# Constant Passage Technique for Undersampling Method with Two Sampling Rates

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***Abstract*-**The present paper deals in the first part with a presentation of the mathematical algorithm proposed, as an undersampling method. In the second part of the paper, the constant passage technique developed as critical area cancellation is presented.

## I. Introduction

Sampling rate parameter specifies how often conversion can take place. Using a faster sampling rate it is acquire more points in a given time, providing a better representation of the original signal. As shown in Figure 1, it is most sample all input signals at a sufficiently fast rate to adequately reproduce the analogue signal.

Obviously, if the signal is changing faster than the data acquisition board is digitising, errors are introduced into the measured data. In fact, data that is sampled too slowly can appear to be at a different frequency. This distortion of the signal is referred to as aliasing. (Figure 2).

According to the Shannon theorem, it is must sample at least twice the rate of the maximum frequency component in the signal to prevent aliasing.

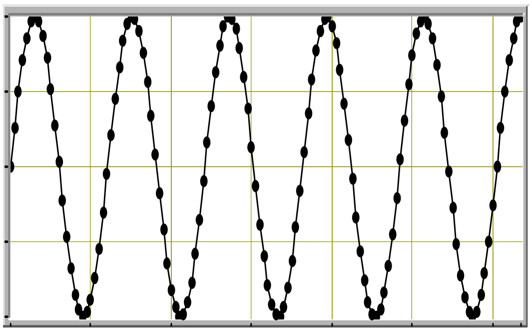

Figure 1. Adequately sampled.

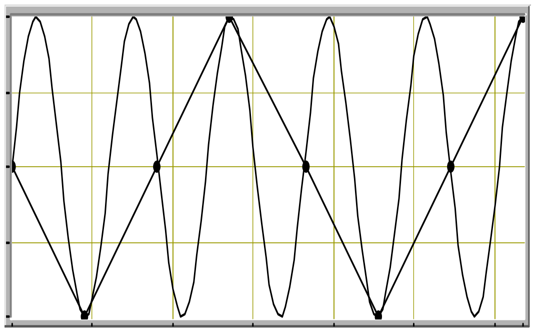

Figure 2. Aliased due to undersampling.

This application is able to calculate the frequency of original signal using the information provided by alias area. The real advantage of the method presented in this paper in fact than it is not necessary knows anything about the frequency of the signal.

## II. Mathematical algorithm

In the mathematical algorithm propose, using a two samples rate (Fe1, Fe2) performs the signal acquisition. The signal frequency (Fs) is up to half the maximum sampling rate (Fs>Femax/2).

The utile information are derived from the alias area. To obtain a large measurement field it is necessary to be a correlation between samples rate. The graphically study performed denote the optimum is Fe2=0.99Fe1.

In order to obtain the information concerning the investigated signal, two samples rate is necessary (Fe1, Fe2). After sample process is obtained two apparent signals, (F1, F2) which are processing in the mathematical algorithm. Practically, for each frequency Fs a pair (F1, F2) is corresponded.

Correlation between investigated signal (Fs) and frequencies pair (F1, F2) is shown in the figure 3, for 7· Femax/2 areas.

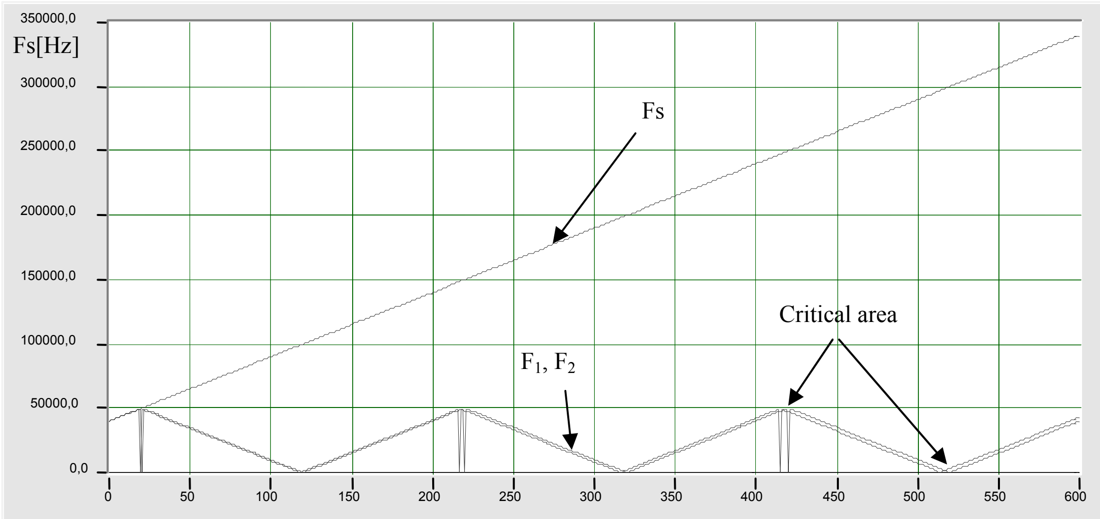


Figure 3. Investigated signal and frequencies pair.

To perform a detail study of this dependence, in the figure 4 it is shown a representative sequence of apparent frequencies scavenging area. This frequencies are scavenge between zero and Fe1/2, respective zero and Fe2/2 in correlation with Fs, thus are obtains field with the value ΔF=F1-F2 as a constant. The fields are separated by critical area.

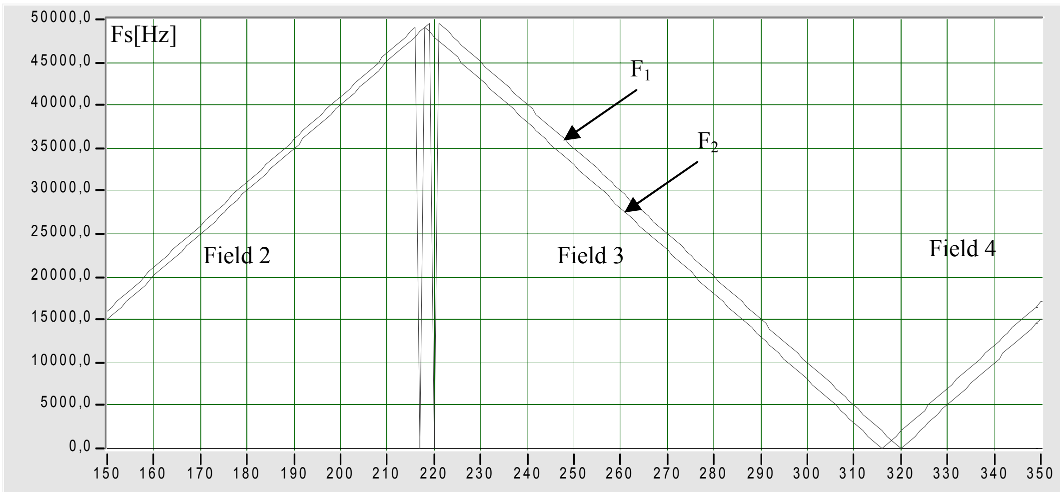


**Figure 4.** Detail of apparent frequencies scavenged area.

The graphical interpretation of apparent frequencies variation involve to perform the mathematical dependent formula Fs= Fs (F1, F2). In order to exemplify this, it is presented the correlation for firsts four fields.

- Field 1: F1-F2= Fe1- Fe2; Fs=Fe1/2+ Fe1/2-F1= Fe1-F1;
- Field 2: F1-F2=-(Fe1- Fe2); Fs=Fe1/2+ Fe1/2+F1= Fe1+F1;
- Field 3: F1-F2=2(Fe1- Fe2); Fs=Fe1+Fe1/2+ Fe1/2-F1=2Fe1-F1;
- Field 4: F1-F2=-2(Fe1-Fe2); Fs=Fe1+Fe1/2+Fe1/2+F1=2Fe1+ F1.

Interpretation of this appearance frequencies and generalization of above relation involve below equivalent formulas for the investigated signal:

$$Fs = \left| \frac{\Delta F}{\Delta Fe} \right| \cdot \frac{Fe_1 \cdot \Delta F - F_1 \cdot \Delta Fe}{\Delta F} \tag{1}$$

$$Fs = \left| \frac{F_1 - F_2}{Fe_1 - Fe_2} \right| \cdot \left( \frac{Fe_1(F_1 - F_2) - F_1(Fe_1 - Fe_2)}{F_1 - F_2} \right) \tag{2}$$

$$Fs = \operatorname{sgn}(\Delta F) \left( Fe_1 \cdot \frac{\Delta F}{\Delta Fe} - F_1 \right) \tag{3}$$

In the above equation is noticing the discontinuity point occurrence, due to denominator cancellation ($\Delta F=0$). In this case it is not possible to calculate the frequency of the original signal. More than that, the discontinuity point is around by critical area.

The critical area weight is congruencies with $\Delta F=F1- F2$, related to the specified field. Periodicity of the critical area is equal with arithmetic mean between one-half the sampling frequencies. The parameter $\Delta F$ is a discrete parameter, it can have only multiples of $\Delta Fe=Fe1-Fe2$ values.

### III. Critical areas cancellations

In order to complete the critical area cancellations, three samples rate are necessary. As in mathematical algorithm, to obtain a large measurement field, it is essential to be a correlation between samples rate. In the critical area, as showed in figure 5, there is a passage with $\Delta F$ constant.

Tacking in to account the marginal samples rate of passage, to establish frequency for investigated signal is possible. In order to calculate frequency of investigated signal, in critical area, a sampling rate pairs selections must be carry out. The procedure consist in selection these sampling rate (two from three) that give maximum in subtractions.

Therefore:

- Position 1 doesn't have critical area, thus (Fe1, Fe2) pair is used;
- Position 2 has critical area for (Fe2, Fe3) pair, thus (Fe1, Fe2) pair is used;
- Position 3 has critical area for (Fe1, Fe3) and (Fe2, Fe3), thus (Fe1, Fe2) pair is used;
- Position 4 has critical area for (Fe1, Fe2) and (Fe1, Fe3) thus (Fe2, Fe3) pair is used;
- Position 5 has critical area for (Fe1, Fe2), thus (Fe2, Fe3) pair is used.

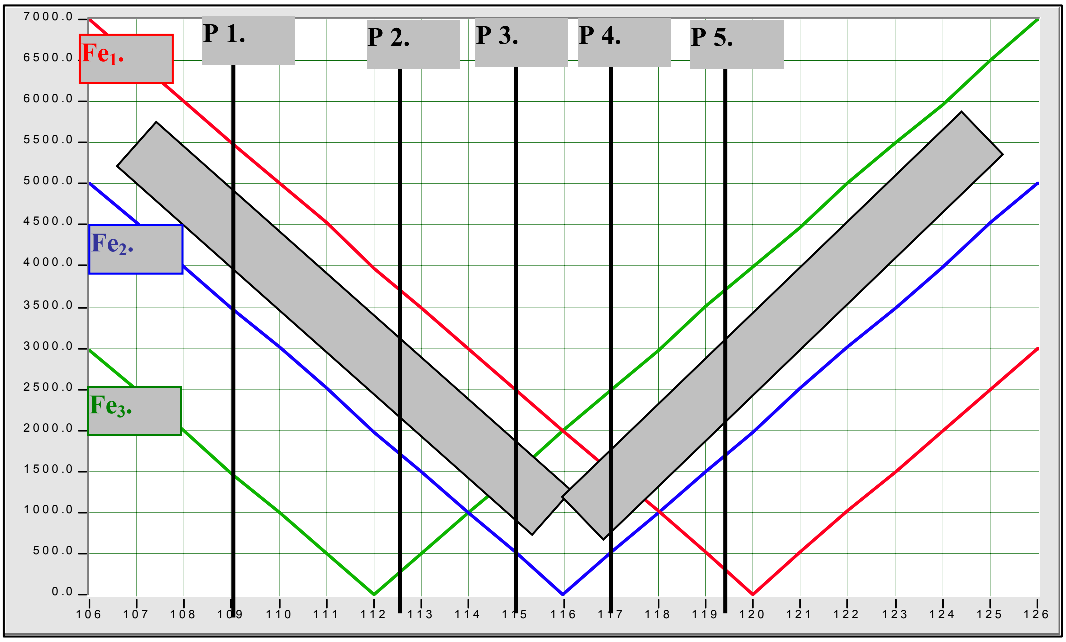


Figure 5. Constant passage method ($\Delta F$=ct)

### IV. Experimental results

The experimental platform is designed as a virtual instrumentation system, including hardware based on data acquisition board and a software component developed in LabView graphical programming language. The data acquisition board used is AT-MIO-16E-10 type with 100kHz max sampling rate.

In this configuration the proposed algorithm is able to measure until 950kHz, thus the high limit frequency imposed by data acquisition board is translated, multiplied by 19 the measurement interval. The amplitude of the input signal is attenuate dint of frequency characteristic amplifier, thus in proximity of 1.3MHz, it does not exist the useful signal at output of amplifier. The maxim relative error is 0.04%, obtained in the field 1. This is the most unfavourable field from all of the fields. A Sony Tektronix AFG 310 numeric generator is used.

## III. Conclusions

This paper presents the mathematical algorithm for an undersampling method, including critical areas cancellations. Method was designed for sinusoidal signals. . It is not necessary knowing anything about the frequency of the original signal. There is no need pick up of one sampling per period or to know the number of samples per period as in the classic undersampling method. In the configuration presented above the application is able to multiply the measurement interval by 19. The maxim relative error is 0.04%.

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