Frequency Properties of Two-Dimensional Median Windows - Phase and Amplitude Responses

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Abstract - The use of medians as filtering windows in digital signal processing is known since many years and applied especially when linear filters could not be used, because of specific character of disturbances which are to be removed. This is the case, if, for instance, impulsive disturbances, having very wide spectra, are observed, and linear filters are not suitable for their elimination. Median procedures, including also weighted medians, are particularly effective in such filtering procedures. According to the applied algorithm, disturbed signal samples are eliminated and replaced by some values resulting from neighbouring nondisturbed samples [2]. In spite of the nonlinear character medians can be characterized in the frequency domain if a certain "harmonic linearization" is done. An analysis of such median characteristics in one-dimensional case was given in [1, 4]. This idea is now extended to two-dimensional (2D) signals and the frequency properties (amplitude and phase) of the most popular 2D median filtering procedures are examined.

I. Introduction

Medians as filtering algorithms in digital signal processing are known since many years; they can be applied especially when linear filters are useless, because of specific character of disturbances which are to be rejected. This corresponds to the case, if, for instance, impulsive disturbances, having very wide spectra, are observed, and linear filtering is not suitable for their elimination. Median procedures, in some cases also weighted medians, are particularly effective in this situation. According to the median algorithm, disturbed signal samples are eliminated and replaced by some values resulting from neighbouring nondisturbed samples [2]. It must be noted here, that in spite of the nonlinear character medians can be characterized in the frequency domain if a certain "harmonic linearization" is done. However, the frequency responses will be differently shaped for different input signals, in particular for inputs being separate sine waves, random sequences etc.(see [4]). An analysis of such median characteristics in one-dimensional case was given in [1, 4]. It seems to be interesting to extend this idea to two-dimensional (2D) signals, images but not only, and examine the frequency – phase and amplitude – responses of the most popular 2D median filtering procedures.
In the following contribution such frequency responses for different shapes and sizes of filtering windows are presented. In particular the phase shift introduced by median filters was calculated and commented.

II. Filtering Algorithm

Two different shapes of median filter windows (mentioned e.g. in [5]) are shown in Figure 1. If, for instance, the scheme "CROSS", for a 5×5 window, is used, the filtering operation can be described by the formula

$$y[m,n] = med\big(x[m,n-2], x[m,n-1], x[m,n], x[m,n+1], x(m,n+2),$$
$$x[m-1,n], x[m-2,n], x[m+1,n], x[m+2,n]\big)$$

where in $x[m,n]$ the first index is the row and the second is the column.

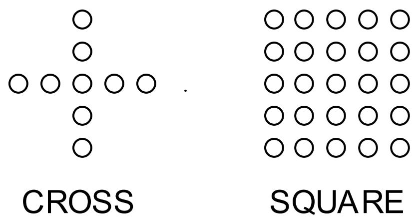

Figure 1. 2D median filter windows

For the “SQUARE” scheme the median is calculated over all pixels belonging to the square around *x[m,n]* (*x[m,n]* is the central point of the square).
In order to determine the frequency response a sequence of N^2 samples of a two-dimensional harmonic signal

$$x[m,n] = A\,sin(2\pi pm / N)sin(2\pi qn / N) \qquad (1)$$

was applied to different median filters and the spectrum of output $y[m,n]$

$$Y(p,q) = F(y[m,n]) \qquad (2)$$

where *F(.)* denotes the 2D-Fourier-transform and *p, q* are harmonic orders, was calculated. The frequency responses were then obtained as quotients of the corresponding component of output and the input amplitude, according to the formula

$$H(p,q) = \frac{1}{A}Y(p,q) \qquad (3)$$

Amplitude responses, as the following absolute values,

$$|H(p,q)| = (1/A)|Y(p,q)| \qquad (4)$$

were calculated and presented in [6]. It was noted there that the frequency properties depend on the window shape and that those of the "SQUARE" window can be more suitable in some cases, namely if low-pass properties are desired within a strictly defined frequency rectangle (see Figures 2 and 3 – copied from [6]). Generally, the passbands, slope, smoothness etc. and can be adapted to the filtered signal.
As it was mentioned, another very important feature is the phase shift introduced by median filters. It can be obtained as

$$angle[H(p,q)] = \arctan\frac{\mathrm{Im}[H(p,q)]}{\mathrm{Re}[H(p,q)]} \qquad (5)$$

and can be immediately calculated from the 2D-FFT values.

III. Simulation Results

Simulation results for both filter windows, “CROSS” and “SQUARE”, are given in Figures 4 and 5. The curves are calculated for 14 2D-harmonics and 5×5 median windows, using MATLAB environment.

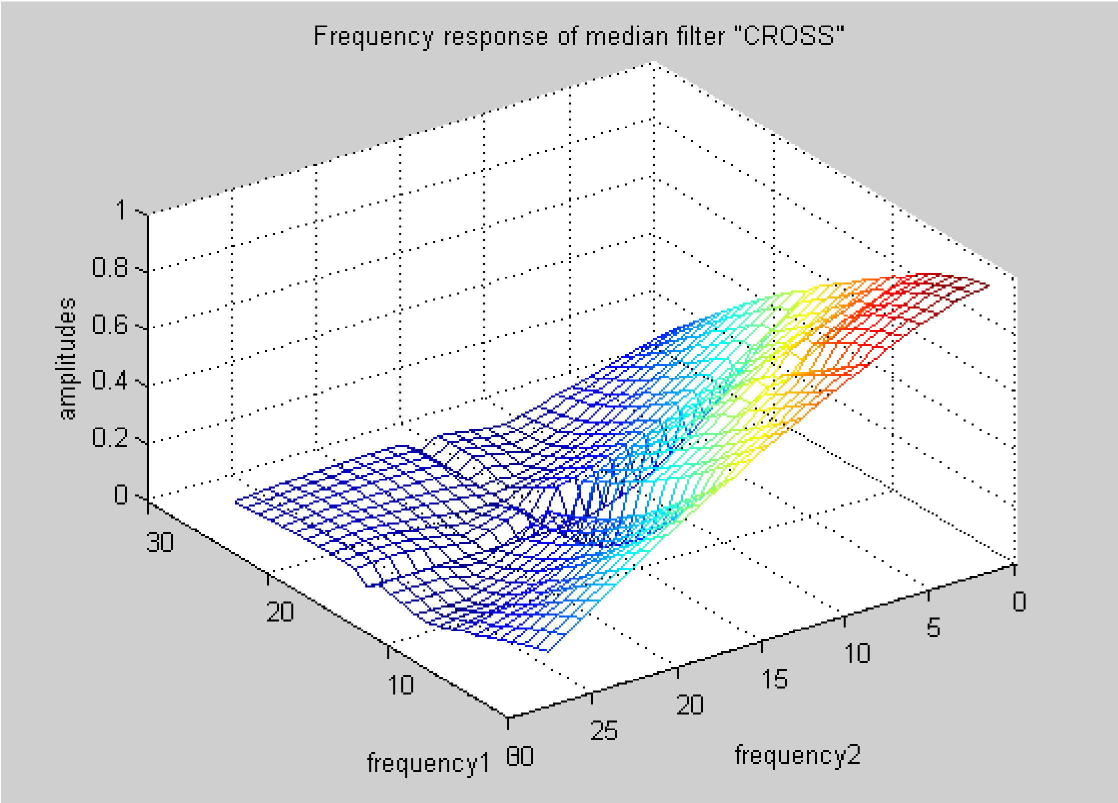

Figure 2. "CROSS" filter amplitude frequency response

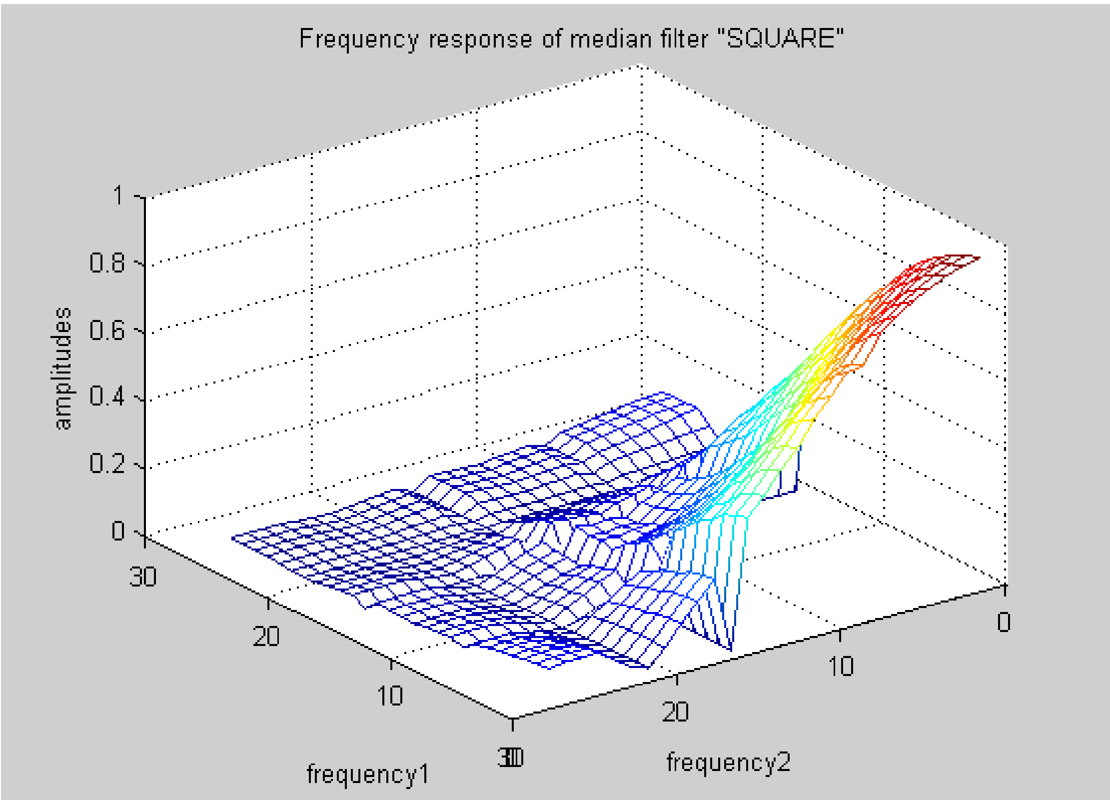

Figure 3. "SQUARE" filter amplitude frequency response

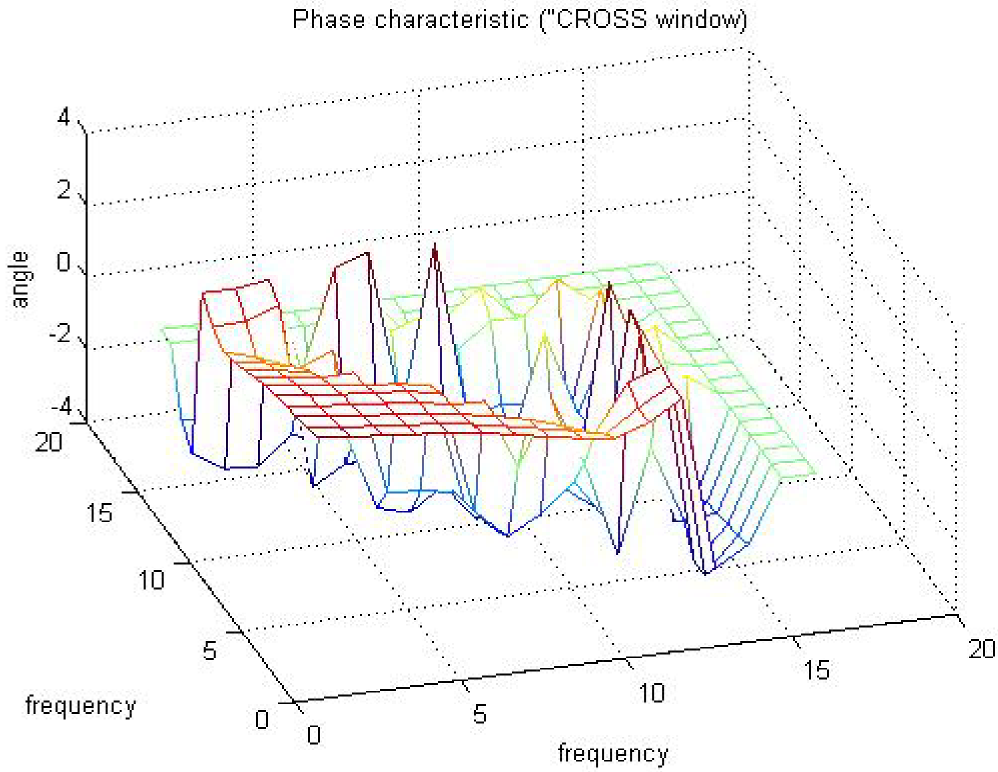

Figure 4. Phase response for the "CROSS" window

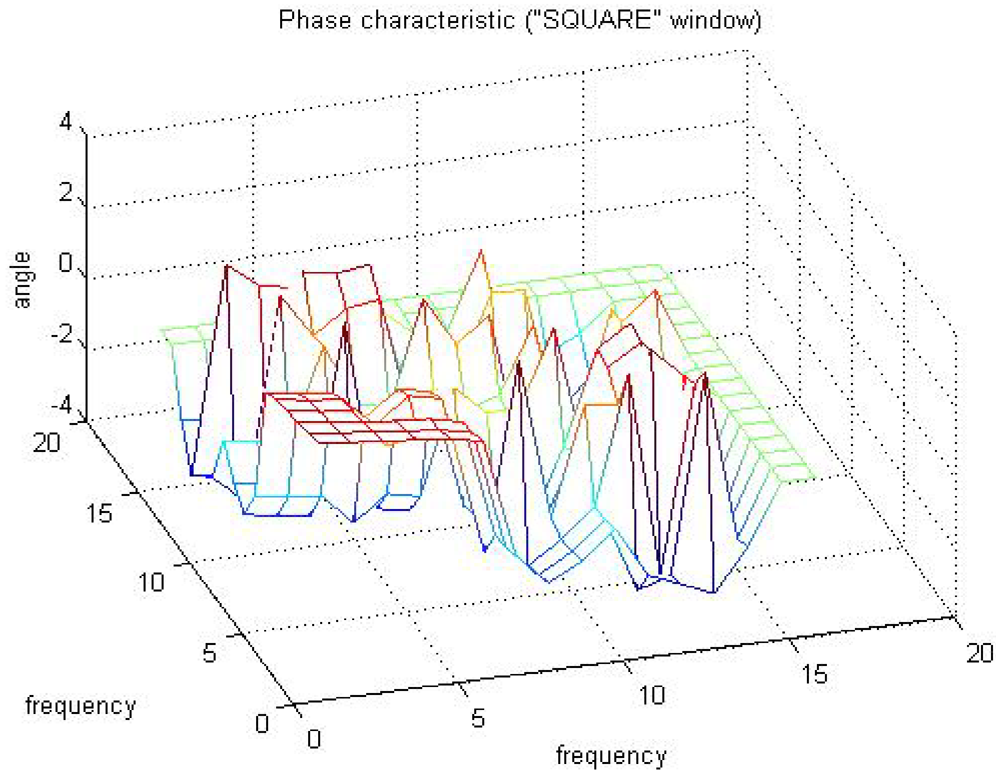

Figure 5. Phase response for the “SQUARE” window

IV. Conclusions

A strong interdependence between the frequency response of the median filter, both phase and gain, and the window shape can be found. Such parameters as the amplitude response slope, constant phase shift area (Figures 4 and 5), certain phase "ripple" should be mentioned here and taken into consideration if any application of median filters is foreseen. Anyway, a trade off between the amount of smoothing and the preservation of details should be the key factor in all design procedures. On the other hand, it must be noted that the window shape influences substantially the computational complexity of median filters.

A thorough analysis of frequency properties for other popular windows, used in image filtering, such as "x-shaped", "circle", "horizontal" and "vertical" should be perhaps done in order to optimize the filtering procedures. Also the use of weighted 2-D medians could be interesting – the weight vector is then an important factor of filtering quality.

References

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