of unbalance functions are in general case different, because r_{30} is different according formulas (13a, b, c). Only initially antisymmetric and unloaded 2J bridge (R_{10}=R_{30}, R_{20}=R_{40}) is in balance on both diagonals, and its both initial sensitivities and as well both output open-circuit resistances are equal, i.e. $t''_0 = t'''_0 = \frac{mR_{10}}{2\,(1+m)}$ and R_{AB0}=R_{CD0}=0,5R_{10}(1+m). It is important also to know that the antisymmetric 4R bridge should be unloaded on both outputs to keep conditions (12a,b) valid.

Currents, voltages and powers of the 1b-d circuits' arms depend on values of all resistances R_{i0} and of their relative changes ε_i. In the unbalanced classic 4R bridge supplied by current I_1=I_2 and I_3=I_4 always if output is unloaded. In the unbalanced and unloaded state double current 4R bridge with equal sources J_1=J_3 currents of the opposite arms are always equal: I_1= I_3 and I_2= I_4. From eq. (5), (6a,b) and (11a,b), (15a,b) it is also obvious, that output voltages differently depend from signs of bridge arms' resistance increments. Examples of signs, of these increments changing output in the same direction are given in Fig. 1b. If absolute values of these increments $|\varepsilon_i|$ are equal, the output voltages are strictly proportional to the number of variable arms, e.g. multiplied by 2 or 4. Linearity conditions of unbalance functions of 2J bridges are different for each output. It is briefly discussed in [10].
Basic formulas for the open-circuits resistances on terminals of both outputs of the 2J bridge are the same as (8) and (9), but their values in balance states should be different to satisfying the balance condition (12a) or (12b). Due the form similarity of the equations (11a,b), (15a,b) to (5), (6a,b), the accuracy analysis of the classic 4R bridge, after transformations of formulas was adopted for 2J bridges [10].
To be continued as Part 2

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More than 20 of author's literature positions in Polish language related to topics of this work are given in the bibliography of the above monograph.