

Comparative Analysis of Lossy Transformation Algorithms for Measurement Signal Compression Based on Limiting Error

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Abstract- The comparative analysis of lossy transformation compression algorithms for measurement signals is presented in the paper. A limiting error criterion and testing signals of synthetic character were proposed for comparing the compression algorithms. The application of such algorithms to test the lossy compression algorithms makes the results obtained in such a way independent of the shape of a single signal and makes them valid for the whole class of measurement signals while the determined limiting errors assume the character of maximum errors. By varying the parameters of the synthetic signals the operation of algorithms can be tested within certain intervals of signal parameter variability, not for single values of these parameters as it occurs for real measurement signals. The DCT, DST, DFT, DWHT, and DWH transformation algorithms were compared for various transformation orders and the classes of signals with single limit (variability range) or two limits (variability range and signal variability rate).

I. Introduction

In the procedure of investigating losses during the implementation of a compression algorithm, the test signal, $u(n)$, undergoes a compression-decompression operation and the resulting signal, $u_r(n)$, is compared to the original signal according to an adequately defined error criterion.

The magnitude of the difference between these signals depends on the algorithm properties (e.g., transformation type, coefficient quantization method), and on the properties of the data (signal) stream. For this reason, the result of the assessment and comparison of compression algorithms will depend on the applied signal (compressed data). In order to make the results of such a comparison independent of the signal shape, a method for assessing compression algorithms was proposed [2,3,4,6] based on synthetic signals specially constructed to this end, of properties corresponding to the selected parameters of measurement signals. The proposed method of study the lossy compression algorithms consists in constructing testing signals that allow to determine limiting errors. The application of synthetic test signals to study the compression algorithms makes the assessment results obtained this way independent of the shape of an individual signal and makes the test signals representative for the whole class of measurement signals.

The error criterion is usually assumed as an error functional determined on the difference $\Delta(n)$ between the test signal and the signal processed during the compression-decompression procedure:

$$\Delta(n) = u_r(n) - u(n), \quad (1)$$

where: $u(n)$ – test signal;

$u_r(n)$ – signal obtained as the result of the compression-decompression procedure made on the $u(n)$ signal

while (1) may be presented in the vector form:

$$\Delta = \mathbf{u}_r - \mathbf{u} \quad (2)$$

where: \mathbf{u} and \mathbf{u}_r are $(N \times 1)$ vectors of test and reproduced signals, respectively.

Assume the following error vector norm Δ , is an error measure (so called maximum value criterion):

$$J = \|\Delta\| = \max_n |\Delta(n)|. \quad (3)$$

The following dynamical limiting error was defined for the assumed error measure J :

$$J_0 = \max_{u(n) \in U} J\{u(n)\}. \quad (4)$$

Constructing a synthetic test signal consists in determining such signal $u_0(n) \in U$ that:

$$u_0(n) = \arg \max_{u(n) \in U} J \{u(n)\}. \quad (5)$$

Next signal $u_0(n)$ is used as test signal for calculating the value of limiting error (4). In measurement applications, the class U of test signals $u(n)$ is assumed as a set of signals with limited variability range:

$$\max_n |u(n)| \leq 1. \quad (6)$$

Test signals $u_0(n)$ obtained from the procedure (4) are the bang-bang type signals [2,6] of infinite speed of rise at the switching time instants and constant in intervals between these instants (the values are the effect of the limited variability range (6)). Fitting the test signal dynamics to the dynamics of real signals under study undergoing the process of compression requires that an additional limitation be imposed upon these signals. In the cases algorithms are investigated, except the variability range limitation, it is natural to introduce for test signals also a limitation of the maximum rate of signal variability rate [6]:

$$\max_n |u(n+1) - u(n)| \leq M_1. \quad (7)$$

II. Limiting errors of transformation compression algorithms

Consider the process of compressing signal $u(n)$ by means of a selected transformation algorithm with real basis functions (e.g., DCT, DST, DWHT, DWH, etc.) or complex basis functions (e.g., DFT). The resulting transformation coefficients are quantized and saved. Making the inverse transformation on the quantized coefficients allow to obtain a reproduced signal $u_r(n)$ where an instantaneous error value may be presented in a vector form (2). Let's accept the following pair of transforms:

$$\Theta = \mathbf{A} \cdot \mathbf{u} \quad (8)$$

$$\mathbf{u}_r = \mathbf{B} \cdot \hat{\Theta} \quad (9)$$

where: \mathbf{A} - is an $(N \times N)$ transformation matrix built from conjugate basis vectors of the transformation,

\mathbf{B} - is a matrix inverse to the transformation matrix,

while for a unitary matrix \mathbf{A} containing complex elements the following holds:

$$\mathbf{B} = \mathbf{A}^{-1} = \mathbf{A}^* = \overline{\mathbf{A}^T}, \quad (10)$$

and quantize with a quantization operation Q the complex coefficients of transformation Θ :

$$\hat{\Theta} = Q(\Theta) = Q(\mathbf{A} \cdot \mathbf{u}). \quad (11)$$

Error (2) when (9) is taken into account will be expressed as:

$$\Delta = \mathbf{B} \cdot \hat{\Theta} - \mathbf{u}, \quad (12)$$

Further, taking into account (8) and (10) through (12), we have:

$$\Delta = \mathbf{A}^* \cdot Q(\mathbf{A} \cdot \mathbf{u}) - \mathbf{u} \quad (13)$$

The quantization level q of the quantization operation Q is chosen depending on the assumed allowable compression error and the expected compression ratio. Let's write the quantization procedure using function *Entier* (integer part) and assuming that for a complex matrix \mathbf{A} the procedure is realized separately for real and imaginary parts of matrix \mathbf{A} .

A detailed method of determining test signal $u_0(n)$ (5) at a single limit (6) is presented in [2], and in [3] for two limits (6) and (7) imposed on the searched signal. In [4], a method is presented that allows to determine a synthetic test signal $u_0(n)$ taking into account the quantization of the transformation coefficients of compression algorithm with real basis functions, and in [5] – with complex basis functions. The methods discussed there lead to the linearization of equation (13) being nonlinear with respect to \mathbf{u} . Equation (13) assumes then the following form:

$$\Delta \cong \mathbf{D} \cdot \mathbf{u} \quad (14)$$

where for matrix \mathbf{A} with real elements, matrix \mathbf{D} can be written as:

$$\Delta \cong \left[\mathbf{A}^T \cdot \text{entier} \left(\frac{\mathbf{A}}{q} \right) \cdot q + \frac{1}{2} \cdot \mathbf{A}^T \cdot \mathbf{E} \cdot q - \mathbf{I} \right] \cdot \mathbf{u} \quad (15)$$

and for matrix \mathbf{A} with complex elements, the equation is:

$$\mathbf{D} = \left[\mathbf{A}^* \cdot \text{entier} \left(\frac{\mathbf{A}_{\text{Re}}}{q} \right) \cdot q + \mathbf{A}^* \cdot \text{entier} \left(\frac{\mathbf{A}_{\text{Im}}}{q} \right) \cdot q + \frac{1}{2} \cdot \mathbf{A}^* \cdot \mathbf{E} \cdot (1+j) \cdot q - \mathbf{I} \right] \quad (16)$$

gdzie: $\mathbf{A}_{\text{Re}} = \frac{1}{2}(\mathbf{A} + \overline{\mathbf{A}})$, $\mathbf{A}_{\text{Im}} = \frac{1}{2}(\mathbf{A} - \overline{\mathbf{A}})$,
 \mathbf{I} – is the unit matrix,
 \mathbf{E} – is a matrix of all its elements being equal 1,
 $j = \sqrt{-1}$.

To determine the limiting error, let's introduce a matrix norm for matrix \mathbf{D} as follows:

$$\|\mathbf{D}\| = \max_{1 \leq i \leq n} \sum_{k=1}^n |d_{ik}|. \quad (17)$$

The matrix norm (17) is consistent and subordinate to the vector norm (3), and the following holds:

$$\|\mathbf{D}\| = \max_{\|\mathbf{u}\|=1} \|\mathbf{D} \cdot \mathbf{u}\|. \quad (18)$$

which with (6) taken into consideration gives for (14) an upper limit of error (3) estimated by norm of matrix \mathbf{D} :

$$\|\Delta\| \leq \|\mathbf{D}\| = \max_{1 \leq i \leq n} \sum_{k=1}^n |d_{ik}|. \quad (19)$$

The matrix norm (17) is equal to the sought value of the limiting error J_0 expressed by relationship (4), i.e.:

$$J_0 = \|\mathbf{D}\|. \quad (20)$$

The synthetic test signal \mathbf{u}_0 (4) with a single limit (6) allowing to calculate the error norm (19) can be calculated based on equation (14) taking into account the norm (17) of matrix \mathbf{D} expressed by (15):

$$u_0(n) = \text{sgn} \mathbf{D}(i_1, n) \quad \text{for } n = 1 \dots N, \quad (21)$$

and for complex matrix \mathbf{D} expressed by (16)

$$u_0(n) = \frac{\overline{d_{i_1, n}}}{|d_{i_1, n}|} \quad \text{for } n = 1 \dots N, \quad (22)$$

where index i_1 is calculated using the equation:

$$i_1 = \arg \max_{i=1 \dots N} \sum_{j=1}^N |\mathbf{D}(i, j)| \quad (23)$$

In accordance with (3), signal $u_0(n)$ allows to determine the limiting error value J_0 . For a complex transformation matrix, the signal will also be complex of unit complex modulus, i.e., it will meet the condition (6) of the signal norm with both sides of (6) equal.

It is possible to determine the synthetic test signal \mathbf{u}_0 (4) basing on equation (14) with two limits (6) and (7) and employing matrix \mathbf{D} (equations (15) or (16)). A detailed algorithm is presented in [3].

III. Comparative analysis of algorithms based on the limiting-error criterion

A comparative analysis has been carried out of transformation compression algorithms with real basis functions (cosine transform DCT, sine transform DST, Walsh-Hadamard transform DWHT, Haar transform DHT) and with complex basis functions (Fourier transform DFT) for various transformation orders and the classes of signals with single limit (variability range (6)) or two limits (variability range (6) and signal variability rate M_1 (7)). The implementation of the algorithms and simulation were made in the Matlab environment [10, 11].

The study was carried out for transformation order (data block length) $N=128$, amplitude limits according to (6), signal variability rate limits $M_1=\infty$ (single-limit case: no signal variability rate limits), $M_1=1.0$, $M_1=0.5$ and $M_1=0.25$, and transformation coefficients quantization level q in the range from $1/2^{16}$ to $2000 \cdot 1/2^{16}$. In Fig.1 the relationship of limiting-error J_0 vs. quantization level q for the listed types of transformation algorithms and four cases of limits M_1 , and transformation order $N=128$ is presented.

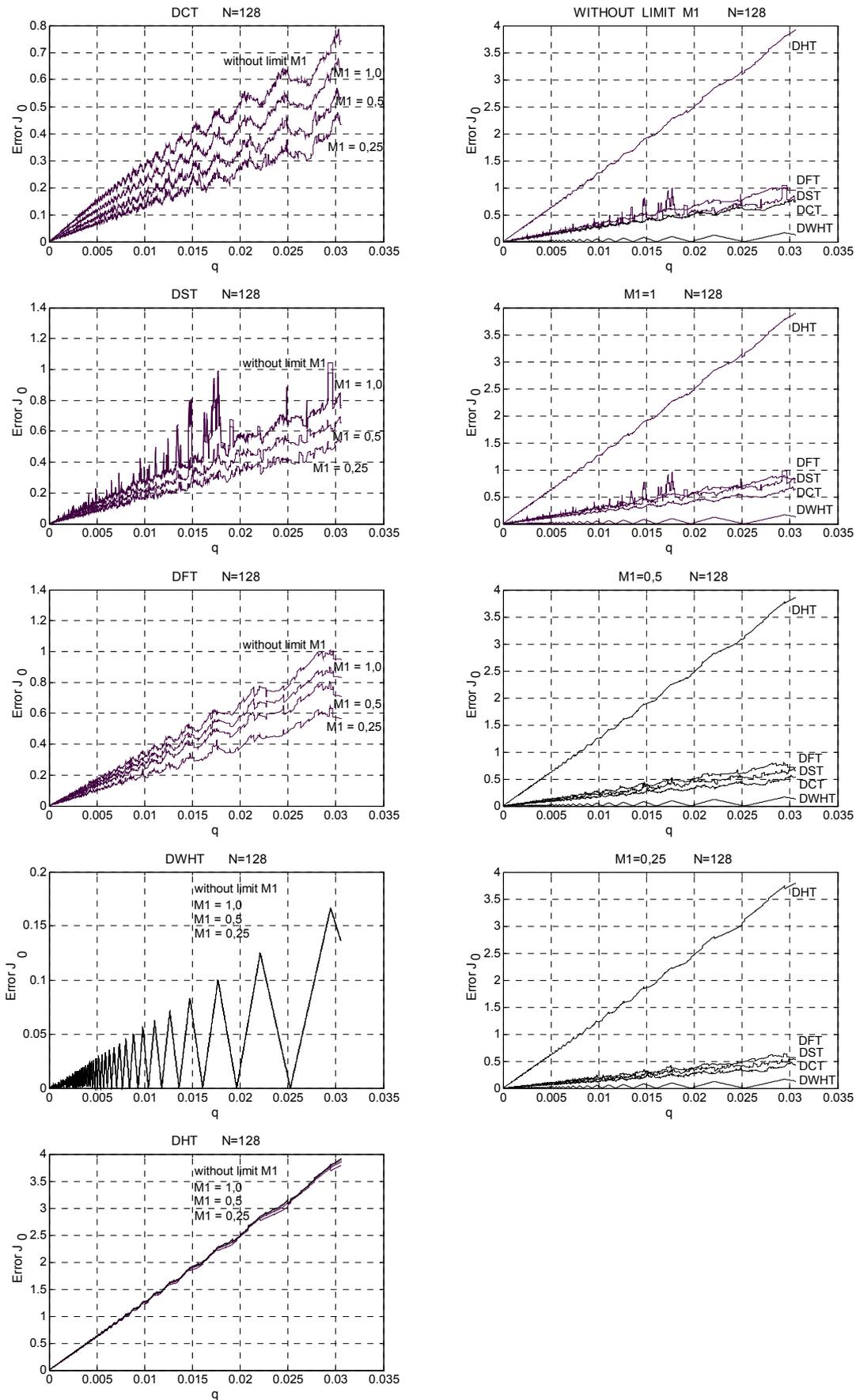


Fig.1. Characteristics of the limiting-error J_0 vs. quantization level q for various types of transformation algorithms and different values of limit M_1 , transformation order $N=128$.

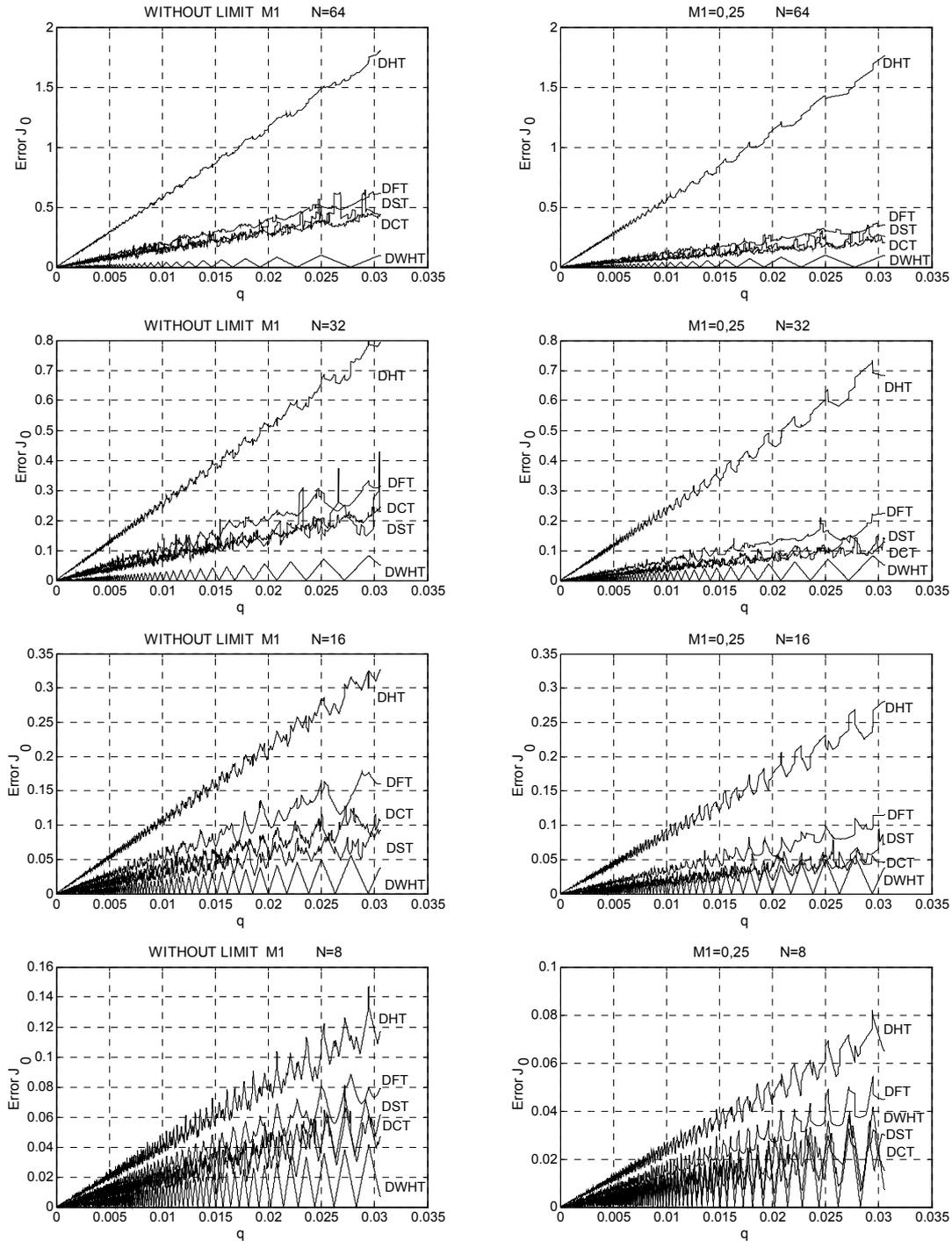


Fig.2. Characteristics of the limiting-error J_0 vs. quantization level q for various types of transformation algorithms and different values of limit M_1 ; comparison for various transformation ($N=64$, $N=32$, $N=16$ and $N=8$).

The relationship of limiting-error J_0 vs. quantization level q for the listed types of transformation algorithms and two cases of limits M_1 ($M_1=\infty$ and $M_1=0.25$) is presented in Fig.2. Also, the influence of transformation order is shown (for $N=64$, $N=32$, $N=16$ and $N=8$).

Based on the characteristics, it is possible to select the quantization level of transformation coefficients, accepting through a compromise an allowable value of the limiting error and the expected compression ratio. This is so because the compression ratio depends on the assumed quantization level of transformation coefficient through the number of bits of the record of transformation coefficients. The result of such an analysis will be valid in the aspect of the limiting error criterion for the accepted signal class meeting the limits (6) and (7) imposed on the test signal $u(n)$.

Based on the characteristics presented, the following conclusion can be drawn:

- if the DWHT transform is used, the least values of limiting errors are obtained (Fig. 1): this results from the fact that the square shape of test signals \mathbf{u}_0 (21) so their span over the basis of square signals is optimal;
- application of other transforms leads to similar limiting error values, except for the Haar transform which differs from the others in its basis vector amplitude variability and hence it produces very large errors;
- based on the characteristics in Fig. 2 it can be seen that it is better to use lower transformation orders (with corresponding lower values of limiting errors) as the signal power is concentrated in a smaller number of expansion coefficients, therefore the coefficients have greater values, and the final results is such that the values of limiting errors are lower at the same quantization level q .

IV. Conclusions

A procedure for compression algorithms comparison was proposed and a method for determining synthetic test signals for limiting-error criterion was presented. The application of synthetic test signals to investigate algorithms makes the assessment results obtained in such a way independent of the shape of a single signal and makes them valid for the whole class of measurement signals.

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