Two Parameter (2D) Measurements in Four Terminal (4T) Impedance Bridges as the New Tool for Signal Conditioning (2)

PART 2 Measurements of Two Variables (2D)

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Abstract- *Two-part paper presents the synthesis of some author's works on unified backgrounds of immitance multivariable (nD) measurements of few jointed variables and on the unconventional signal conditioning in four terminal (4T) bridge circuits used for that. In the Part 1 four different circuits for 2D measurements was proposed. Short description of equations and main parameters of the 4R bridge as* ***X*** *type twoport with variable parameters and then as circuit 4T unconventionally supplied by two current sources, named double current bridge (2J) are given and shortly discussed. In the Part 2 application of both types of 4R bridges for 2D measurements is presented and discussed in details.*

PART 2 Measurements of Two Variables (2D)

In several situations the simultaneous information about changes of two or more arm resistances of the 4R bridge circuit is needed for testing, diagnostics or monitoring and it is possible to find them indirectly only by measurements of input and output bridge terminal parameters. Measured internal parameters could additionally depend on two or more variables. As typical examples are 2D unselective 4T sensors, monolithic or integrated ones as a bridge and measurement of the distribution of spatial components even of one quantity only by a couple of the sensors.

4. Two variable (2D) measurements by the 4R bridge as twoport supplied by current source

4.1 Variable two arms of the bridge

As the first representative situation is 2D measurement of increments of only two resistances, e.g. of arm R_1 and R_2 of the 4R bridge no 1 on Fig 1a. It should be done at its terminals as in unloaded twoport, i.e. when $R_L >> R_{CD}$. From formula (5) and (7) it is

$$U'_{DC} = JR_{10}\frac{mn}{(1+m)(1+n)}\,\frac{\varepsilon_1-\varepsilon_2}{1+\dfrac{\varepsilon_1+m\varepsilon_2}{(1+n)(1+m)}} \qquad (16)$$

$$R^{\infty}_{AB} = R_{10}\frac{n(1+m)}{(1+n)}\,\frac{1+\dfrac{\varepsilon_1+m\varepsilon_2}{(1+m)}}{1+\dfrac{\varepsilon_1+m\varepsilon_2}{(1+n)(1+m)}} \qquad (17)$$

From (7b) relative increment of this input bridge resistance is:

$$\varepsilon_{AB} = \frac{n(\varepsilon_1+m\varepsilon_2)}{(1+n)(1+m)+\varepsilon_1+m\varepsilon_2} \qquad (17a)$$

where: $\varepsilon_{AB} \equiv \dfrac{R_{AB}-R_{AB0}}{R_{AB0}}$

And after solving the set of two linear to ε_1, ε_2 equations (16), (17a), such solutions are obtained

$$\varepsilon_1 = \frac{1+n}{n-\varepsilon_{AB}}\left(\varepsilon_{AB}+\frac{U'_{DC}}{JR_{10}}\right) \qquad (18)$$

$$\varepsilon_2 = \frac{1+n}{n-\varepsilon_{AB}}\left(\varepsilon_{AB}-\frac{U'_{DC}}{m\,JR_{10}}\right) \qquad (19)$$

Thus, both increments linearly depend on voltage U_{DC}' and are nonlinear but univocal functions of the bridge input resistance increment ε_{AB}. These two terminal parameters could be measured, for example, in the two-bridge cascade circuit of Fig 1. Both voltage signals should be converted into digital form and processed according to (18) and (19) to obtain values of ε_1 and ε_2. If resistance increments are small enough, i.e. $\varepsilon_{\Sigma R}$ >>1 ($|\Delta R_1+\Delta R_2|<<\sum R_{i0}$) then ε_{AB} <<n and (18), (19) are simplified and become linear to ε_{AB}. Formulas (18), (19) are simpler also if $m=n$, $m=1$ or $n=1$. These both cases could happen together.

Let us now analyse situation when arm resistances of the 4R bridge depend differently on two variables: x_1, x_2. Then terminal parameters of AB and CD are jointed and different also. They could be applied to simultaneous 2D measurements of these variables [7, 9 and 10]. If only two resistances of neighbouring arms $R_1(x_1,x_2)$ and $R_2(x_1,x_2)$ are variable and with very frequent in practice simple relation between their increments as given below

$$\varepsilon_1(x_1, x_2)= \varepsilon'(x_1) + \varepsilon''(x_2)$$
$$\varepsilon_2(x_1, x_2)= \varepsilon'(x_1) - \varepsilon''(x_2) \qquad (20a,b)$$

Then, from (18) and (19) one gets:

$$\varepsilon' = \frac{\varepsilon_1+\varepsilon_2}{2} = \frac{1+n}{n-\varepsilon_{AB}}\left(\varepsilon_{AB}+\frac{m-1}{m}\frac{U_{DC}'}{2JR_{10}}\right) \quad (21) \qquad \varepsilon'' = \frac{\varepsilon_1-\varepsilon_2}{2} = \frac{1+n}{n-\varepsilon_{AB}}\,\frac{1+m}{m}\,\frac{U_{DC}'}{2JR_{10}} \quad (22)$$

Very similar to (18) and (19) are formulas for variable resistances R_3, R_4. For pairs of resistances R_1, R_4 or R_2, R_3, vertically located in bridge 1, output resistance R_{CD}^{∞} should be measured instead of R_{AB}^{∞}, as the formula of ε_{AB} having components of products $\varepsilon_i\varepsilon_j$, makes solution not so simply and of less accuracy. 2D measurement of increments of two opposite bridge resistances is even more complicated.

4.2 2D measurements with four variable 4R bridge arms

It follows from (5) that output voltage U'_{DC} of the unloaded and current supplied 4R bridge in arbitrary case depends nonlinearly on all four increments ε_i of resistances. This function becomes much more complex if the source resistance R_G and load R_L are not considered to be infinite. Bridge open circuit terminal parameters are related practically linearly only on very small values of independent increments ε_i. But it is possible also to obtain the linear unbalance function $f'(\varepsilon_i)$ for large values of ε_i too if they are respectively jointed. In this case increments should take place simultaneously at least in two bridge arms and they are not fully independent. It happens so if increments of the sum of resistances of neighboring arms are equal to each other, i.e. $\varepsilon_{12}=\varepsilon_{34}$, or $\varepsilon_{14}=\varepsilon_{23}$. In these cases from (7b) and (9) or from (8b) and (9) comes: $\varepsilon_{AB}^{\infty}=\varepsilon_{12}=\varepsilon_{\Sigma R}$ or $\varepsilon_{CD}^{\infty}=\varepsilon_{14}=\varepsilon_{\Sigma R}$ respectively, and generalized bridge linearity conditions could be formulated as:

$$\varepsilon_1+m\varepsilon_2=\varepsilon_4+m\varepsilon_3 \quad (23) \qquad \text{or} \qquad \varepsilon_1+n\varepsilon_4=\varepsilon_2+n\varepsilon_3 \quad (24)$$

For absolute increments it is

$$n(\Delta R_1+\Delta R_2)=\Delta R_3+\Delta R_4 \quad (23a) \qquad \text{or} \qquad m(\Delta R_1+\Delta R_4)=\Delta R_2+\Delta R_3 \quad (24a)$$

With every one of the formulas the bridge terminal parameters become simpler. If condition (23) is valid then it is possible to find from (5) that:

$$r_{21}=R_{10}\frac{n}{1+n}(\varepsilon_1-\varepsilon_4) \qquad (25)$$

and from (7a):

$$\varepsilon_{AB}^{\infty}=\frac{\varepsilon_1+m\varepsilon_2}{1+m} \qquad (26)$$

In this case, output resistance R_{CD}^{∞} depends nonlinearly on increments ε_i of arm resistances.

If on the other hand condition (24) is satisfied then it is:

$$r_{21} = R_{10}\frac{m}{1+m}(\varepsilon_1 - \varepsilon_2) \tag{27}$$

and respectively

$$\varepsilon_{DC}^{\infty} = \frac{\varepsilon_1 + n\varepsilon_4}{1+n} \tag{28}$$

Open-circuit input resistance R_{AB}^{∞} is now nonlinearly depended on increments ε_i.

If each one of the linearity conditions (23), (24) is to be separately satisfied, at most three increments ε_i can be independent, and the forth one follows from them according to above formulas. It is difficult to implement such cases in practice. Furthermore, in any pair of terminal parameter formulas (25) - (28) three increments ε_i are present and additional equation is needed to find all of them from the terminal measurements. It is not so easy because the third terminal parameter (R_{CD}^{∞} or R_{AB}^{∞}) depends on ε_i nonlinearly. Much easier is to obtain two pairs of increments related to each other within each pair. It can be done forming respectively the relations to common measured variables. For example the stationary resistance connected in series to the arm resistance (or parallel to arm conductance) proportionally decreases its increment. In force or pressure sensors it is also possible to place two strain gages in points stressed differently but at known ratio. Then for condition (23) it could be:

$$\varepsilon_1 = m\varepsilon_3, \qquad \varepsilon_2 = m\varepsilon_4 \tag{29}$$

or

$$\varepsilon_1 = -m\varepsilon_2, \qquad \varepsilon_4 = -m\varepsilon_3 \tag{30}$$

Of course voltage U'_{DC} is higher when increments of neighboring arms are of opposite signs, i.e. $sign\ \varepsilon_4 = -sign\ \varepsilon_1$ and $sign\,\varepsilon_2 = -sign\,\varepsilon_1$. For the (29) relations even could be $\varepsilon_i > 1$ and there are limitations only from the permissible dissipating power of arm resistances or of maximum voltage of the current supply source. Formula (26) is now changing to:

$$\varepsilon_{AB}^{\infty} = \frac{\varepsilon_1 + \varepsilon_4}{1+m} \tag{31}$$

From (25) and (31) it is possible to find increments ε_1, ε_4 and then also ε_2, ε_3 from (29).
Relations (29) are easy to obtain in practice for the bridge initially symmetrical to the diagonal CD, i.e. m=1, $\varepsilon_1 = \varepsilon_3$ and $\varepsilon_2 = \varepsilon_4$. Sensors of each pair of opposite arms should have only the same relative sensitivity to influencing quantities and their initial values could be different ($n \neq 1$).

As an example of the case (30) is the bridge built by connection of two potentiometers with output on their slides. Extreme resistance increments are limited by both end positions of these potentiometers' slides and they are: $|\varepsilon_i| \leq 1+m$. The input resistance R_{AB}^{∞} of this bridge is constant and the output resistance R_{CD} nonlinearly depends on out of balance increments, but it should be measured instead.

Linear relationships of the bridge transmittance r_{21} and one of its terminal resistance could be used to obtain simultaneously two variables (2D) measured indirectly at bridge terminals, e.g. in circuit 1a. From values of U'_{DC} and ε_{AB}, if (29) or (30) is valid, equations of ε_1, ε_4 are as follow:

$$\begin{aligned} \varepsilon_1 &= 0{,}5(1+m)\,\varepsilon_{AB} + 0{,}5\frac{1+n}{n}\frac{U'_{DC}}{JR_{10}} \equiv w_E\ \varepsilon_{AB} + w_U\ U'_{DC} \\ \varepsilon_4 &= 0{,}5(1+m)\,\varepsilon_{AB} - 0{,}5\frac{1+n}{n}\frac{U'_{DC}}{JR_{10}} \equiv w_E\ \varepsilon_{AB} - w_U\ U'_{DC} \end{aligned} \tag{32 a, b}$$

where coefficients: $w_E \equiv 0{,}5(1+m)$; $\quad w_U \equiv 0{,}5\frac{1+n}{n}\frac{1}{JR_{10}}$.

Indirectly measured two external quantity x_1, x_2 can be found from inverse functions to the ones describing their influence on bridge arm increments.

Linearity conditions could be also useful in the correction of the influence of another variable on the sensor unit in one variable (1D) measurements. As an example is the temperature compensation of the strain gage bridge sensitivity made by changes of its input resistance.

5. 2D measurements by double current bridges

Principles of two types of 2D measurements by the double current bridges 2J of Fig 1b-d will be now discussed in details for comparison with the classic bridge.
The antisymmetric 2J bridge ($m=n$), i.e. initially balanced in both diagonals, and two variable arms R_1, R_2 only are taken into consideration. From basic equations (11a,b), (15a,b) of 2J bridge output voltages it is:

$$U''_{DC} = T_0 \frac{\varepsilon_1 + \varepsilon_2 + \varepsilon_1 \cdot \varepsilon_2}{1 + \frac{\varepsilon_1 + m\varepsilon_2}{2(1+m)}} \quad (33a) \qquad U_{AB} = T_0 \frac{\varepsilon_1 - \varepsilon_2}{1 + \frac{\varepsilon_1 + m\varepsilon_2}{2(1+m)}} \quad (33b)$$

where: $T_0 = J\frac{mR_{10}}{2(1+m)}$ - initial voltage sensitivity equal for both outputs

For $m=1$, $T_0=0{,}25\, JR_{10}$.
If absolute incremental values $|\varepsilon_1|$, $|\varepsilon_2|$, are small enough, i.e. $|\varepsilon_1\varepsilon_2| << |\varepsilon_1+\varepsilon_2|$ and $|\varepsilon_1 + m\varepsilon_2| << 2(1+m)$, or $|\Delta R_1 + \Delta R_2| << 2(R_{10} + R_{20})$, above formulas simplify to:

$$U''_{DC} = T_0(\varepsilon_1 + \varepsilon_2) \quad (34a) \qquad U_{AB} = T_0(\varepsilon_1 - \varepsilon_2) \quad (34b)$$

The first output voltage is proportional to the sum and the second one to the difference of increments.
Both increments can be find directly from basic 2J bridge equations (11a,b), (15a,b) as follow

$$\varepsilon_1 = \frac{m+1}{m}\,\frac{U''_{DC} + U_{AB}}{JR_{10} - U_{AB}} \quad (35a) \qquad \varepsilon_2 = \frac{m+1}{m}\,\frac{U''_{DC} - U_{AB}}{JR_{10} + U_{AB}} \quad (35b)$$

These solutions are univocal. For the bridge of the double initial symmetry ($m=n=1$) first coefficient of (35a,b) is (m+1/m)=2.

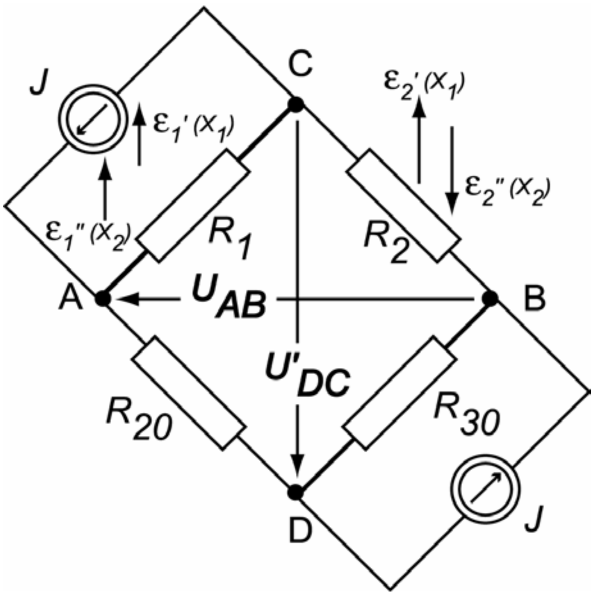

Fig. 2. Principle of two variable x_1, x_2 measurements by the antisymmetric double current bridge of $R_1=R_{30}$, $R_{40}=R_{20}$.

This method of 2D measurement may be useful in the testing and diagnostics of the sensor bridge when it is not possible to disconnect it. If relations between sensors' increments and measured external quantities: x_1, x_2 are known, then after the reverse transformations it is possible to obtain their values. The example of such relations, the same as before for the classic bridge supplied by current is considered in details below.

On the Fig 2 the operation idea of the antisymmetric 2J bridge in 2D measurement is shown. It is, in practice, very frequent particular case of increments, described by equations (20a,b), i.e.:

$\varepsilon_1'(x_1)=\varepsilon_2'(x_1)=\varepsilon'$, and $\varepsilon_1=\varepsilon'+\varepsilon''$,
$\varepsilon_1''(x_2)=-\varepsilon_2''(x_2)=\varepsilon''$ $\varepsilon_2=\varepsilon'-\varepsilon''$.

Two output signals are as follow:

$$U''_{DC} = JR_{10}\frac{m}{(1+m)}\frac{\varepsilon' + \dfrac{(\varepsilon')^2-(\varepsilon'')^2}{2}}{1+\dfrac{\varepsilon'}{2}+\dfrac{\varepsilon'(1-m)}{2(1+m)}} \quad (36a) \qquad U_{AB}= JR_{10}\frac{m}{(1+m)}\frac{\varepsilon''}{1+\dfrac{\varepsilon'}{2}+\dfrac{\varepsilon'(1-m)}{2(1+m)}} \quad (36b)$$

Initial sensitivities and denominators of the above formulas are equal. If ε' and ε'' are not too high, the difference of their squares in the last nominator of the equation (33) is negligible. If additionally both sensors are linear for both measured quantities, i.e. $\varepsilon'_1(x_1) = k_1x_1$; $\varepsilon''(x_2) = k_2x_2$, then from (11a,b) and (15a,b) these signals could be written in slightly different forms.

$$U''_{DC}=\frac{J R_{10}R_{20}}{\sum R_{i0}+\sum \Delta R_i}2k_1x_1 \quad (37a) \qquad U_{AB} = \frac{J R_{10} R_{20}}{\sum R_{i0} + \sum \Delta R_i}2k_2x_2 \quad (37b)$$

Sensitivities for the above two signals are equal and could only slightly depend on x_1 or x_2, if $\sum \Delta R_i \neq 0$. When $\varepsilon_1<<1$, $\varepsilon_2<<1$, these formulas simplify more up to:

$$U''_{DC}=JR_{10}\frac{m}{1+m}\varepsilon'(x_1) \quad (38a) \qquad U_{AB}=J R_{10}\frac{m}{1+m}\varepsilon''(x_2) \quad (38b)$$

Each of the above voltages depends linearly only on one component of the resistance increments, which depends only on one of the influencing variables. For higher values of ε', ε'' output voltages become non-linear functions, but always both sensitivities are proportional to each other. Classic bridges haven't such futures as ones given above.
In such simple method as the above one, it is possible to realise simultaneous measurements of two variables: x_1, x_2 by one or by two pairs of sensors. For higher resistance increments: $0<|\varepsilon_i|\leq 1$ it is also possible to obtain separately and nearly linear relations of output voltages of the double bridge, if both components $\varepsilon_i'(x_1)$ and $\varepsilon_i''(x_2)$ separately fulfil linearity conditions of the unbalance functions [9, 10].

It is also possible to obtain nearly similar pair of relations from the output voltage of the classical supplied bridge and of only one of outputs of circuit1b-d, but in this case supply sources have to be switched at least three times. With Anderson current loop [1] it is also possible to obtain two parameters' measurements, but only for all similar initial resistances of the two-terminal sensors connected in series. The circuit is also much more complicated, because for every pair of sensor it includes three special double input differential units with few operational amplifiers each and needs seven connecting lines.

6. Summary and conclusions

- Four different 2D-measurement circuits containing four-terminal (4T) four-arm resistance (4R) bridge of variable parameters, are presented and discussed in details. They all are supplied by current sources, classically to diagonal or unconventionally, and provide two output voltage signals differently dependent on increments from values of bridge arms resistances in balance.
- From measured values of both output signals it is possible to obtain two arm increments or two their relations and furthermore two external quantities if they differently modulate bridge arm resistances and if the set of describing functions is known and if solutions are univocal or physically acceptable.
- In the first circuit, build as cascade of two 4R brides (see Fig 1a) measured are two open-circuit terminal parameters of bridge 1, i.e. its output voltage for current supply and increment of input resistance by bridge 2. Both bridges are classically working as twoports. Limitations for measurements of arm increments are also discussed, e.g. opposite increments of neighbouring arms couldn't be measured.

- As the next ones are presented three variants of double current (2J) bridges. They have voltage outputs on both diagonals and are unconventionally supplied - in parallel to opposite bridge arms. Bridge on Fig 1b has two equal current supply sources, bridge 1c - two unequal ones but mutually replaced, and bridge 1d - only single source also switched between opposite arms. In last two circuits each pair of results obtained with switching are averaged.
- 2J bridges can be successfully used as new type of measuring circuits especially for 2D signal conditioning of immitance sensors. Their balance conditions – equivalence of products of pair resistances neighbouring to each output, are different for both bridge diagonals, and different then those of classic bridge.
- 2J bridge is in balance on both diagonals when it is antisymmetric i.e. ($R_{10}=R_{30}$, $R_{20}=R_{40}$).
- Equations describing the 2J bridge output voltages have the same form as current supplied classic one, but immitances of arms take another places in equations according to different balance conditions. Opposite increments of neighbouring arms could be measured.
- Initial sensitivity depends on relative resistances differently than in the classic current bridge.
- In some applications double current bridges could be an alternative to existing circuits, in others – they give new possibilities. The main application is in measurements of the immitance increments of the bridge and of other topology 4 terminal circuits, which can not be disconnected.
- It is also possible to measure indirectly two variables by a pair of non-selective sensors and measure separately increments of any two of the bridge arm resistances.
- Changing one of the bridge supply current it is possible to create new double current 2D measurement circuits, e.g. with resistance to frequency converters, with feedback, with multipliers, Hall sensors etc.
- Very large and perspective field of applications of double current bridges is in the domain of AC circuits. For every conventional AC bridge it is possible to create 2 (or even 4) such double bridges. Balance could be achieved in 4 arms bridges, which are structurally not balanced for conventional supply because phases' sums of the pairs of opposite arms are always different, e.g. in the simple RC bridge as an AC phase shifter [10].
- It will be also possible to create new AC measurement circuits of few variables using a number of various immitance sensors.

The author would like to present additionally at the Symposium some of new applications and it is also possible to find them in monograph [10]. Accuracy analyses of all types of DC 4T bridges are also there.

Presented in this work original idea of supplied bridges by double current sources opens wide new possibilities to signal conditioning of multivariable measurements not only by DC and AC 4T circuits but also by many other too.

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    More than 20 of author's literature positions in Polish language related to topics of this work are given in the bibliography of the above monograph.