

Accelerated Impedance Spectrum Measurement via Multisine Perturbation and Digital Filter Banks

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Abstract—The paper presents accelerated impedance spectrum measurement method, oriented for measuring the quality of anticorrosion coating, modelled by a linear equivalent circuit. The method is based on applying multisine stimulation signal. Both stimulation and object response are analysed in frequency domain by digital filter bank. The theoretical description of method and a proposition of measurement methodology are presented. The simulation results are discussed, concerning spectrum calculation error and acceleration of measurements for several shape-designed multisine signals.

I. Introduction

Vector impedance measurements and impedance spectroscopy are nowadays widely used in science, medicine and industrial applications to identify parameters of objects and phenomena modelled by electric circuits. Among other fields, they have been used to performance estimation of anticorrosion coatings [1]. The performance of coating can be estimated by its equivalent electrical circuit parameters [2]. Impedance spectroscopy is the most popular way of finding equivalent circuit parameter values. It relies on measurement of object impedance for selected frequencies in a specified frequency range and fitting of parameter-dependant model to measurement data, frequently by Complex Non-linear Least Squares (CNLS) method [3]. Point-by-point impedance spectrum measurements lengthens the experiment time, especially when using very low frequencies, necessary to identify parameters of thick coatings [2].

Modern impedance analysers are DSP based – they compute transforms of stimulus and response to calculate impedance in frequency domain [4]. In order to accelerate impedance spectrum measurement of linear objects, the method based on additive property of DFT has been proposed in [5]. The method uses multisine perturbation and DFT analysis to calculate impedance values for several frequencies at one time. However, the method is difficult to implement in practice, due to problems with both multisine perturbation synthesis and response analysis (assuming proper DFT length). To circumvent these disadvantages the method of measuring impedance frequency spectra of non-stationary objects by means of filter banks has been proposed [6]. The digital filter bank method also possesses great possibilities of optimisation, to accelerate stationary object impedance spectra measurement and decrease numeric complexity.

II. DSP based impedance measurements

A. Single frequency DFT impedance measurement

Modern DSP based impedance analysers calculate the impedance frequency spectrum via multiple single-frequency DFT–method based measurements. A single frequency DFT method is carried out as follows: the sinusoidal stimulation signal and object response are sampled coherently, thus producing the digital, discrete $u_x[n]$ and $i_x[n]$ signals, proportional to voltage stimulation and current response. The information about magnitude and phase of analysed signals (complex amplitudes of sinusoids) can be calculated by DFT or (more efficiently) using the Goertzel algorithm. If the analysing window length has been assumed a multiplication of period of the stimulation frequency, then, both voltage and current spectra contains only one non-zero frequency bin. It was proven [4], that even one period of sinusoidal stimulation is sufficient to do the measurement. Time necessary to measure the impedance spectrum is greater or equal to sum of periods of the frequencies in spectrum.

$$t_{meas.} \geq \sum_{i=1}^N T_i = \sum_{i=1}^N \frac{1}{f_i} \quad (1)$$

Eq. 1 leads to a conclusion, that measuring the impedance spectrum with low and very low frequency signals (order of mHz), necessary to identify the parameters of high-impedance objects (e.g. thick anticorrosion coatings) is very time consuming [2].

B. Multisine DFT impedance spectrum measurements

The multisine measurement methodology is similar to single frequency DFT method. According to superposition principle, the response of object to sum of stimuli is equal to sum of responses to single signals. If the poliharmonic stimulus signal is applied to CUT, the corresponding spectra bins in transforms of current and voltage should allow calculating the impedance spectrum in single measurement. Although the method looks simple, it is hard to implement in practice. If the DFT length is not integer multiplication of periods of analysed signals, instead of few non-zero bins corresponding to frequencies present in multisine, the spectrum has multiple non-zero bins (Fig. 1), which values (according to Parseval relation) are not equal to amplitudes of multisine and they change with every measurement, depending on which part of multisine has been captured by DFT window.

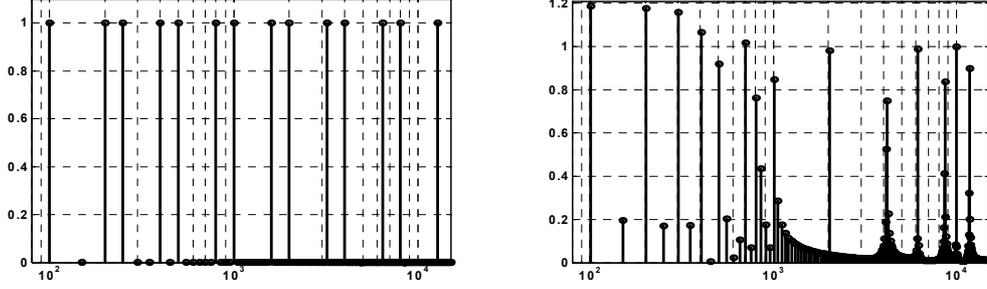


Fig. 1. The results of DFT with proper (left) and improper (right) length.

It was proposed [5] to repeat the experiment several times (256) to average the calculated spectra. The method was advantageous (faster), when compared with analogue FRA impedance analysers of the seventies. However, in comparison with modern single-frequency DSP analysers [4], the measurement would last longer than single frequency, point-by-point method.

To circumvent disadvantages of multisine DFT impedance measurement, a new method of obtaining time varying frequency spectra has been proposed, by means of digital filter banks.

III. Filter bank based impedance spectrum measurement

The DFT transformation can be realized by L-channel filter bank representation [7]. Filter banks implement in DSP the idea of synchronous amplitude detection. The methodology of accelerated impedance spectrum measurement via multisine perturbation and filter bank is presented in Fig. 2.

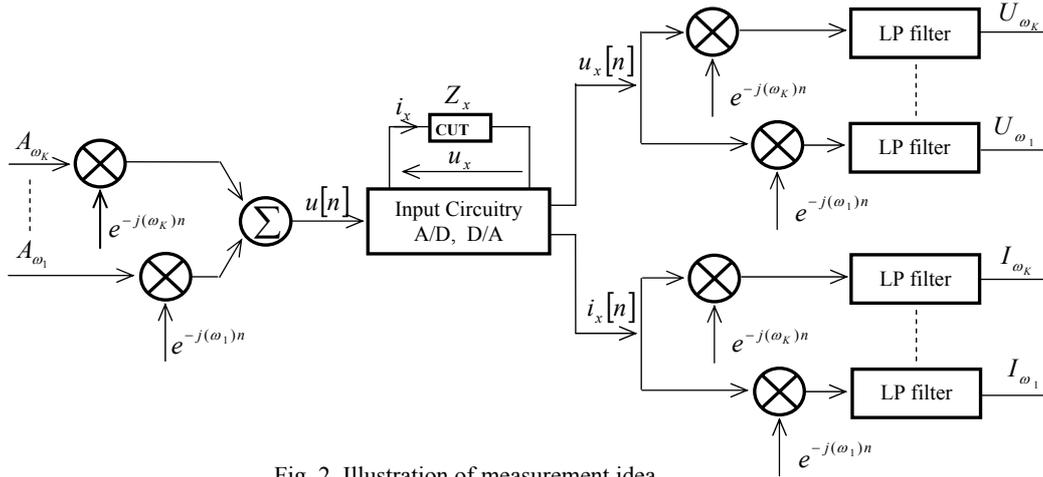


Fig. 2. Illustration of measurement idea.

A summation node synthesise multisine perturbation signal $u[n]$ with amplitude and phase relations given by complex amplitudes A_ω . As this operation can be done sample-by-sample, there is no need to prepare a single AWG data record of extensive length equal to Least Common Multiple (LCM) of all periods of multisine signal components. Input circuitry D/A converter provides analogue stimulation signal u_x and gathers analogue current response i_x . Both stimulation and response signals are sampled coherently with A/D converters, thus producing digital signals $u_x[n]$ and $i_x[n]$. In order to calculate impedance spectrum, the complex amplitudes of frequency components in both signals are calculated

with two identical filter banks. Every filter bank consists of several parallel structures, each of them built with modulator and low-pass filter. Modulator shifts the spectrum in frequency domain according to modulation principle:

$$\begin{aligned} x(t) &\leftrightarrow X(j\omega) \\ x(t) \cdot e^{j\omega_s t} &\leftrightarrow X(j(\omega - \omega_s)). \end{aligned} \quad (2)$$

After shift, the bin corresponding to complex value of desired frequency ω_s is placed at DC, and its value can be extracted from modulator's output by low-pass filtering. It is worth noticing, that modulators' harmonic signals are the same that are used to synthesise the stimulations signal.

If we denote K-component multisine signal (e.g. $u_x[n]$ and $i_x[n]$) as:

$$x[n] = X_0 + \sum_{k=1}^K (X_k e^{j\omega_k n} + X_k^* e^{-j\omega_k n}), \quad (3)$$

than the signal after modulation by ω_s in s-th frequency filter bank channel is:

$$\begin{aligned} x_{\omega_s}[n] &= x[n] \cdot e^{j\omega_s n} = X_0 \cdot e^{-j\omega_s n} + \sum_{k=1}^K (X_k e^{j(\omega_k - \omega_s)n} + X_k^* e^{-j(\omega_k + \omega_s)n}) = \\ &= X_0 \cdot e^{-j\omega_s n} + \sum_{k=1}^{s-1} (X_k e^{j(\omega_k - \omega_s)n} + X_k^* e^{-j(\omega_k + \omega_s)n}) + X_{k=s} + \sum_{k=s+1}^K (X_k e^{j(\omega_k - \omega_s)n} + X_k^* e^{-j(\omega_k + \omega_s)n}). \end{aligned} \quad (4)$$

Eq. 4 shows, that if modulator pulsation ω_s is equal to multisine component pulsation ω_k , the DC value at output of modulator will be the amplitude $X_{k=s}$ of component with frequency $f_{k=s}$. The low-pass (LP) filter extracts the DC value from output of modulator. Corresponding complex amplitudes of voltage U_ω and current I_ω can be divided by each other, in order to calculate complex impedance (admittance) for the frequencies present in perturbation signal.

IV. Implementation of method

The filter bank method of accelerated impedance spectrum measurement has been examined by means of simulation in Matlab using simulator of transient states for models defined in s-domain.

A. Circuit Under Test

As a test engine for simulation and experimental verification of method, a simple 3 element two terminal network (Fig. 3) has been chosen. It is widely used to test and compare impedance spectrum estimation methods.

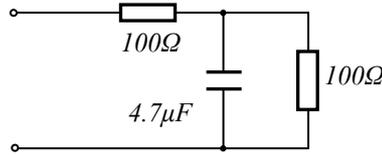


Fig. 3. Circuit Under Test.

The object impedance and admittance in s-domain are:

$$Z(s) = \frac{(R_s R_p C_p)s + (R_s + R_p)}{(R_s R_p C_p)s + 1}, \quad Y(s) = \frac{(R_s R_p)s + 1}{(R_s R_p C_p)s + (R_s + R_p)}. \quad (5)$$

The CUT parameters have been chosen to obtain significant impedance spectrum changes in 2 decades frequency range: from 100Hz to 12.8kHz. Magnitude and phase of impedance is presented in Fig. 4.

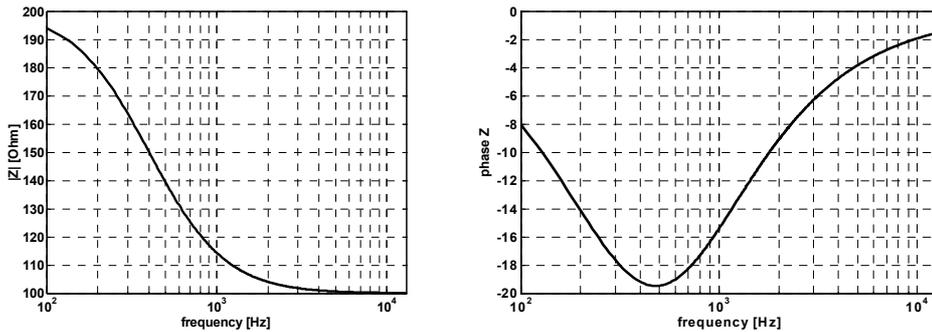


Fig. 4. Circuit Under Test impedance spectrum.

B. Multisine stimulus

The filter bank method has been tested with 3 multisine stimulation signals: S1, S2 and S3, designed to measure impedance in 2 decades range, from 100Hz to 13kHz, with 14 sinusoids. Sampling frequency has been assumed 128kHz. A dedicated frequency optimisation algorithm has been used to obtain integer number of samples for every component in stimulation signals.

Table 1: Stimulation signal S1 – sum of 2 linearly spaced multisines.

f [kHz]	0.101	0.203	0.304	0.409	0.510	0.707	0.815	1.008	2	4.129	6.095	8533	9.846	11.64
T_k	1268	631	421	313	251	181	157	127	64	31	21	15	13	11

First stimulation signal examined is a sum of two linearly spaced multisines. It was formed by two arithmetic progressions: with $\sim 200\text{Hz}$ step in frequency range 100-1000Hz and with $\sim 2\text{kHz}$ step in 2kHz to 12kHz range. The periods of signal S1 components are not in relation with each other – none of them is a divider of the other one.

Table 2: Stimulation signal S2 – optimised sum of 2 linearly spaced multisines.

f [kHz]	0.1	0.2	0.3	0.4	0.5	0.699	0.8	1	2	4	6.095	8	9.846	11.63
T_k	1280	640	427	320	256	183	160	128	64	32	21	16	13	11

Signal S2 is LCM-optimised version of S1. Although the multisine frequencies are similar to S1, 7 of 14 component periods have common multiple – the T_l period. It is desirable to have a small LCM of multisine periods, as the LP filtering in filter bank works best with LCM length filters.

Table 3: Stimulation signal S3 – alternated 2 logarithmically spaced multisines

f [kHz]	0.1	0.2	0.25	0.4	0.5	0.8	1	1.6	2	3.2	4	6.4	8	12.8
T_k	1280	640	512	320	256	160	128	80	64	40	32	20	16	10

The S3 signal frequencies are different: they were formed by alternating two $q=2$ geometric progressions with $a_i=10$ and $b_j=16$. The signal was designed to realize fully the LCM-criterion: a block of 2056 samples contains an integer number of periods of all components in multisine stimulus.

C. Low-pass filter

The Low-Pass filter has to be used to measure the DC value at output of modulator. Although the task seems to be frequency domain related, there is a need to use a time-domain optimised filter. The main goal is to accelerate impedance measurement, so it is required to calculate the DC value at output of filter as soon as possible – the filter with fast impulse response and sharp step response is demanded.

It was decided to use a Finite Impulse Response (FIR) filter, due to the stability, performance and ability to forecast filter output stabilisation time (equal to filter length) [8]. As the Moving Average (MA) filter has been chosen, the main disadvantage of the FIR filters, a slow execution time is circumvented, due to possibility of MA recursive implementation:

$$y[n] = \frac{1}{M} \sum_{k=0}^{M-1} x[n-k] \quad \Leftrightarrow \quad y[n] = \frac{1}{M} x[n] - \frac{1}{M} x[n-M-1] + y[n-1]. \quad (6)$$

In order to achieve measurement acceleration, the filter length M is limited by single-frequency measurement time (1). The MA-filters with length M equal to T_l and $2T_l$ have been tested, where T_l is period of lowest frequency component in signals S1, S2, S3. The signals at output of current filter-bank channels ($M=2560$, signal S3) and single voltage channel are presented in Fig. 5.

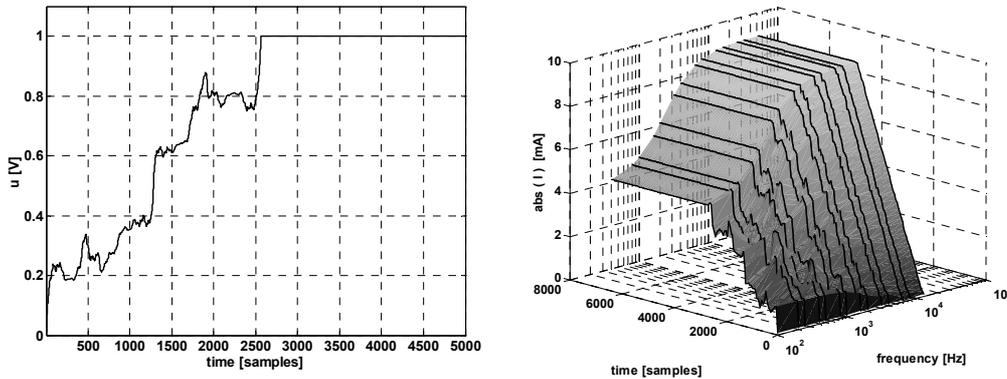


Fig. 5. Output of voltage (left) and current (right) LP filters.

It can be seen, that after feeding the filter with number of samples equal to filter length (2560), the filter bank channel output stabilises – the DC value of signal, equal to complex amplitude of selected frequency can be measured. In Fig. 5 the stimuli voltage 1V is calculated properly, same as current magnitude, changing from 5mA to 10mA.

V. Results

The impedance spectra reconstruction has been conducted for 3 stimuli S1, S2 and S3. All three signals allowed measuring the impedance faster than single frequency DFT method.

Table 4: Time necessary to measure 2 decades wide impedance spectrum of CUT.

Signal	Measurement time [samples]		Acceleration ratio	
	single frequency DFT	filter bank		
S1	sum of 2 linearly spaced multisines	≥ 3504	1268	2.7634
S2	optimised S1	≥ 3551	1280	2.7742
S3	2 logarithmically spaced LCM multisines	≥ 3558	1280	2.7797

The results are presented on 3D plots. Every plot shows the magnitude and phase of reconstructed impedance spectrum, and for clarity, the time axis has been scaled in milliseconds and the filter stabilisation period (equal to measurement time) is not shown. The variation of measured values in time afterwards the filter stabilisation is caused by insufficient filter selectivity for non-LCM signals. Despite the expected DC value, influences of multiplication of other multisine components in modulator can be seen at filter output.

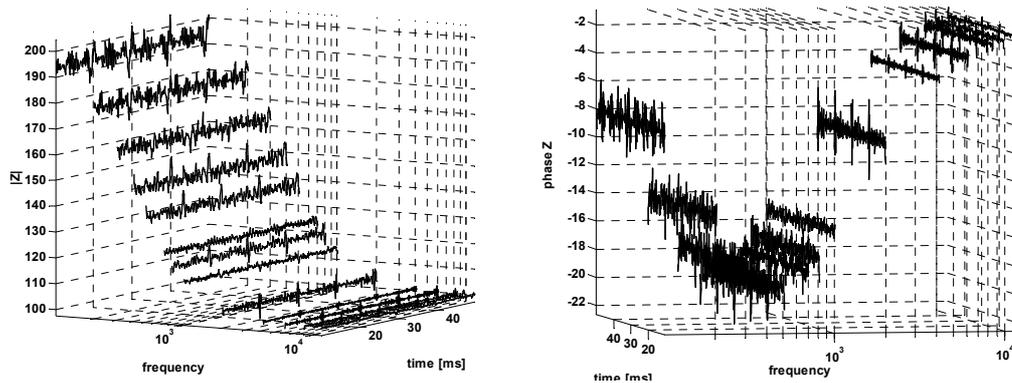


Fig. 6. Reconstructed impedance spectrum – signal S1.

The results for S1 signal are presented in Fig. 6. It can be seen, that although both magnitude and phase characteristic of CUT have been reconstructed similar to Fig. 4, the selectivity of MA filter has been insufficient for using stimuli S1 – the output of filter fluctuates. However, the magnitude error does not exceed 10% while the measurement time is 2.7 faster than single frequency DSP method.

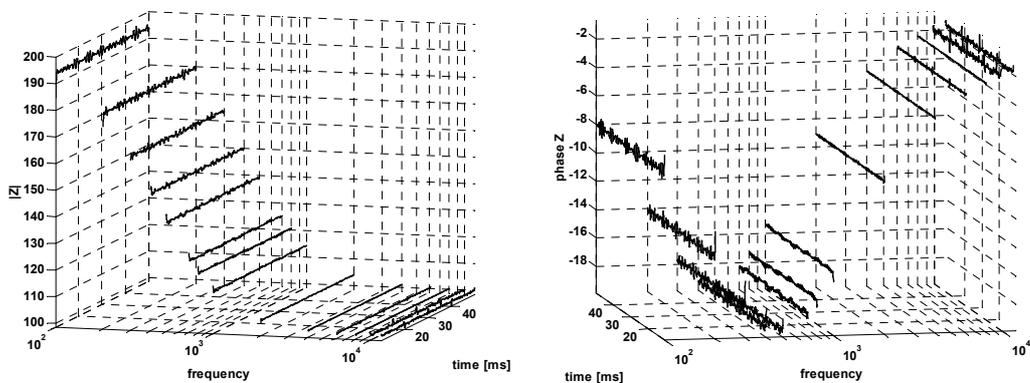


Fig. 7. Reconstructed impedance spectrum – signal S2.

The situation is much better, when the optimised S2 stimulus is applied. The magnitude of Z is calculated with error less than 4%, while maintaining the measurement acceleration ratio 2.77.

The frequencies in signals S1 and S2 are quite undistinguishable, thus showing the need of multisine signal shape-design. In both cases, using longer MA filter did not improve the selectivity.

The results for signal S3 show the superiority of LCM signals. The MA filter calculates the DC value at output of filter without fluctuations, as the length of filter is common multiple of component periods, equal to $2T_l$ (2560 samples). However, single frequency method also takes longer time for signal S3 and the measurement acceleration ratio is kept the same: 2.77.

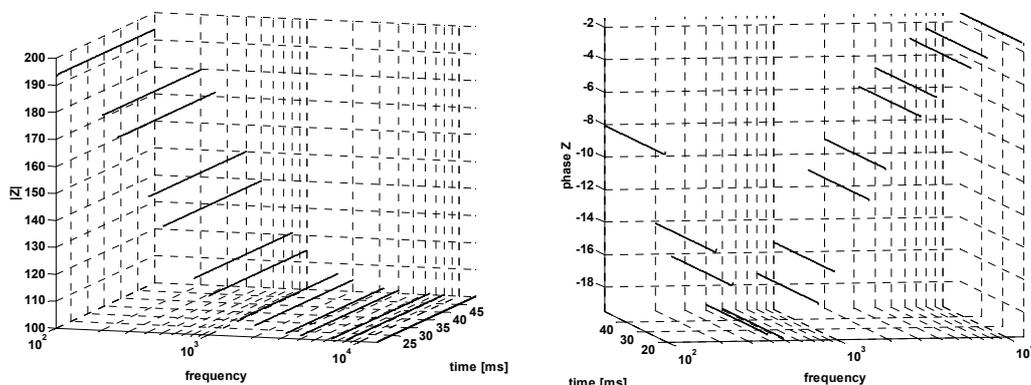


Fig. 8. Reconstructed impedance spectrum – signal S3.

VI. Conclusions

Numerical simulations in Matlab environment have confirmed the possibility of accelerating impedance spectrum measurements via applying multisine perturbation and filter bank analysis of stimulation and response signal. The Filter Bank approach to multisine impedance spectrum measurement has been proved to be superior to single frequency DFT method – the presented methodology allows performing single-shot impedance spectrum measurement more than two times quicker.

Although best results are achieved with LCM signal (S3), where the MA filter length is equal to LCM of periods of all multisine perturbation components, the method can be used with arbitrary selected frequencies. However, in that case its accuracy is limited by low-pass filters selectivity.

The simulations and calculations have shown, that the method is sensitive to distribution of frequencies in stimulation signal. That leads to a conclusion, that multisine stimulation signal should be shape-designed: frequency and phase optimisation algorithms need to be developed.

Furthermore, in order to improve accuracy of method for non-LCM signals, consecutive low-pass filters and their influence on measurement accuracy should be compared by means of numeric criteria.

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