# Calculation of Uncertainties in Harmonics Determination

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***Abstract***- The main purpose of the paper is to present a new approach to calculation of uncertainties in harmonic contents, which are induced due to imperfection of acquired values of samples. The acquired samples are representing true values but every sample is bearing errors caused by all front end elements wired to Analogue to Digital Converter, ADC, and errors of ADC converter itself. The method is similar to uncertainty distribution through measurement model, but the difference is in generation of distribution. The distribution is not generated for each sample itself and each all possible variants are calculated, but all variants for 10 millions of points distributed uniformly, normally and combination of normally and uniformly were applied. The results are very promising from the point of view of time consumption for calculations. Something a bit unrespectable at the first glimpse, the result, results seems to be a bit surprises as uniform probability distribution of errors at the input is resulting with similar to normal distribution at the output of Fast Fourier Transform procedure. It was proved applying test data of one single, just only fundamental harmonic. The acquired data of current consumption in a real electrical power net will be presented in a full appear.

## I. Input samples, their imperfection – probability distribution functions of every input data

The samples are not perfect, but are representing measured values at the instant moment. Samples are bearing errors, of which distribution and limits of values of errors may be calculated based on declaration of manufacturers of used elements. These elements are the very front elements of measuring part of the measurement system may be recognised a multiplicative and additive components to every instantaneous value.
In Fig. 1 the raw data are presented and in the Fig 2 data after pre-processing to define how errors are distributed in sampled data. The two components can be distinguished: one related to type A and the second related to type B, classifications according to GUM.

The values gain from real system were used as errors which are corrupting a ideal signal, as errors, as errors scattered around acquired every data. Uniform distribution and Normal distribution were assumed as, and looking at the histogram in Fig. 3 let say that "more or less" we may agree. This not a point in this paper to identify real shapes of probability distributions functions.

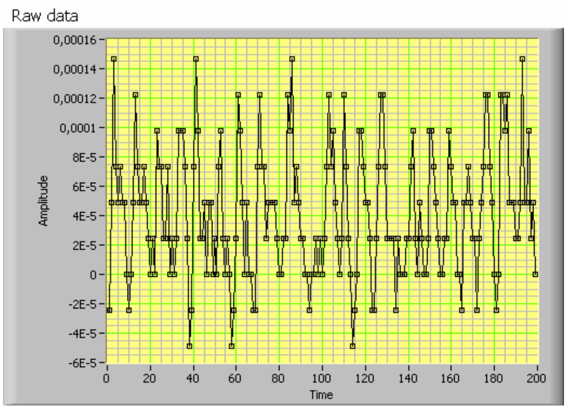



Fig. 1 Raw data with marked uncertainty interval for samples at confidence interval of 95

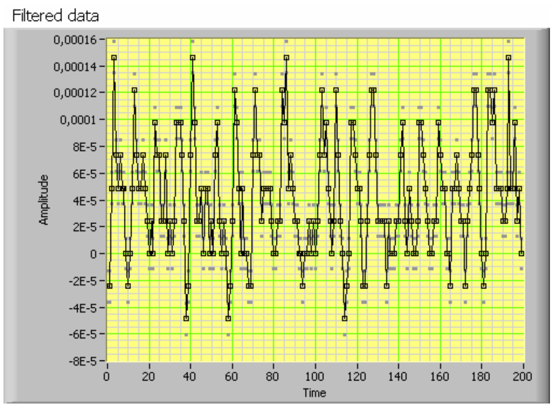



Fig. 2 Filtered data (prepossessed) with marked uncertainty interval for samples at confidence interval of 95

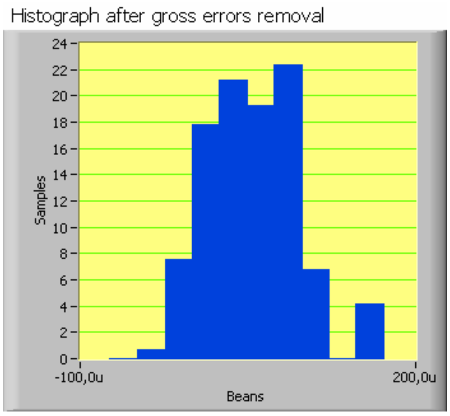



Fig 3. The histogram which manifests that the probability distribution function is a result of convoluted two components: uniform distribution and random effect

## II. Distribution of uncertainties through FFT procedure

Although the FFT algorithm is very well known, still the problem of calculation of uncertainties in harmonic contents is under deliberation, as the calculation of uncertainties is a hot subject itself.
The author is widely applying Law of Propagation of Uncertainty through measurement model to calculate uncertainties, but in this case, calculation of propagation of uncertainties in the measurement model through FFT procedure is not so convenient as the Monte Carlo Method.

The uniform distribution of errors and Normal and combination of both distributions seems the most basic and the most likely at the input. Such three cased of distributions were tested over a very simply input signal – caring one single fundamental harmonic, in order to easy conclusion if any other harmonic appears at the output of FFT. It will allow to make conclusions of evidence of being induced harmonics, which are unwanted.

## III. Results of calculation of uncertainties due to FFT procedure

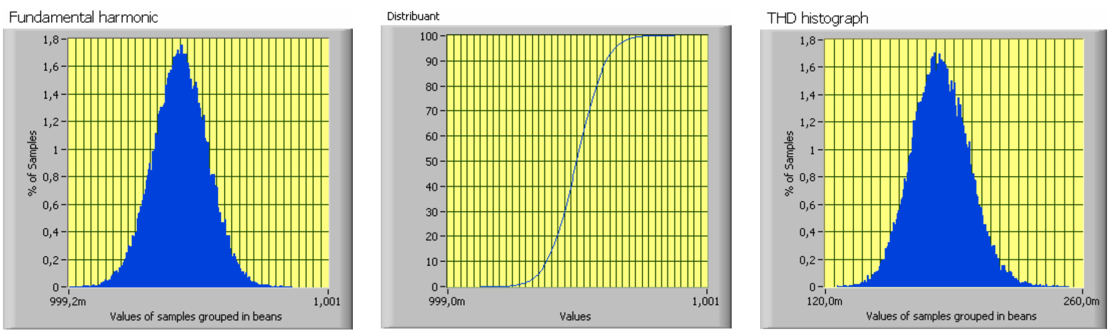
The example was calculated for the very basic signal – just one fundamental harmonic of 50 Hz and unit amplitude was tested within a window 200 ms, 5 Hz resolution. It is not difficult to predict, that the input data contaminated by errors of which distribution is a consequence of imperfection of instrumentation before data were acquired for processing, the components of 5 Hz will be generated at the harmonic contents. The question is, how the error which at the input is although uniform or normal shape of probability density function, are effecting on harmonic contents.

Fig 4 representing Histogram (a), cumulative histogram(b) and histogram of THDrms (c) for uniformly distributed error over samples.

Fig 5 representing Histogram (a), cumulative histogram (b) and histogram of THD rms (c) for normally distributed error over samples.

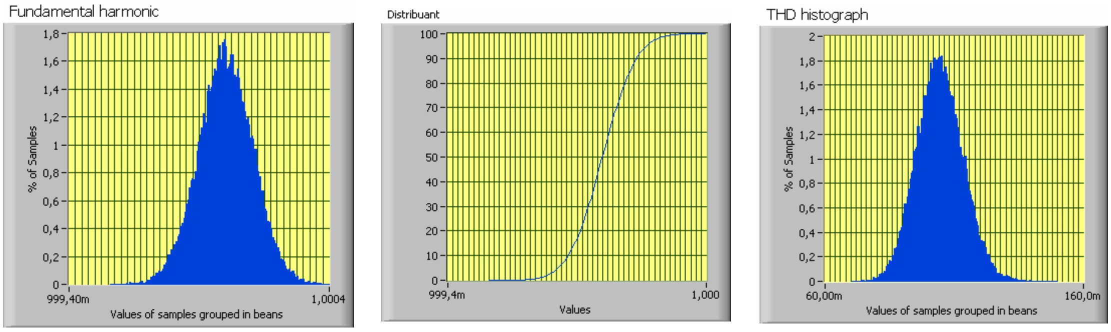
Fig 6 representing Histogram (a), cumulative histogram(b) and histogram of THD rms for uniform distributed error.

The calculation of harmonic contents were completed for 10 mln of randomly distributed values over 2048 point performed FFT with a 5 Hz resolution.



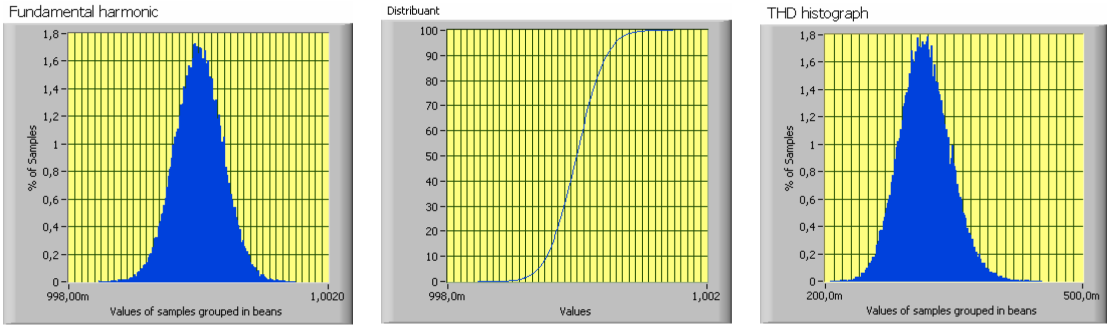

(a) (b) (c)

Fig 4 representing Histogram (a), cumulative histogram(b) and histogram of THD rms for uniformly distributed error over samples. For 95 % of coverage probability the coverage interval for fundamental harmonic is <0,9993; 1,0007> (mean value equals to true value which is 1) and for THD rms coverage interval is <0,13 0,25> mean value 0,18%




(a) (b) (c)

Fig 5 representing Histogram (a), cumulative histogram(b) and histogram of THD rms for Normally distributed error over samples. For 95 % of coverage probability the coverage interval for fundamental harmonic is <0,9996- 1,0004> (mean value equals to true value =1) and for THDrms coverage interval is 0,073 - 0,15 mean 0,10%




(a) (b) (c)

Fig 6 representing Histogram (a), cumulative histogram(b) and histogram of THD rms for uniform distributed error for 1 % od additive and 2 % of multiplicative components of errors distributed over samples. For 95 % of coverage probability the coverage interval for fundamental harmonic is <0,998 - 1,002> (mean value equals to true value which is 1) and for THDrms coverage interval is <0,21 - 0,45>, at mean 0,32 %.

## IV. Conclusions

Uniformly distributed errors in acquired samples are resulting by a distribution which is Normal distribution at the harmonic contents for both fundamental, DC and any other element of harmonic content, but for a very simple signal only.

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