

## New Ratio Transformer Bridges for Intercomparing Standards of RLC Bridges

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**Abstract** - Auto balance RLC bridges are the most common instruments for impedance measurements on low frequencies. Both national metrology laboratories (NMLs) and secondary laboratories (SLs) maintain the scale of ac resistance, inductance and capacitance with sets of artifact standards. In NMLs, the traceability chain for the impedance scale involves a set of coaxial ac ratio bridges, the primary standard being the calculable capacitor. The illustrated new ratio transformer bridges may operate using as reference a capacitor standard covering the whole required frequency range. Experimental bridges have been used in the intercomparison of self- mutual- inductors, resistance and capacitor. The achievable intercomparison uncertainty may not exceed a few  $10^{-7}$ .

### I. INTRODUCTION

Commercial RLC automatic bridges can be used in a very efficient way for 1:1 calibration with good accuracy. The accuracy of these bridges is inadequate for top-level calibrations, but the sensitivity [1] and the short-term stability are quite good; therefore, calibration by direct substitution become feasible. In the following, we will call  $Z_s$  the standard impedance, and  $Z_x$  the impedance to be calibrated. The RLC bridge is first connected to  $Z_s$  giving the reading  $Z_s^{RLC}$  (a complex number), and immediately after to  $Z_x$  giving the reading  $Z_x^{RLC}$ . Calling  $Z_s$  the traceable value of  $Z_s$  and  $Z_x$  the value of  $Z_x$  being evaluated, we have,

$$Z_x = Z_x^{RLC} - Z_s^{RLC} + Z_s \quad (1)$$

The type B uncertainty in such  $Z_x$  calibration is essentially dependent on the uncertainty of  $Z_s$ , on the linearity of the RLC bridge near  $Z_s$  ( $\approx Z_x$ ) and on the stability of the RLC bridge; its absolute accuracy does not enter the uncertainty budget. The effect of the short-term instability of the RLC bridge can be overcome by repeating the  $Z_s$ - $Z_x$  measurement cycle several times and by reducing the dead time.

Autobalance RLC bridges [2] are the most common instruments for impedance measurements on low frequencies. The main attribute of autobalance bridges is connection of the measured impedance between the source of measured signal and virtual ground of electronic measuring instrument. Both national metrology laboratories (NMLs) and secondary laboratories (SLs) [3] maintain the scale of ac resistance, inductance and capacitance with sets of artifact standards. In NMLs, the traceability chain for the impedance scale involves a set of coaxial ac ratio bridges, the primary standard being the calculable capacitor. In SLs, the full set of standards is periodically calibrated by NML. Hence, NML and SL impedance calibration process involves direct comparison of impedance standards with equal nominal values (1:1 calibration).

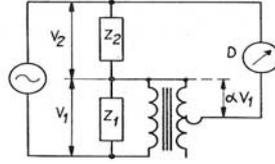


Fig. 1. Schematic diagram of PCRTB.

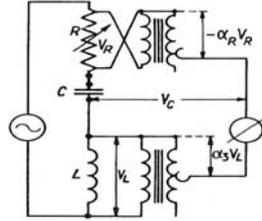


Fig. 2 – Schematic diagram of PCRTB for comparing self-inductors and capacitors.

For resistor standards for calibration of RLC meters [4] there exist other requirements than for classical resistors with calculable frequency characteristic. The most important is as wide as possible frequency range. The requirements for accuracy and stability are much lower. For calibration of RLC meters, which are commercially available, it is enough to have accuracy in the order of tens to 100 ppm. For such uses the full range of values is from 10  $\Omega$  to 100 k $\Omega$ . The illustrated bridges may operate using as reference a capacitor standard covering the whole required frequency range.

The primary standard being the calculable capacitor, the at present, available impedance calibration process appears lacking on an intercomparison method of standards of unequal nominal values with the capacitance standard. The illustrated new ratio transformer bridges respond to this need. Two structures of the new bridges denoted the, "parallel connected ratio transformer bridge (PCRTB)" and "voltage in quadrature bridge" (VQB) are illustrated.

## II. Basic principle of PCRTB

The schematic diagram of PCRTB is illustrated in Fig. 1 –  $Z_1$  and  $Z_2$  are the compared impedances -,  $V_1$   $V_2$ , and  $\alpha V_1$  - respectively are, the voltage drops at  $Z_1$  and  $Z_2$  and the output voltage of the decades ratio transformer, and  $l$  is the input self-inductance of the decades ratio transformer. At balance,

$$\frac{Z_2}{Z_1} = \alpha \frac{1}{\left(1 + \frac{Z_1}{\omega l}\right)} \quad (2).$$

## III. Applications of PCRTB

This section is devoted to illustrate the use of PCRTB for comparing, self-inductors and capacitors and mutual inductors.

### A. Bridge for Comparing Self-Inductors and Capacitors

The schematic diagram of PCRTB for comparing self-inductors and capacitors is illustrated in Fig. 2. -  $L$ ,  $C$ , and  $R$  - are the, compared self-inductor and capacitor and a decades resistor.  $L$ ,  $C$ , and  $R$  are series connected. A decades ratio transformer is parallel connected with  $L$ . Its output voltage  $\alpha_3 V_L$  is compared with the voltage drop  $V_c$ , at  $C$ , series connected with the output  $-\alpha_R V_R$  of the decades ratio transformer in parallel to  $R$ . This ratio transformer needs to change to sign of  $V_R$ . If  $L$  and  $C$  are pure components at balance is,

$$L = \frac{1}{\alpha_3 \omega^2 C} \quad (3).$$

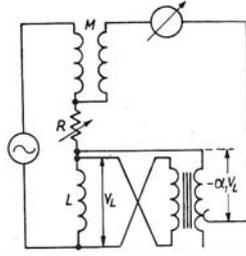


Fig. 3. The schematic diagram of PCRTB for comparing self and mutual inductors.

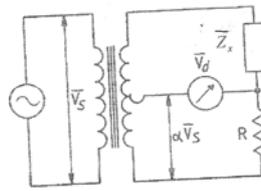


Fig. 4. Schematic diagram of VQB.

If L and C have equivalent series resistance respectively  $R_L$   $R_c$  at balance is,

$$\alpha_3 = \frac{1}{\omega^2 LC} \text{ and } \alpha_3 R_L = \alpha_1 R - R_c \quad (4)$$

### B. Bridge for Comparing Self and Mutual Inductors

The schematic diagram of PCRTB for comparing self and mutual inductors is illustrated in Fig. 3 - M, L, and R - are the, compared self- and mutual- inductor and a decades resistor. L, the primary of M, and R are series connected. A decades ratio transformer is parallel connected with L. Its output voltage  $\alpha_1 V_L$  is compared with the secondary e.m.f. of M series connected with the voltage drop at R. With  $R_L$ , L, and M, the equivalent series resistance and inductance of self-inductor, and mutual inductance of the mutual inductor, at balance is,

$$M = \alpha_1 L \text{ and } R_L = R / \alpha_1 \quad (5)$$

### C. Bridge for Comparing Mutual Inductors and Capacitors

In Fig. 3 L is replaced by the capacitor having capacitance C and equivalent series resistance  $R_c$  with  $\alpha_2$  the ratio of the deca-des ratio transformer, at balance is,

$$M = \frac{\alpha_2}{\omega^2 C} \text{ and } R_c = \frac{R}{\alpha_2} \quad (6)$$

## IV. Basic Principle of VQB

VQB applies for comparing a resistor R with a capacitor or an inductor. The schematic diagram of VQB is illustrated in Fig. 4. -  $\bar{Z}_x$  and R are the compared components - (self-inductor or capacitor and resistor). The VDB balance occurs when the voltage at the null detector diagonal,  $V_d$ , is in quadrature with the

supplying voltage  $V_s$ . With,  $\overline{Z}_x = R_x + j\overline{X}$ ,  $\overline{V}_d = \alpha\overline{V}_s - \frac{R\overline{V}_s}{R + \overline{Z}_x}$ . The quadrature is evaluated by zeroing the real part of  $\overline{V}_d$ . It results,

$$R(1 + \frac{R_x}{R}) = X\sqrt{\frac{\alpha}{(1-\alpha)}} \quad (7).$$

*Practical Details-* To detect the quadrature between  $\overline{V}_d$  and  $\overline{V}_s$  a proper method has been devised. Let us assume  $\overline{V}_1 = \overline{V}_d + \overline{V}_s$  and,  $\overline{V}_2 = \overline{V}_d - \overline{V}_s$ . When  $\overline{V}_d$  and  $\overline{V}_s$  are in quadrature  $\overline{V}_1$  and  $\overline{V}_2$  have the same rms value and then  $|\overline{V}_1| - |\overline{V}_2| = 0$ . If  $\varepsilon$  is the phase defect from quadrature,

With  $\varepsilon \ll 1$ , the difference  $|\overline{V}_1| - |\overline{V}_2| \cong 2\varepsilon V_d$  and  $\varepsilon = \frac{|\overline{V}_1| - |\overline{V}_2|}{2V_d}$ . The error in the voltage ratio  $\alpha = \frac{\Delta V_2}{V_s}$  is,

$$\Delta\alpha = \frac{\Delta V_2}{V_s} = \varepsilon \frac{V_d}{V_s} = \frac{|\overline{V}_1| - |\overline{V}_2|}{2V_d} \quad (8).$$

$\overline{V}_1$  and  $\overline{V}_2$  may be obtained by parallel connecting to  $\overline{V}_s$  the primary coil of a three winding transformer, with twinned wires. The two secondary voltages nearly equals in magnitude and phase. One terminal of each of the two secondary is connected (with opposite sign) to the decades ratio transformer's variable tap.  $\overline{V}_1$  and  $\overline{V}_2$  are available between the common tap of R and  $\overline{Z}_x$  and the free terminals of the secondary windings. The difference  $|\overline{V}_1| - |\overline{V}_2|$  may be detected within  $2.5 \cdot 10^{-7}$  and  $V_s$  ranges from 21 V at 60 Hz to 350 V at frequencies  $\geq 1$  kHz. With  $V=10$  V, the error  $\Delta\alpha$  in the detected  $\alpha$  is within  $1.25 \cdot 10^{-8}$ .

- *Estimation of uncertainties of PCRTB or VQB* – Type B uncertainties are due to the ratio error with which is known  $\alpha$  and has been evaluated by the illustrated either PCRTB or VQB bridges the ratio  $\frac{R_x}{R}$  and  $\Delta\alpha$  in evaluating quadrature. Current calibrations may assign results within few  $10^{-7}$ .

## V. Applications of VQB

This section is devoted to illustrate the use of VQB for comparing, resistors and capacitors and self-inductors.

### A. Bridge for Comparing Resistors and Capacitors

$\overline{Z}_x$  and R are the compared capacitor and resistor. With,  $\overline{Z}_x = R_x - \frac{j}{\omega C}$ , (7) is still valid assuming  $X = \frac{1}{\omega C}$ . If a gas-dielectric is used as capacitance transfer standard,  $R_x$  is nearly zeroed. Quadrature bridges for comparing capacitors and resistors have been earlier illustrated [6, 7]. Nevertheless if compared with VQB they appear highly intricate, Requiring e.g. auxiliary bridges, quadrature voltage and adjustable current sources, passive filter, and injection voltage, current equalizer, and iterative procedures.

### B. Bridge for Comparing Resistors and Self-inductors

$\overline{Z}_x$  and R are the compared inductor and resistor. With,  $\overline{Z}_x = R_x - j\omega L$ , (7) is still valid assuming  $X = \omega L$ . With self-inductors  $R_x$  cannot be disregarded. Nevertheless the ratio  $\frac{R_x}{R}$  may be evaluated within any required accuracy.

## VI. Accuracy and Test Results

In order to investigate and discuss the illustrated bridges' accuracy and performances, it must be outlined that this may concern their assigned use, namely, for intercomparing standards of RLC bridges. PCRTB bridges have performed an intercomparison of self- mutual- inductor and capacitor. A mutual inductor  $M \cong 17 \text{ } \mu\text{H}$  has been compared both with a self-inductor,  $L \cong 0.1 \text{ mH}$  and a capacitor  $C \cong 10 \text{ } \mu\text{F}$  and than the self-inductor  $L$  has been compared with the capacitor  $C$ . By (3), (5), and (6),  $\alpha_1 = \alpha_2 \alpha_3$ . Tests have been performed at 5 kHz known within  $2.5 \cdot 10^{-7}$ . The decades ratio transformers had: input self-inductance  $l \cong 500 \text{ H}$ , seven decades, and linearity 0.5 ppm. Ratio decades were calibrated within  $\pm 0.2 \text{ ppm}$ ,  $\pm 0.1 \text{ ppm}$ , and  $\pm 0.01 \text{ ppm}$  of input for settings respectively of, 0.1 to 1.0, 0.01 to 0.1 and below 0.001. Obtained test results,  $\alpha_1 = 0.1700000$ ,  $\alpha_2 = 0.1677993$ , and  $\alpha_3 = 1.013114$ .  $\alpha_3$  evaluated by intercomparison tests is,

$$\alpha_3 = \frac{\alpha_1}{\alpha_2} = 1.013115, \text{ whose difference with respect to the direct test result is } < 1 \text{ ppm, corresponding to}$$

one step of the seventh decade of the used ratio transformer.

An intercomparison by VQB bridges has been performed assuming as reference standard a resistor. The quadrature between  $V_d$  and  $V_s$  voltages has been detected using as auxiliary three winding transformer (see practical details in Section 4) a three wires winding of 1000 wires on a super-mumetal core, 162 mm length,  $24 \text{ mm}^2$  surface. The intercomparison uncertainty has been assigned by the corresponding  $\Delta\alpha \cong 10^{-7}$  in evaluating quadrature. These test results prove the effectiveness of the illustrated new PCRTB and VQB bridges mainly for intercomparing the standards of RLC bridges or of test laboratories. The achievable intercomparison uncertainty may not exceed a few  $10^{-7}$ .

## VII. Conclusions

Autobalance RLC bridges [2] are the most common instruments for impedance measurements on low frequencies. In national metrology laboratories the traceability chain for the impedance scale involves the use of the calculable capacitor as primary standard. The illustrated new ratio transformer bridges may calibrate the standards of the autobalance RLC bridges using as reference a capacitor standard covering the whole required frequency range. Experimental bridges have been used in the intercomparison of self-mutual- inductors, resistance and capacitor. The achievable intercomparison uncertainty may not exceed a few  $10^{-7}$  proving the usefulness of their use for intercomparing standards of RLC bridges.

## References

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## Biographies

Franco Castelli was born in Milan, Italy. He received the M.S. degree in electrical engineering from the "Politecnico di Milano", Milano, in 1958. He worked one year at C.G.E. (Compagnia Generale di Eletticit ), Milan, in the servomechanism field. In 1961 he joined the Polytechnic of Milan as Assistant Professor of Electrical Measurements. He was awarded the "Angelo Barbagelata" premium in 1965 and the "Lorenzo Ferraris" premium in 1967 on the basis of the publications on electrical measurements. In 1971 he has been qualified for university teaching, on "Electrical Measurements". Since 1974 he has been an Associate Professor of "Advanced Electrical Measurements" at the Polytechnic of Milan. He has published various aspects of electrical measurements.

Marco Faifer was born in Bormio (Italy) on July 28, 1978. In 2003 he received his M.Sc. degree in Electronic Engineering at the Politecnico of Milan. Currently he is an assistant professor of Electrical and Electronic Measurements at the same University. His scientific activity is mainly concerned with the DSP techniques, the development of industrial sensors and the devices for High Voltage measurements.