Exploiting the Accuracy of Measurement Systems

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***Abstract*-**In this paper we present a technique to improve the accuracy of a measurement system. This technique is based on the arithmetic of the integer numbers in finite fields and represent an extension of the well known "calliper rule". The said technique is explained by using some simple measurement examples, in particular length, time and mass measurements. Some related possibilities of improvement for the modern instrumentation are suggested and discussed.

I. Introduction

The accuracy of a measurement system is a feature which depends on some characteristics of the system. As an abrupt simplification we can consider :
a) - the chosen measurement method,
b) – the goodness of the whole involved hardware;
c) – the smallest division of the scale;
d) - the attention of the operator.

Here we fix the attention over the scales. Often the smallest division of the scale is related to the system possibilities: a more small division would not improve the accuracy, i.e. not reduce the uncertainty related to the chosen measuring method and its hardware implementation. But, in other cases, the smallest division of the scale is determined by different constraints: the realization of small scale divisions can be difficult or its observation can become unpractical. The smallest division of the scale obviously affects the uncertainty of the measure.

In a length measurement system, e.g. a simple meter divided in millimeters, it is clear that the smallest uncertainty due to the observation of the scale is of the order of 1 mm.

In a time measurement, it is commonly retained that the smallest time interval which can be resolved by a clock correspond to the period of the reference periodic physical phenomenon. I. e. in a mechanical clock the period of the oscillating pendulum or of a mass-spring system or, in an electronic clock, the period of a vibrating quartz plate.

We find an analogue situation in the case of mass measures. Usually, a pair of scales is furnished together with a set of reference masses and the order of the obtainable uncertainty is related to the smallest mass of the given set.

In this cases, when the intrinsic accuracy of the measurement system is not fully exploited, an introduction of a comparison between two or more scales can be very useful, as will be shown in the following.

II. Proposed technique

The basic underlying theory of the proposed technique was already proposed by the author [1]. Here are briefly reported, for the sake of clarity, the useful features of this theory which are used for the proposed improvement of the accuracy. The proposed improved measurement system make use of a comparison between "scales". The concept of "scale" was recently discussed in the literature devoted to the fundamentals of measurement [2]. The said comparison between the scales determines a "virtual unit" that not exists physically but determines the available accuracy of the system [1]. This unit is referred in the following as "basic unit" (BU) of the measurement system under consideration. We consider here scales of the type ϕ^1 [2], i.e. scales which have the same origin but different steps u_i , $i = 1, 2, ..., N$ that have prime values in terms of BU. The value and the residues with respect to the u_i , $i = 1, 2, ..., N$ of the value of an unknown quantity X can be determined by shifting one of the scales of a quantity X with respect to the others. Since the values of the lengths u_i are integer, they can be interpreted as the "moduli" of a RNS System [3, 4]. In this system the residues with respect to the given moduli can represent each integer value measured (in terms of BU) $X \mid \quad 0 \leq X < M$.

So the residues of this unknown quantity X can be obtained by inspection of the relative position of the first scale with respect to the others. This inspection determines firstly the steps of the scales that are more next each other. Then, a simple difference between two lengths gives the residues [1].

III. Brief recalling of the used theory

Suppose that to perform the length measurements we have available some physical segments having unknown lengths u_i , $i = 1, 2, ..., N$ and the property that a multiple or a sub multiple of u_i cannot "correspond", in terms of our available accuracy, to u_j, for all the i, j . The expression "prime each other" cannot be used here because the values of the lengths are still unknown. By iteratively reporting these lengths over a straight line we can create a set of scales, of the type ϕ^1 , following the notation given in [1], i.e. having the same origin but different units. Let n_i, $i = 1,..., N$ be the integer counters of this iterative procedure. Then, for some particular values $\tilde{n}_i$ of these counters, we have that

$$\left|\tilde{n}_i u_i - \tilde{n}_j u_j\right| = \min \textit{ for all } i, j \qquad (2)$$

Condition (1) is cyclic, so we choose the lower values for the $\tilde{n}_i$. On the basis of the given example, we can suppose that, in the general case, a set of values of the counters $\{\tilde{n}_i\}$ exists so that

$$\tilde{n}_i u_i \equiv \tilde{n}_j u_j \quad \textit{for all } i, j \qquad (3)$$

Where the symbol "$\equiv$" means "corresponds in terms of our accuracy" and will be replaced in the following by a simple "=". Recalling the property imposed to the u_i, condition (5) is verified when

$$\tilde{n}_i u_i = \prod_j^N u_j = M \quad , \; i = 1, 2, ..., N \qquad (4)$$

So we have that the following quantity:

$$\frac{n_i u_i}{\prod_j^N u_j} = 1 \quad , \; i = 1, 2, ..., N \qquad (5)$$

can be assumed as the "Basic Unity" (BU) of this measurement system. However, this is a "virtual unity", very small with respect to the u_i, which not exists physically. Nevertheless, as happens in the calliper, the measurements can be performed with an accuracy related to this unity. This simply considering the n_i, given by direct observation, and some of their differences. From (4) it follows that:

$$\tilde{n}_i = \prod_{j \neq i}^N u_j \qquad (6)$$

Expression (8) means that also the u_i must be integer, as can be easily proved. Indeed the n_i are integer, so that the sentence is surely valid for $N = 2$. An induction procedure over N shows that if we have a system of order N and the related u_i, $i = 1, 2, ..., N$ that are integer, also a further $(N+1)^{th}$ length, inserted in the preceding system to obtain a system of order $N+1$, must be integer.
Now

$$\prod_i^N \tilde{n}_i = \prod_i^N \prod_{j \neq i}^N u_j = \left(\prod_j^N u_j \right)^{N-1} \qquad (7)$$

in such a way that

$$\prod_j^N u_j = M = \sqrt[N-1]{\prod_i^N \tilde{n}_i} \qquad (8)$$

and finally, from (2), the unknown values of the lengths u_i can be determined in terms of BU as:

$$u_i = \frac{M}{\tilde{n}_i} \text{ , } i = 1,...,N \qquad (9)$$

IV. Examples of simple applications

In a true RNS system, the product of the involved prime moduli determines the dynamic range. I.e. the range in which a set of residues corresponds univocally to a quantity X. In a measurement system, the observation of the operator or some automated actions can reduce to only two scales the comparison which realizes the measure. In this simple case, the BU becomes the difference $u_2 - u_1$. An example was given in [1] showing that the "caliper rule" in measuring lengths can be considered as a particular case of the cited theory. Other examples can be given considering time and mass measures. We consider firstly the time measure.

Also in this case, by realizing a comparison between two suitable time scales, we can greatly reduce the uncertainty of our measure. Consider for example a system of two pendula having the first one the oscillation period of 1 sec. and the second one .99 sec respectively, giving a BU of 1/100 sec. Using this system we can measure, as an example, the elapsed time between the start and the arrive of a runner over the 100 meters distance with an uncertainty of the order of 1/100 sec., i. e. well below the smallest reference period. We operate as follows: firstly we pose both the pendula to the maximum elongation, then at the start, liberate the first pendulum and begin a count of its complete oscillations; at the arrive we stop the count over the first pendulum, liberate the second pendulum and begin a new count of this second oscillations. We stop the second count when both the pendula oscillate in phase. In this way the first count give the seconds (coarse measure) and the second count give the cents of second (fine measure) of our time measure. This described measure appears to be particularly simple due to the suitable choice of the periods of the two pendula. However any couple of periods prime each other can be used for this measure, which appears in terms of BU in the general case [1].

In similar way we can perform accurate mass measurements. To this purpose we can use a classical pair of scales balance. Also in this case however, we can shown that the uncertainty of the measure can be reduced well below the smallest available mass. To perform the measure we suppose that two sets of masses are available: both the sets are formed by equal masses and the values of the masses of the two sets, expressed in terms of BU, namely m_1 and m_2, are different prime numbers. To perform the measure we firstly pose the unknown mass X on the first pan and attempt to obtain the balance by posing some of the masses of the first set on the second pan. We annotate as n_1 the maximum number of these masses that can maintain the unbalance on the X-side (coarse measure). Then add some other masses m_2 on the X-side pan and some other masses m_1 on the opposite pan until the minimum unbalance is reached (fine measure). Let $n_1^{'}$ be the number of the further masses m_1 and n_2 the number of the masses m_2. Then, again in terms of BU, the value of the measure X can be evaluated as:

$$X = (n_1 + n_1^{'}) \cdot m_1 - n_2 \cdot m_2 \qquad (10)$$

Some remarks related to the statistical distribution of the values of the "equal" masses and the resulting uncertainty and , in general, the possibilities of improvement for the modern instrumentation by this technique will be suggested and discussed in the next section.

V. Uncertainty evaluation

It is interesting the evaluation of the uncertainty involved in the proposed method. Consider firstly the time measure. Suppose that the periods of the two pendula are calibrated by means of a measure over N periods, having an uncertainty u1(N) and u2(N) respectively. Now in our measure suppose that the involved periods are n1 and n2 respectively. Then the uncertainty involved in the proposed comparison between scales becomes

$$u_c = u_1(N) \cdot \frac{n_1}{N} + u_2(N) \cdot \frac{n_2}{N} \qquad (11)$$

Thus the proposed measure appears to be significant if the BU, i.e. the difference between u1 and u2, must be greater than the value given by the (11). Only in this case we can detect the univocally the steps of the two scales which are "nearest" each other.

A similar reasoning line can be followed in the case of mass measurements. Suppose that the values of the masses of the two kind are given with an uncertainty of u1 and u2 respectively. Then the uncertainty of a set of n_1 or n_2 masses is $n_1 \cdot u_1$ and $n_2 \cdot u_2$ respectively. However, we can consider that the masses are produced by independent processes regarding the same reference sample, so the whole uncertainty can be divided by n_1 and n_2 respectively. In this case, the total uncertainty to be considered can be the uncertainty related to a single mass. And the total uncertainty involved in the proposed comparison can be simply

$$u_c = u_1 + u_2 \qquad (12)$$

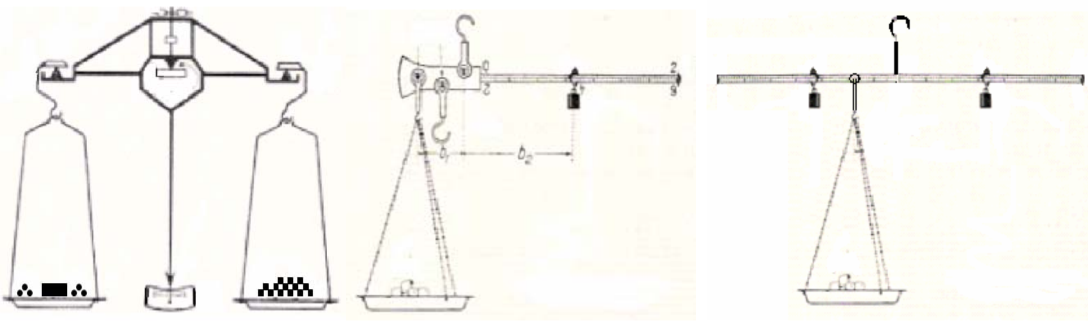
Also in this case the BU must be chosen greater than the value given by (12).

VI. Advantages of the proposed method

The mass measure above described is depicted in figure 1-a). Another possibility is the use of a single pan balance, o the type shown in figure 1 – b), replacing the two set of masses by two single masses, also equal each other, and two graduated branches. The two graduate branches are realized respecting the calliper rule, i.e. a particular case of the above summarized theory, which is recalled here in short.
In the calliper, the two scales are determined in such a way that the same interval *l* (length in the calliper, again length in this case, since the chosen masses are equal each other) is divided by *n* in the fixed scale and by 1 . *n* in the mobile one. Usually, in the calliper, l/n is the coarse unity, ($l = 19mm$ and $n = 19$), so 1/20 of the coarse unity ($= 1/20mm$) can be resolved. The result X of the measure is given by:

$$X = \frac{l}{n} \cdot (k_1 + \frac{1}{n+1} \cdot k_2) \qquad (13)$$

Where k_1 is the number of integer mm s, i.e. the number of marks of the fixed scale involved in the translation of the origin of the mobile scale, and k_2 the number of marks in the mobile scale between its origin and the marks of the two scales that are coincident each other. In the present case, in which we measure masses using a balance, the coincidence between two marks of the two scales is given when, operating by the two mobile masses, the minimum unbalance is reached. Then the value of the measured mass is given by (10), taking for $n_1 + n_1^{'}$ and n_2 the number of the involved marks over the two branches, respectively, and for m_1 and m_2 the value m of the equal mobile masses. A balance of this type is shown in figure 1-c).



a) *b*) *c*)

Figure 1 – a) measurement performed using two set of masses and a two pan balance; b) single pan balance; c) single pan balance arranged applying the calliper rule.

The arrangement of figure 1-c), unpractical from some points of view, is used here to shown that, using the calliper rule, the complexity of the involved hardware can be greatly reduced. Indeed, the use of the virtual unity gives two different advantages: the first one is that the "virtual unity" is not physically present and not be managed. This is particularly advantageous when this "virtual unit" is very small: in the simple case of single pan balance used as example the marks over the two branches can be conveniently spaced each other. The second advantage is that the number of the marks is greatly reduced. To obtain $n \cdot (n+1)$ comparison possibilities, corresponding to $n \cdot (n+1)$ BU, only $n + (n+1)$ marks are necessary.

VII. Possible applications in modern instrumentation

Possible applications to modern instrumentation appear to be very wide. We cite here as examples only two possible applications.

The first one is related to SONAR and RADAR techniques: the measure of the distance of the target is usually determined from the elapsed time between the launch of the exploring wave and the arrive of the echo related to the target. This procedure can be used as the "coarse measure" of the distance. Following the theory above described, a further "fine measure" can be obtained using two slightly different frequencies for the exploring wave and detecting the phase difference between them. However, similar procedures are already in use [5]. In this case the first of the above evidenced advantages is exploited. Note that, following the Residue Number System (RNS) theory, using a convenient number of frequencies having the related periods prime one another, the detection of all the phase relations between these waves in the echo signal allows the possibility of determining the whole distance. In this case, the periods of the given frequencies are interpreted as "moduli" and the related phases are interpreted as "residues" of a RNS. However, a coarse measure of the type recalled above seems to be more practical.

The second proposed example is completely new [6] and described in particular in another parallel paper [7]. The example regards the realization of "full flash" or "subranging" Analogue to Digital Converters (ADC). An inconvenience in such realizations is the great number of resistors necessary for the realization of a reference scale of voltages. Indeed 2^n resistors are needed if a $n-bit$ word length is requested in the conversion. In this case, the second of the cited advantages of this proposed technique can be exploited. A virtual voltage unity (the BU cited above) can be created using a comparison between two scales of voltages "fix" and "mobile" respectively. In this way, only about $3 \cdot 2^{n/2}$ instead of 2^n resistors are needed. This, for example, means that for $n = 10$ the 1024 resistor of a "classical" realization are reduced to about 100. The details of an architecture of this kind are presented in the parallel paper [7].

VIII. Conclusions

A technique to improve the accuracy of a measurement system was presented. This technique is based on the RNS theory and allows the possibility of improving the accuracy of a measurement system. This is obtained by reducing the uncertainty of the measure induced by the reference scale. Two possible advantages are discussed and a new application is proposed. The detail of this new proposed application are reported in a parallel paper [7].

References

[1] – R. Lojacono "The caliper rule as a particular case of residue number system arithmetic", 10th TC7 Symposium, St Petersburg, June 30-July 2, 2004.

[2] - J. Barzilai, "On the foundations of measurement" Systems, Man, and Cybernetics, 2001 IEEE International Conference on ,Volume: 1 , 7-10 Oct. 2001 Pages:401 – 406.

[3] - N.Szabo, and R. Tanaka, "Residue Arithmetic and its Applications to Computer Technology", New York: McGraw-Hill, 1967.

[4] - M. A. Soderstrand, W. K. Jenkins, G. A. Jullien, and F. J. Taylor, "Residue Number System Arithmetic: Modern Applications in Digital Signal Processing". New York: IEEE Press

[5] - Gjessing, D.; Hjelmstad, J.; Lund, T, "A multifrequency adaptive radar for detection and identification of objects: Results on preliminary experiments on aircraft against a sea-clutter background", Antennas and Propagation, IEEE Transactions on [legacy, pre - 1988] Volume 30, Issue 3, May 1982 Page(s):351 - 365

[6] - R. Lojacono , *Italian Patent n° RM 2005 A000086, 28/02/05*

[7] - R. Lojacono "Reduced complexity full-flash ADCs", 10th Workshop on ADC Modelling and Testing – 12-15 September 2005 – Gydnia and Jurata - Poland