Measurement of Phase Difference Using DSP Algorithms by Non-Coherent Sampling

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Abstract- The paper compares nine methods of measurement of the phase difference of digitized sinusoidal noised signals according to their sensitivity to sampling non-integer number of signal periods. The investigated methods can be classified into four groups - modifications of classical zero-crossing based measurements, DFT-based measurements, virtual vector voltmeter and modifications of sine-wave fit algorithm. Results of both simulations and measurements are presented.

I. Introduction

Measurement of phase difference of two harmonic signals is used e.g. by measurement of phase frequency characteristics of linear circuits or measurement of impedances. The classical approach is based on detection of zero crossings of signals [1], [2]. Other methods are based on virtual vector voltmeter, on DFT (e.g. [3]) and on sin-wave-fit algorithms (e.g. [4]). Altogether 9 methods based on the above mentioned principles were investigated. The methods are compared from the points of view of their sensitivity to sampling non-integer number of signal periods ("non-coherent sampling"). The paper is in a sense a continuation of [5] where comparison was based on coherent sampling and windowed and interpolated DFT [6, 7] and a recently published modification of sine-wave-fit [8] were not included. MATLAB environment was used for both simulations and measurements.

II. The investigated methods

A. Methods based on zero-crossings detection (ZCRF and ZCRR)

Detections of crossing a zero level by both the measured signals is used for phase difference estimation e.g. using universal counters or dual-channel oscilloscopes. Its principle is shown in Fig.1 and in (1). Local linear interpolation around zero crossings allows resolution increase. Signal pre-processing before detection of crossings of zero level is required to prevent additional zero-crossings caused by noise and higher harmonics. We have investigated two types of this signal pre-processing – LP filtering (IIR or FIR) and moving averaging (method denoted here "ZCRF") or moving average and linear regression in surroundings of zero crossings (method "ZCRR").

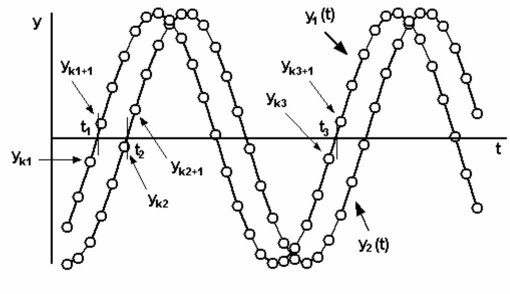

Fig.1 Principle of phase difference estimation by zero crossings detection

$$\varphi = 2\pi \frac{t_2 - t_1}{t_3 - t_1} \quad \text{(rad)} \tag{1}$$

B. Method based on virtual vector-voltmeter (VVV)

This method uses multiplication of the signal by a reference signal (sinusoid of the same frequency as frequency of the measured signals) and finding mean value (DC component) of this product. This mean value is proportional to cosine of the phase difference between the fundamental harmonic component of the measured signal and the reference signal, the phase shift of which is considered to be zero. This mean gives also the real part of the phasor of the measured signal. Multiplying the signal by cosine reference signal (i.e. reference signal shifted by 90° with reference to the first one) and finding the mean (e.g. by low-pass filter) gives the imaginary part of the phasor $\mathbf{U}_1$ of measured signal. Phase difference of the measured signal and the reference signal can be found afterwards by means of $arctg(\mathrm{Im}\mathbf{U}_1/\mathrm{Re}\mathbf{U}_1)$ functio. Using the same procedure for the second measured signal allows finding the phase difference between the second signal and the reference signal. Difference of phase shifts of the both signals referred to reference signal is the measured phase difference (phase difference of their fundamental harmonic components in case of sinusoidal signals distorted by higher harmonic components). The both signals' amplitudes are found together with phase difference.

C. Methods based on DFT (DFT, IDFT)

The phase difference between fundamental harmonic components of the two measured signals is found as the phase difference of the fundamental harmonics DFT phase spectrum values (method "DFT"). Owing to the spectral leakage in case of non-coherent sampling, signal windowing (using the Hann window or higher-order Rife-Vincent windows of the first class) and using frequency-domain interpolated DFT allows to gain better results (method "IDFT"). As is well known, DFT is in practice computed by FFT algorithms and frequency, amplitude and phase can be found using this method.
Our intention was to investigate also two modifications of the two-point DFT interpolation in frequency domain without signal windowing (i.e. using rectangular windows) published recently [9] but results of our programming the interpolation algorithms presented in [9] were not satisfactory and are therefore not included here.

D. Methods based on sine-wave-fits (SWFM, SWF3p, SWFF4p, SWFR)

A powerful tool for phase difference measurement are the sine-wave-fit techniques, based on least-square error (LSE) between (the samples of) the measured signal and (samples of) an ideal sinusoid. This sinusoid is characterized by 3 (magnitude, frequency and phase) or 4 (nonzero DC component added) parameters found for getting minimum LSE. If applied on both the measured signals, phase difference is obtained as the difference of phases of the both measured signals. These methods provide apart from phase also values of signal amplitude, frequency and in case of 4-parameter methods also the DC component, and are described in [4] (method designed especially for measurement of phase difference -"SWFM") or in the standard [10] (realization of this method named here "SWFF3p" and "SWFF4p" use MATLAB function *fminsearch* for finding minimums of functions of several variables with 3 or 4 parameters). Recently the 7-parameters sine-wave-fit modification supposing the same frequency of both signals and finding all 7 parameters of the two sinusoids with identical frequency was published [8] (method denoted here as "SWFR"). All 7 parameters are being found in a common iteration process, and this method is compared in [8] with 3 and/or 4 parameter sine-wave-fit procedures applied separately to each measured signal.

III. Graphical user interface used

Since many simulations and measurements had to be performed for comparison of the sensitivity of the above mentioned methods to different influences, a graphical user interface was designed in MATLAB allowing easy modification of parameters of signals, range and resolution of the ADC (in simulations), number of signal samples processed and sampling frequency (desired non-integer number of periods can be selected), noising the signals by additive noise for selected SNR and distorting signals with higher-order harmonic components (up to the order 50, with random phases and magnitudes related to the fundamental harmonic components according to compatibility levels given by the international LF EMC standards (e.g. [11])). This GUI allows selection of methods to be applied for given simulation or measurement and selection of quantity that should be used as independent variable for presentation of results. The output graphs can be easily modified in MATLAB, and 7 different output figures are at the user disposal after finishing the measurement or simulation.

IV. Results of simulations

Many simulations were performed, but only some results concerning the comparison of the investigated methods from the point of view of their sensitivity to sampling of non-integer number of signal periods ("non-coherent sampling") are presented here because of lack of place. All figures are shown for 12-bit signal quantization, both signals magnitudes 5 V, zero signals DC components, frequency 50 Hz and phase difference 50 deg. Number of repeated simulations was 50, sampling frequency 6400 Hz (128 Sa/per), *SNR* = 30 dB, *THD* = 0 (resp. 0.04% corresponding to the *THD* of the signals generated by generators in measurements – see section V) and for 4 to 12 periods sampled. The investigated methods results of which are shown in Fig.2 to Fig.7 are grouped according to Section II into three groups (method VVV was added to DFT, since their results are identical; the reason of this at first sight surprising fact is explained in section VI –see [5]).

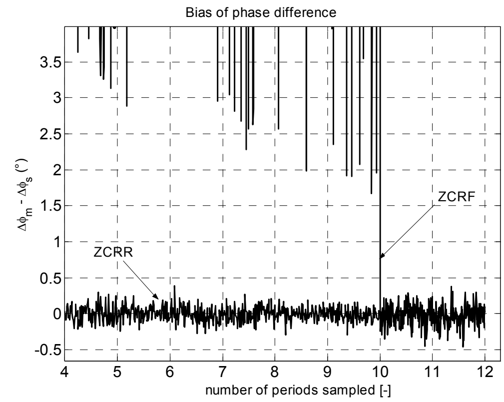

Fig.2 Bias of ZCRR and ZCRF methods

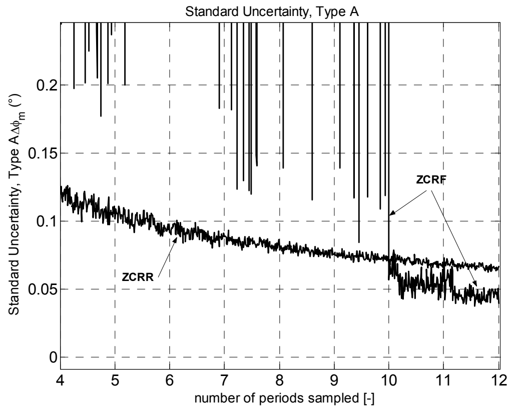

Fig.3 Standard deviation of ZCRR and ZCRF methods

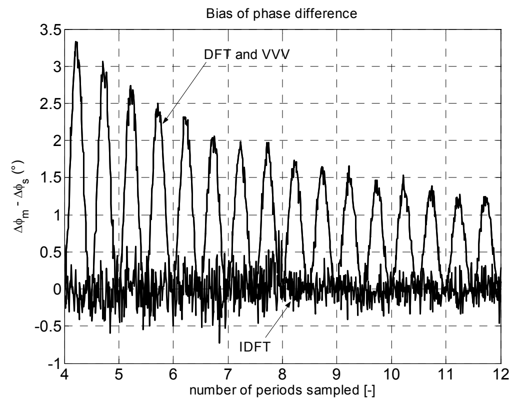

Fig.4 Bias of DFT, VVV and IDFT methods

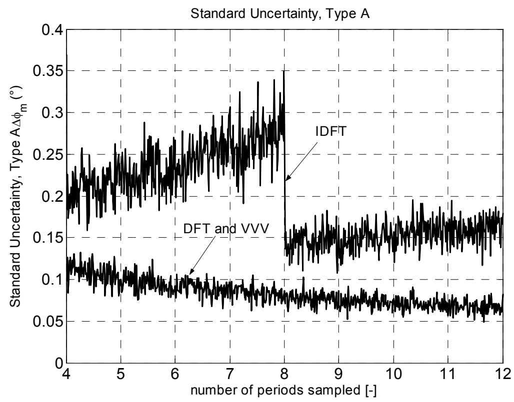

Fig.5 Standard deviation of DFT, VVV and IDFT methods

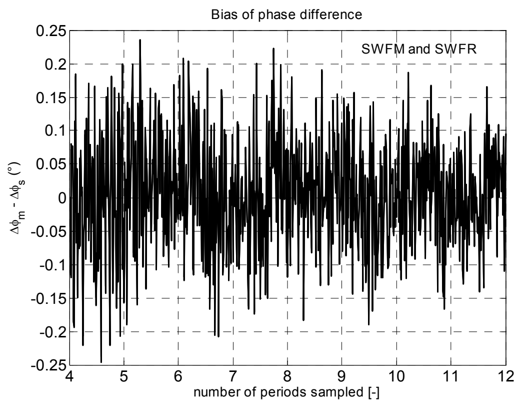

Fig.6 Bias of SWFR and SWFM methods

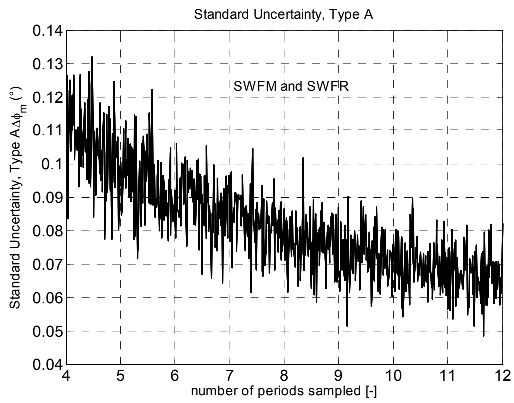

Fig.7 Standard deviation of SWFR and SWFM methods

V. Results of measurements

Measurements were performed on sinusoidal signals generated from two 20 MHz function/ arbitrary generators Agilent 33220A, equipped with external frequency reference and phase-locked loop options, allowing phase adjustment via computer interface. The generators were controlled via GPIB interface in MATLAB (using Instrument Control Toolbox). *THD* of generators is 0.04 % and there was very low additive noise, since coaxial cables were used for signal connections. The both measured signals had frequency 50 Hz, amplitudes 5 V and no DC component and their phase difference was set to 50 deg, number of measurement the results of which are presented in Fig. 8 to Fig. 13 is 10.
The measurement results therefore correspond to the case of practically pure sinusoid contrary to simulations (simulations were performed for comparatively large external noise *SNR* = 30 dB). No LP filter was here used in ZCRF method in addition to moving average and no change in signal processing algorithm for sampling more than 10 signal periods was used.
Both the measured signals were sampled by a 12-bit multifunction DAQ PC plug-in board National Instruments NI 6023E (usable for maximum sampling frequency 100 kHz), using a multiplexer on its front end and a one common ADC so that the measured signals were sampled with frequency equal to half of the sampling frequency set on the plug-in board. The board sampling frequency was therefore selected to be twice the sampling frequency used in simulations, and was 12800 Hz (256 Sa/per). The plug-in board setting and signal sampling was controlled by means of Data Acquisition Toolbox of MATLAB.
Methods SWFF3p and SWF4p are not included in Fig 12 and Fig. 13, their results were worse than results of the other sine-wave fit methods (SWFM and SWFR) for given sampling frequency and their computation time is much higher than by the rest of the investigated methods. For the used sampling with 128 Sa/per their results had bias about 1.4 deg (see Fig. 14), which corresponds very good with the dependence of their results on sampling frequency (Fig. 15).

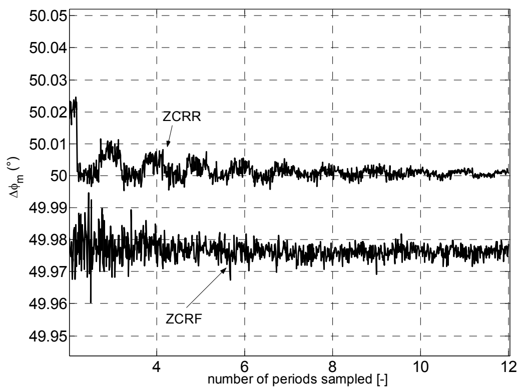

Fig.8 Phase difference by ZCRR and ZCRF methods

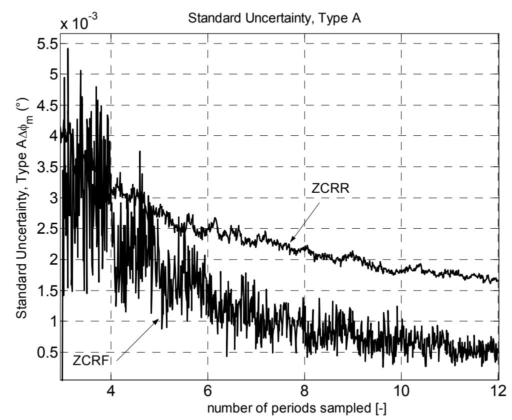

Fig.9 Standard deviation of ZCRR and ZCRF methods

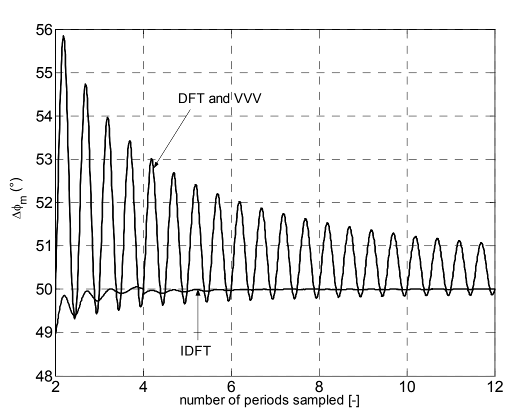

Fig.10 Phase measured by DFT, VVV and IDFT methods

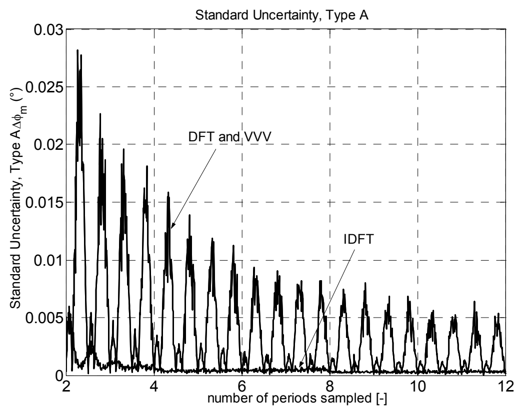

Fig.11 Standard deviation of measurements by DFT, VVV and IDFT methods

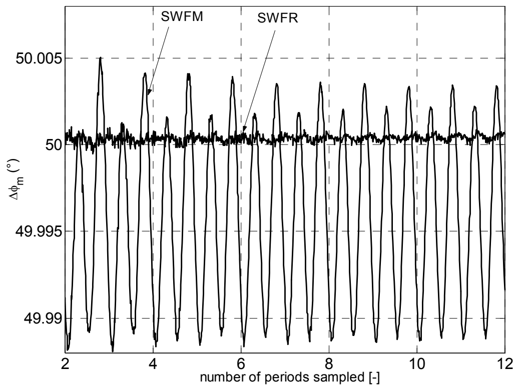

Fig.12 Phase measured by SWFM and SWFR methods

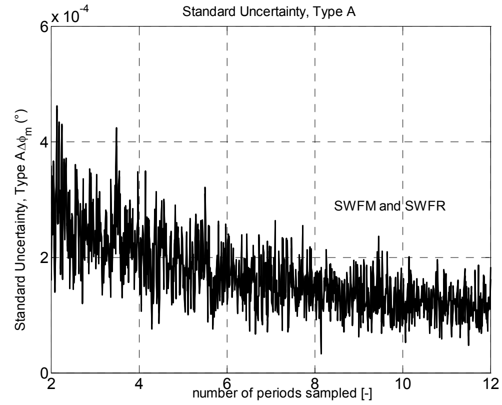

Fig.13 Standard deviation of measurements by SWFM and SWFR methods

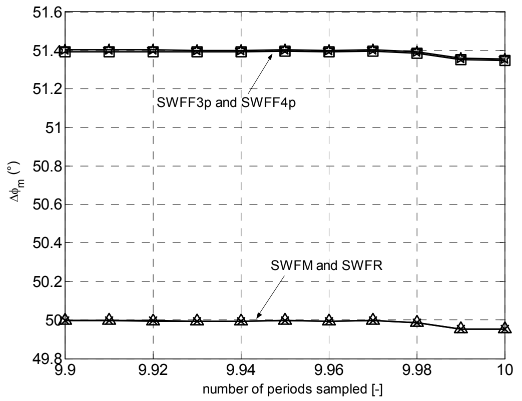

Fig.14 Phase difference by SWFF3p and SWFF4p methods (SWFR shows the true value), measurement

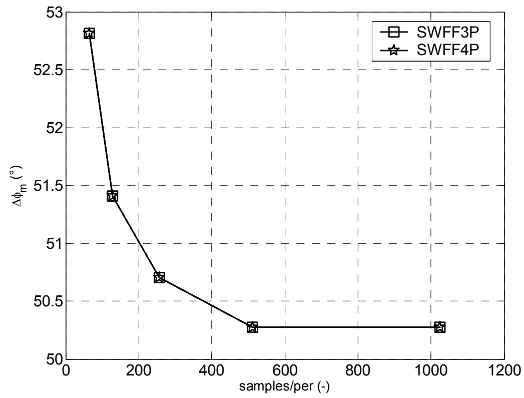

Fig.15 Influence of number of samples on SWFF bias (phase difference is 50 deg, measurement)

The figures Fig.8 to Fig. 13 present the same methods as Fig.2 to Fig.7 for simulations, but since the true phase shift is not known in general by measurements (and the MATLAB program for measurement is respecting this fact), measured values of phase difference are presented here instead of the bias of the phase difference shown by simulations.

VI. Conclusions

As can be seen from the figures above, methods DFT and VVV are not suitable for non-coherent sampling (sampling not integer number of periods). Sensitivity of the DFT method to sampling non-integer number of periods (i. e. to leakage effect) is substantially reduced by signal windowing and interpolation (see IDFT method results). Bias caused by leakage is decreasing with number of periods sampled. The results of IDFT method shown in Fig. 4, 5, 10 and 11 are plotted for Hann window used. If higher-order windows were used for signal windowing in IDFT method, so many signal periods have to be sampled that the main lobes of window spectrum placed at frequencies of neighbouring signal harmonic components do not overlap.

Methods DFT and VVV give the same results, since mathematical processing of signal samples is in fact the same for these methods [5] (multiplication of signal sample sequence by exponential with imaginary exponent by DFT computation corresponds to multiplication of signal by sinusoidal and cosinusoidal reference signals by VVV method for computation of real and imaginary parts of signal phasor). Therefore both graphs of phase difference and standard deviations (type A standard uncertainties) of these methods overlap in all figures.

Method of ZCRF used with LP FIR filter to suppress the additive noise before zero-crossing detections is usable in our implementation only for more than 10 periods sampled (see Fig. 2 and Fig. 3) to exclude the filter output transient. If used without additional signal pre-filtration, this limitation is not applied (see Fig. 8 and Fig. 9).

The presented results of simulations and measurements show that from the point of view of *measurement bias* the sequence of the investigated methods is for non-coherent sampling for the same sampling frequency and number of samples ordered from the best one to the worst one following: SWFR, SWFM, ZCRR, IDFT (depending on number of periods sampled and window order used in signal windowing), ZCRF, SWFF3p and SWFF4p.

As can be seen from the figures above, from the point of view of *measurement uncertainty (type A)* the sequence of the investigated methods is for non-coherent sampling is similar to the sequence ordered by phase bias.

From the point of view of *computation time* (programmed in MATLAB) the sequence of the investigated methods is approximately (from the best one to the worst one, computation times given in relative scale): SWFM (1), DFT (1.1), VVV (1.6), SWFR (11), ZCRF (16), IDFT (36), ZCRR (59), SWFF3p (350), SWFF4p (610).

The slowest – SWFF - methods require moreover larger numbers of samples to reach bias comparable to other methods (see Fig. 14 and Fig. 15).

Sensitivity of the investigated methods to the *additive noise* can be seen by comparing figures from section V (pure sinusoid) to those from section IV (SNR = 30 dB).

The sine-wave-fit methods SWFR [8] and SWFM [4] are the best ones from the compared methods from the point of view of both measurement bias and type A uncertainty both for sampling integer and non-integer number of periods. The SWFM method is at the same time the fasted one (by programming the methods in MATLAB).

Acknowledgement

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