

Determination of an Impedance Operator of Time-Varying Dynamic Two Terminals on the Basis of the Terminal Signals Measurements

Radosław Kłosiński¹

¹ *University of Zielona Góra, Electrical Metrology Institute 65-246 Zielona Góra, ul. Podgórna 50, Poland, phone +48 68 3282233, +48 68 3282222, Fax +48 68 3254615, r.klosinski@ime.uz.zgora.pl*

Abstract – This paper presents the derivation and effects of a genuine algorithm for identification of circular parametric operators describing performance of periodically time-varying systems, of one input – one output type. The periodic steady state is taken into the consideration, while the period of the change of the system parameters overlaps the period of the changes of input and output signals. Within the domain of discrete time, such operators take the form of a real element matrix. A circular parametric operator may be transformed into the frequency domain. In this version, the quantitative assessment of mixing and generating of harmonics of input and output signals is possible. The original identification algorithm is used for determination of time-varying two-terminal impedance. Some results of computer simulations are presented.

I. Introduction

The notion of complex impedance means a parameter of a two-terminal supplied with voltage or sinusoidal current, and can be applied to a steady state of systems – linear time invariant (LTI). The situation is different in case of deforming two-terminal networks, i.e., non-linear or linear time-varying (LTV). Such systems deform signals, that is, they are capable of mixing and generating harmonics. The mathematical description of such phenomena by means of one complex number is impossible. Linear two-terminals of periodically varying parameters, synchronically with power signal, constitute an important group of elements (e.g.: switching converters or thyristor droved loads). Such systems can reach a periodical steady state, which allows to analyze them in the frequency domain.

The relation between terminal signals of the periodically time-varying two-terminal network at a steady state is described with a circular parametric operator. Within the domain of discrete time, such operator takes the form of a real element matrix [3]. An electric two-terminal is treated as one input – one output type system. The method for determination of an impedance operator presented in this paper is based on sets of periodical voltage and current signal samples measured on the two-terminal clamps. A uniform sampling of signals is assumed, that is, the period of signal and parameter changes is an integral multiplicity of the sampling period. The sampling frequency is high enough to avoid the phenomenon of aliasing. The genuine identification algorithm is used for determination of time-varying two-terminal impedance circular parametric operator.

A circular parametric operator may be transformed into the frequency domain. In this version, the quantitative assessment of phenomenon of mixing and generating harmonics of input and output signals is possible.

The paper does not take in account the problems of measurement errors and interference occurrence.

II. The circular parametric operator

Within the domain of discrete time, the relation of the input signal $x(n)$ to the output signal $y(n)$ for a time-varying system may be described with a difference equation of variable coefficients [3]:

$$\sum_{i=0}^q A_i(n) y(n-i) = \sum_{i=0}^q B_i(n) x(n-i) \quad (1)$$

The equation (1) may be solved with the operator set in the form of the following sum:

$$y(n) = Hx(n) = \sum_{m=-\infty}^{\infty} h(n, m) x(m) \quad (2)$$

The operator's kernel $h(n, m)$ is the pulse response to the Kronecker's pulse applied at the moment m . In the case of a time-varying system, it is a function of two variables, i.e. it depends on the current time n , and on the moment the pulse which has been applied m .

For the N -periodical input signal $x(n+N) = x(n)$ the response may be determined using the formula:

$$y(n) = \tilde{H}x(n) = \sum_{m=0}^{N-1} \tilde{h}(n,m)x(m) \quad (3)$$

where:

$$\tilde{h}(n,m) \equiv \sum_{p=-\infty}^{\infty} h(n,m-pN) \quad (4)$$

For a system of N -periodically variable parameters the pulse response fulfills the property [3,4]:

$$h(n+N,m+N) = h(n,m) \quad (5)$$

and:

$$\tilde{h}(n+N,m) = \tilde{h}(n,m) \quad (6)$$

This means an N -periodic response $y(n)$ to N -periodic coercion $x(n)$:

$$y(n+N) = \sum_{m=0}^{N-1} \tilde{h}(n+N,m)x(m) = \sum_{m=0}^{N-1} \tilde{h}(n,m)x(m) = y(n) \quad (7)$$

N -periodic response of the system of N -periodically variable parameters to N -periodic coercion may be determined with the use of the so-called circular parametric operator given in the form of the following matrix [3,4]:

$$\begin{bmatrix} y(0) \\ y(1) \\ \dots \\ y(N-1) \end{bmatrix} = \begin{bmatrix} \tilde{h}(0,0) & \tilde{h}(0,1) & \dots & \tilde{h}(0,N-1) \\ \tilde{h}(1,0) & \tilde{h}(1,1) & \dots & \tilde{h}(1,N-1) \\ \dots & \dots & \dots & \dots \\ \tilde{h}(N-1,0) & \tilde{h}(N-1,1) & \dots & \tilde{h}(N-1,N-1) \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ \dots \\ x(N-1) \end{bmatrix} \quad (8)$$

or, shorter:

$$\mathbf{y} = \tilde{\mathbf{H}}\mathbf{x} \quad (9)$$

where: \mathbf{x} , \mathbf{y} - vectors of samples of one period of coercion and response signals,

$\tilde{\mathbf{H}}$ - the circular parametric operator (circular parametric matrix).

For time independent systems, the pulse response is one variable function:

$$h(n,m) = h(n-m) \quad (10)$$

Then, the circular parametric matrix introduced equalization (8) passes to form of cyclical matrix:

$$\begin{bmatrix} y(0) \\ y(1) \\ \dots \\ y(N-1) \end{bmatrix} = \begin{bmatrix} \tilde{h}(0) & \tilde{h}(N-1) & \dots & \tilde{h}(1) \\ \tilde{h}(1) & \tilde{h}(0) & \dots & \tilde{h}(2) \\ \dots & \dots & \dots & \dots \\ \tilde{h}(N-1) & \tilde{h}(N-2) & \dots & \tilde{h}(0) \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ \dots \\ x(N-1) \end{bmatrix} \quad (11)$$

Where cyclical impulse response:

$$\tilde{h}(n) = \sum_{p=-\infty}^{\infty} h(n+pN) \quad (12)$$

Samples of N -periodic signal can be expressed by means of Fourier row of complex harmonic amplitudes [3,5]:

$$x(n) = \frac{1}{2} \sum_{m=0}^{N-1} X_m w^{nm}, \quad n = 0, 1, 2, \dots, N-1, \quad (13)$$

where: X_m - for $0 < m \leq \frac{N}{2}$ - the m^{th} harmonic complex amplitude,

- for $\frac{N}{2} < m \leq N-1$, $X_m = X_{N-m}^*$ - the $(N-m)^{\text{th}}$ harmonic adjoint complex amplitude,

- for $m = 0$ - double value of constant composition,

$$w = e^{j \frac{2\pi}{N}}$$

N - number of samples in period of signal.

Values of complex amplitudes may be calculated on the ground of samples from the dependence [3,5]:

$$X_m = \frac{2}{N} \sum_{n=0}^{N-1} x(n) w^{-nm} \quad (14)$$

The expressions (13) and (14) may be described in the matrix form:

$$\mathbf{x} = \frac{1}{2} \mathbf{F} \mathbf{X} \quad (15)$$

$$\mathbf{X} = 2 \mathbf{F}^{-1} \mathbf{x} \quad (16)$$

where: \mathbf{x} – vector of samples of one period of signal,

\mathbf{X} – vector of complex amplitudes and adjoint complex amplitudes,

\mathbf{F} – Fourier matrix [3,4], $[\mathbf{F}]_{nm} = w^{nm}$.

To find spectral equivalent of circular parametric matrix, one should multiply both sides of equation (9) by $2\mathbf{F}^{-1}$ and substitute the equation (15). Taking into account the equalization (16) with reference to signal y , it yields:

$$\mathbf{Y} = \mathbf{F}^{-1} \tilde{\mathbf{H}} \mathbf{F} \mathbf{X}, \quad (17)$$

Then, the spectral circular parametric operator [3,4]:

$$\tilde{\mathbf{H}}(\omega_y, \omega_x) = \mathbf{F}^{-1} \tilde{\mathbf{H}} \mathbf{F}, \quad (18)$$

where: ω_x, ω_y - harmonics pulsations of coercion and response signals of system.

The spectral circular parametric operator creates the possibility for the quantitative assessment of mixing and generating of harmonics of input and output signals. Coefficients of spectral matrix lying outside the main diagonal qualify manner of transformation coercion harmonics into other harmonics of response of system. For linear time invariant system (LTI) a spectral circular parametric operator takes a form of diagonal matrix.

Graphs of example of an impedance circular parametric operator $\tilde{\mathbf{Z}}(n, m)$ and of module of a spectral circular parametric operator $|\tilde{\mathbf{Z}}(\omega_U, \omega_I)|$ are presented in figure 3.a) and 3.f).

III. Identification algorithm

The identification of a circular parametric operator consists in the determination of coefficients of matrix $\tilde{\mathbf{H}}$ on the basis of a K set of coercive signals \mathbf{X} and the K signals of the response of system \mathbf{Y} . The following relation implied by equation (9) is used here:

$$\tilde{\mathbf{H}} \mathbf{X} = \mathbf{Y} \quad (19)$$

where: $\tilde{\mathbf{H}}$ - matrix $N \times N$ - the sought circular parametric operator;

\mathbf{X} - matrix $N \times K$ of the K input signals, each column is a vector of one input signal samples;

\mathbf{Y} - matrix $N \times K$ of the system response signals.

In the case where $K = N$, i.e. the number of coercion and response signals of the system K equals the size N of the square circular parametric matrix, the problem has an unequivocal solution:

$$\tilde{\mathbf{H}} = \mathbf{Y} \mathbf{X}^{-1} \quad (20)$$

The solution depends on:

$$\det \mathbf{X} \neq 0 \quad (21)$$

which means the linear independence of coercive signals. It is not simple to generate N linear independent N -periodic coercive signals. Usually the number of such signals is lower.

In the case where $K < N$, the matrix equation (19) has an infinite number of solutions. One solution, the most optimal one, should be chosen from among the possible solutions. One can propose to seek an operator which describes a system of the smoothest possible changes of parameters. With consideration of the relation (19), the optimization problem may be originally defined as follows:

$$(\Delta \mathbf{h}_n)^T \Delta \mathbf{h}_n \rightarrow \min \quad (22)$$

$$\mathbf{X}^T \mathbf{h}_n = \mathbf{y}_n \quad (23)$$

where:

\mathbf{h}_n - vector including elements of the n^{th} line of matrix $\tilde{\mathbf{H}}$

\mathbf{y}_n - vector of n^{th} samples of all K response signals of the system (elements of the n^{th} line of matrix \mathbf{Y});

$\Delta \mathbf{h}_n$ - vector of increments of matrix $\tilde{\mathbf{H}}$ elements for the n^{th} line of the matrix, defined as follows:

$$\Delta h_{n,m} = h_{n,m} - h_{n(-),m(-)} \quad (24)$$

(-) - subtraction mark of modulo N ;

The optimization criterion (22) means the minimization of increases of coefficients of matrix on direction of main diagonal. Such criterion choice originates from the fact, that in case of linear time invariant system a relationship between signals of coercion and answer, in periodic steady state, are described by means of cyclical matrix. Then, increases of elements defined by means of (24) equal zero. Choice criterion (22) means the research of circular parametric operator describing the system of least variability of parameters, realizing equalization (19). In the situation, that variability of parameters is not indispensable, the result of optimization (22) (23) will become a cyclical matrix describing time-invariant system. Equation (23) results from (19).

The optimizing problem (22), (23) may be solved using the Lagrange method in the manner given by the author of [3,4]. The method is described in [1,2,4]. The result is the below shown iterative solution:

$$\mathbf{h}_n = \left(\mathbf{1} - \mathbf{X}(\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \right) \mathbf{P} \mathbf{h}_{n-1} + \mathbf{X}(\mathbf{X}^T \mathbf{X})^{-1} \mathbf{y}_n \quad (25)$$

where: $\mathbf{1} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{bmatrix}$ - an unit matrix, $\mathbf{P} = \begin{bmatrix} 0 & \dots & 0 & 1 \\ 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & 1 & 0 \end{bmatrix}$ - a circular delay matrix.

IV. Numerical experiments

The identification algorithm has been accomplished in a computer program. The following experiments have been conducted in order to examine the correctness of performance and properties of the algorithm. A sample circular parametric matrix was generated, which was an impedance operator of a parallel connection of a capacitor and a real induction coil (Fig. 1.), where parameters $r(t)$, $l(t)$, $c(t)$ might periodically change in time. Then, various periodical current signals were affected by this operator's performance, which brought the voltage signals referring to them. The groups of current signals (coercion) and voltage signals (network response) were the data for the identification procedure.

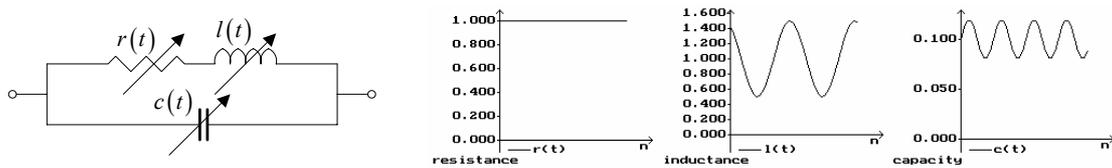


Figure 1. A scheme of a branch described with a impedance circular parametric operator and graphs of functions of parameters changes.

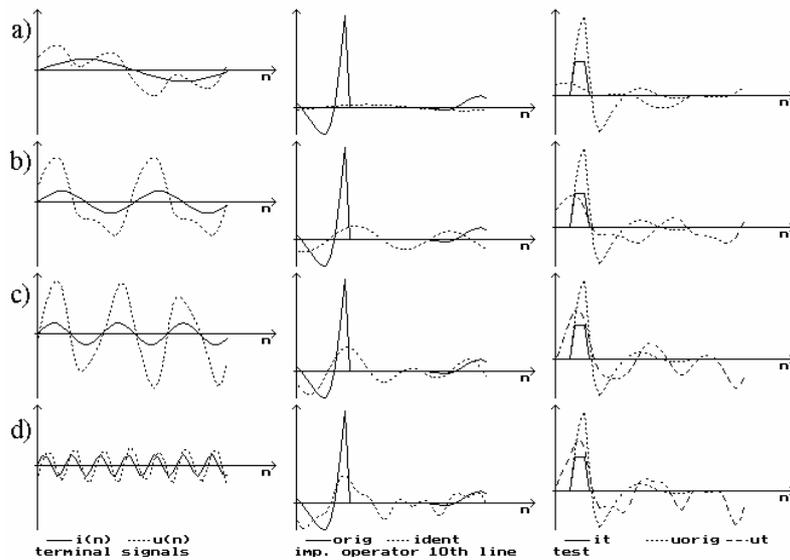


Figure 2. A time-varying two-terminal impedance operator identification. A set of current and voltage signals is filled up by the next harmonic current and its response voltage signal. For better comparison, line of matrix is display like a continuous signal (orig – original operator line, ident – obtained from identification).

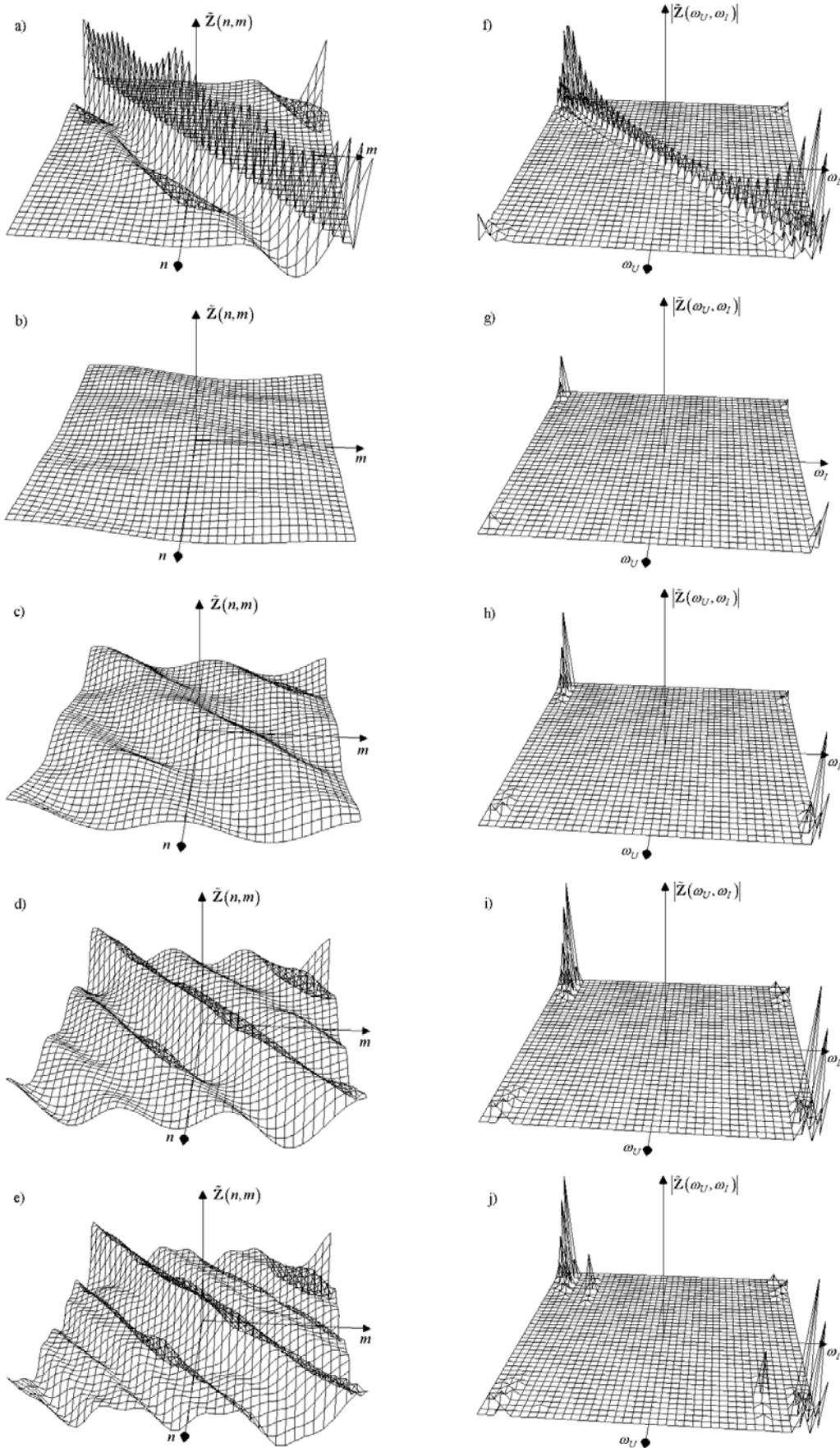


Figure 3. The circular parametric operators and spectral operators obtained during the identification.

Figures 2. and 3. present the results of identification an impedance circular parametric operator of a time-varying two-terminal. A set of current and voltage signals is filled up by the next harmonic current $i(n)$ and its response voltage signal $u(n)$. For a better comparison, the line of matrix is displayed as a continuous signal (orig – original operator line, ident – line of operator obtained from identification). Test results: it – test coercion current signal, uorig and ut – output voltage signal received by the use of the original operator and of the operator obtained from identification. The impedance circular operators and spectral circular operators obtained during the identification are presented in figure 3. It is clearly visible how the addition of another harmonic to the set of forcing signals (Fig.2. a), b), c), d)) causes the appearance of non-zero coefficients in the columns of the spectrum operator corresponding to these harmonics (Fig.3. g), h), i), j)).

IV. Conclusions

The relation between terminal signals of the periodically time-varying two-terminal network at a steady state is described with a circular parametric operator. Within the domain of discrete time, such operator takes the form of a real element matrix. The genuine identification algorithm may be used for determination of time-varying two-terminal impedance circular parametric operator. Such identification algorithm works properly. However, using it may bring the expected results only if the identification data i.e. input and response signals include the necessary portion of information about the performance of the network being identified.

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