Characterization and Compensation of Dynamic Non-Linearities in Digital Data Acquisition Channels by Means of the Discrete-Time Convolution Model

P.A. Traverso[1], G. Pasini[2], A. Raffo[3], D. Mirri[1], G. Iuculano[4], F. Filicori[1]

[1] *DEIS – Department of Electronics, University of Bologna. Viale Risorgimento, 2 – 40136 Bologna, Italy. Ph.:+39-051-2093080 Fax:+39-051-2093073 E-mail: ptraverso@deis.unibo.it*
[2] *DIE – Department of Electrical Engineering, University of Bologna, Bologna, Italy*
[3] *Department of Engineering, University of Ferrara. Via Saragat 1, 44100 – Ferrara, Italy*
[4] *Department of Electronics, University of Firenze. Via S. Marta, 3 – 50125 Firenze, Italy.*

Abstract- A technology-independent, behavioural non-linear dynamic model, which has been previously proposed by the authors for the accurate experimental characterization of digital acquisition channels, is considered in this paper for the compensation of the overall non-idealities within these measurement sub-systems. It is shown how the non-linear dynamic equations of the model, which inherently describe both the static and the dynamic non-linearities affecting the channel, can be numerically inverted, allowing for the reconstruction of the spectrum at the input of the system from the corrupted samples available at its output. Different recursive algorithms for the solution of the non-linear problem involved into the procedure proposed are described. The choice among these can be accomplished according to the application-dependent best trade-off between numerical convergence robustness and computational cost. Experimental results are provided, which validate the technique proposed.

I. Introduction

The characterization of non-idealities associated with the operation of digital acquisition channels is not only an essential step in order to estimate the performance of these sub-systems when exploited within modern sampling instrumentation and evaluate the uncertainty in the response of the overall system, but can also allow for the compensation of such phenomena, if a *suitable* model of the channel is also identified starting from the empirical investigation. This is a critical issue, since the ever-increasing speed which is requested to the channels introduces the need for modelling approaches, which are not exclusively based on classical data describing the quasi-static input/output relationship of the system (i.e., information about the actual position of code transition levels of ADCs with respect to the nominal ones under slow-varying input excitations) and its linear frequency response. In fact, even though refined methods have been proposed in the recent literature in order to accurately and efficiently describe the static response of ADC devices, non-linear *dynamic* effects should be definitely taken into account for an accurate, exhaustive characterization of the channel, and properly modelled in order to proceed to a successful compensation at its output.

II. The Discrete-Time Convolution Approach

In order to describe the channel non-idealities from a more complete standpoint, which takes into account (along with linear dynamic effects) both the *static* and *dynamic* non-linearities within the system behaviour, the authors have proposed and developed a technology-independent, "black-box" non-linear dynamic model (namely, the Discrete-Time Convolution Model, DTCM)[1-3]. Following this approach, the digital channel with input $s_I(t)$ can be represented as an *ideal* system, which acquires and converts to digital the output of a non-linear dynamic system (Fig. 1). The latter "embeds" the channel overall non-idealities, and can be adequately described by the truncation to the first-order term of a modification [4] of the classical Volterra series, once that the "long-lasting" memory effects (basically due to the input signal conditioning circuitry and the sample/hold process) have been separated from those (non-linear) associated with "short" memory times, which are typical of active electron devices within the channel blocks. As shown in Fig. 2, the response $y(t)$ of the non-linear dynamic system is obtained as the summation of two contributions, the first one ($y^{(S)}(t) = z_0\left[s(t)\right]$, i.e., the zero-order term of the modified series) simply being the experimental static characteristics of the ADC, and the second one being expressed by the following equation

$$y^{(D)}(t) = \int_{-T_A}^{T_B} w\left[s(t),\tau\right]\cdot\left[s(t-\tau)-s(t)\right]d\tau \cong \sum_{\substack{p=-P_A \\ p\neq 0}}^{P_B} w_p[s(t)]\cdot[s(t-p\Delta\tau)-s(t)] \qquad (1)$$

where $w\left[u,\tau\right]$ is the first-order kernel of the modified series, which is non-linearly controlled by the instantaneous values of signal $s(t)$ and integrated along a memory time with "short" (with respect to the typical dynamics of channel input signals [1,4]) duration $T_M = T_A + T_B$. Contribution (1) is a non-linear *functional* with respect to the input signal, which models all those non-linear dynamic effects otherwise not suitably described neither through the static response z_0 (that can be embedded within the overall model according to virtually any approach available in the literature [3]) nor the linear block. Equation (1) also shows the time-domain discretisation which leads to the DTCM operative formulation.

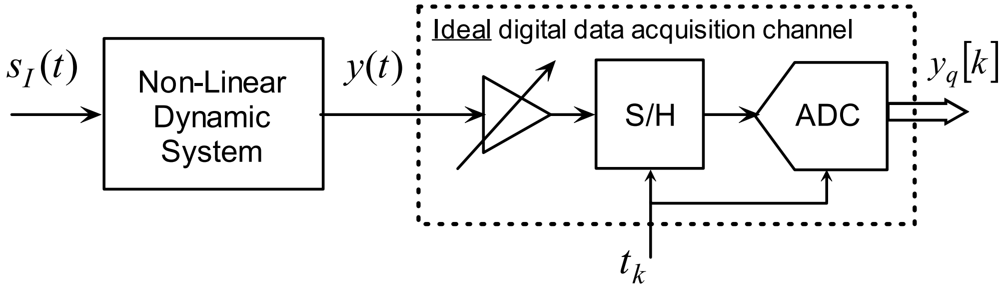

Figure 1. Functional representation of an actual digital data acquisition channel by means of an ideal one, suitably driven by a non-linear dynamic system describing the channel overall non-idealities.

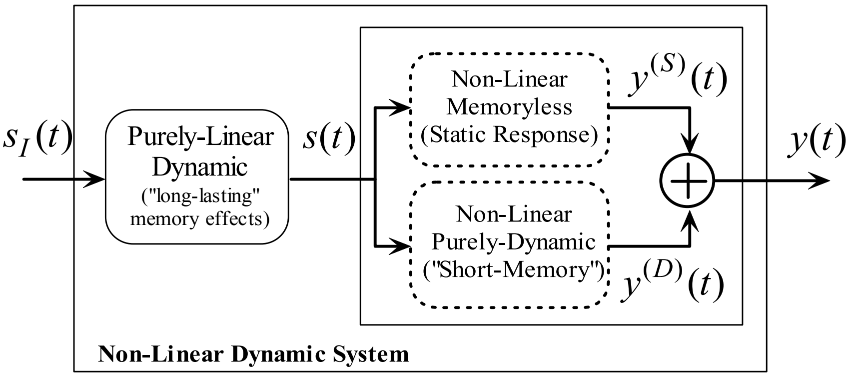

Figure 2. Cascade of sub-blocks in which the non-linear dynamic system of Fig. 1 can be further subdivided, according to the Discrete-Time Convolution approach.

III. Compensation techniques based on the DTCM characterization

In order to point out the main idea at the basis of the numerical techniques, which make use of the DTCM characterization, aimed to the compensation of non-idealities within the digital data acquisition channel a periodic, finite-spectrum ($N+1$ harmonics) signal $s_I(t)$ with fundamental frequency ω_0 can be considered at the input of system. Independently of the architectural complexity and accuracy of the front-end circuitry (i.e., mostly anti-alias filters), which is usually exploited in order to limit and make well-defined the bandwidth of the input signal, the system operation suffers from a non-negligible spectral re-growth due to the presence of the overall (both static and dynamic) non-linearity effects. Thus, without any loss of generality, the spectrum of all the signals involved into the scheme of Fig. 2 can be considered up to the *M*th harmonic (with $M > N$) and the output $y(t)$ of the non-linear dynamic system written according to the Fourier series expansion, as follows:

$$\sum_{k=-M}^{+M} Y_k e^{jk\omega_0 t} = \sum_{k=-M}^{+M} Z_{0,k}[\mathbf{S}] e^{jk\omega_0 t} + \sum_{k=-M}^{+M} Y_k^{(D)}[\mathbf{S},\omega_0] e^{jk\omega_0 t} \quad with$$
$$Y_k^{(D)}[\mathbf{S},\omega_0] = \sum_{\substack{p=-P_A \\ p\neq 0}}^{P_B} \sum_{\substack{q_1,q_2 \\ q_1+q_2=k}} W_{p,q_1}[\mathbf{S}] S_{q_2}\left(e^{-jq_2\omega_0 p\Delta\tau} - 1\right) \quad k = (-M,\ldots,-1,0,1,\ldots,+M) \tag{2}$$

where $\mathbf{S}$ is the vector of harmonics S_k for $s(t)$ and $[Y_k, Z_{0,k}, Y_k^{(D)}, W_{p,k}]$ being the *k*th harmonic of $[y(t), y^{(S)}(t) = z_0[s(t)], y^{(D)}(t), w_p[s(t)]]$, respectively. Following the well-known Harmonic-Balance (HB) methods (e.g. [5,6]), which were originally proposed for the efficient numerical analysis of non-linear circuits but can be applied to a wide family of non-linear dynamic problems, the relationship of Eq. (2) can be considered satisfied, for every time instant *t*, if the equality

$$Z_{0,k}[\mathbf{S}]+Y_k^{(D)}[\mathbf{S},\omega_0] \doteq Z_{0,k}[\mathbf{S}]+\sum_{\substack{p=-P_A\\ p\neq 0}}^{P_B}\sum_{\substack{q_1,q_2\\ q_1+q_2=k}} W_{p,q_1}[\mathbf{S}]S_{q_2}\left(e^{-jq_2\omega_0 p\Delta\tau}-1\right)=Y_k \tag{3}$$

is imposed at each harmonic order *k*. Since real signals are concerned, the attention can be focused on the positive side of the spectra: expression (3) can be interpreted as a non-linear system of $M+1$ complex equations with respect to the vector of $M+1$ unknowns **S**. By introducing the vector **X**, whose elements are the real and imaginary parts of harmonics S_k (S_0 being real), the overall problem becomes a system of $2M+1$ non-linear *real* equations with respect to **X**, which can be re-written into the compact form

$$\mathbf{G}(\mathbf{X})=\mathbf{Y} \tag{4}$$

where the vector **Y** (real and imaginary parts of Y_k 's) has been introduced.

The solution $\mathbf{X}^*$ of the non-linear algebraic problem (4) provides the estimate of the spectrum **S,** with the compensation of both static and dynamic non-linearities, through the *inversion* of the non-linear dynamic laws describing the behaviour of the digital channel. System (4) is fully defined, since the known term **Y** can be estimated through a time-to-frequency analysis performed on the samples at the output of the digital channel, while the non-linear relationships $[Z_{0,k}(\mathbf{S}), W_{p,q}(\mathbf{S})]$ are available once the DTCM experimental characterization has been carried out. The spectral components of signal $s_I(t)$ at the input of the system can be then easily derived from spectrum **S** by simply inverting the transfer function $H(\omega)$, which describes the linear response of the dynamic block at the input of Fig. 2. This step, which completes the DTCM-based compensation procedure by taking into account the long-lasting memory effects associated with the channel, can be performed as an independent task at the end of the non-linear problem.

Some consideration should be made, which concern the evaluation of the known term **Y** in (4). When the bandwidth of the signal $s_I(t)$ at the input of the channel begins to be comparable with the digital bandwidth of the latter, which is imposed by the maximum sampling rate it can be operated with, the spectral re-growth due to the system non-linearity can not be accurately estimated if a conventional real-time periodic sampling is adopted. In fact, not only the computation of the spectrum of $y(t)$ suffers from the truncation at the Nyquist frequency, but the few harmonic components Y_k which can be obtained from the samples at the output of the channel are strongly affected by aliasing effects. In such conditions, an equivalent-time sampling strategy is to be preferred for an exhaustive estimate of the spectral content of signal $y(t)$, in order to proceed to a more accurate compensation through a larger system of HB equations (4). If the channel can be operated directly at the time-base level, and the sequence of sampling instants can be imposed according to custom algorithms, more refined techniques can be further adopted in order to increase the bandwidth investigated at the *output* of the channel, such as those based on randomized sampling strategies (e.g.,[7-9]). It is worth to notice that the limitation to the full knowledge of components Y_k is not due to the modelling approach adopted, but is instead inherent to the non-linear properties of the system under investigation, or better, to the way the acquisition channel is operated through.

An accurate application of the HB approach to the DTCM characterization and modelling of the digital channel can thus allow for the compensation of the overall non-ideality affecting the operation of the system. A conventional, quasi-static representation of the non-linearity can be embedded within the DTCM representation, while the contribution deriving from the first-order term of the modified Volterra series provides the estimation of non-linear dynamics. Finally, the straightforward characterization of the linear block at the input of the model leads to the compensation also of delays and attenuation within the channel operation. In the next subsections, different recursive approaches are proposed for the numerical inversion of problem (4), which can be adopted according to the trade-off between convergence robustness and computational efficiency.

A. Newton-Raphson algorithm

The Newton technique [5] for the solution of systems of non-linear equations is an *iterative* algorithm, according to which the linearization of the problem is performed around the result descending from the *n*th iteration step, in order to obtain the (*n*+1)th guess for $\mathbf{X}^*$**.** More precisely, the recursive recipe for the solution of system (4) becomes:

$$\mathbf{X}_{n+1}=\mathbf{X}_n+\underline{\mathbf{J}}^{-1}(\mathbf{X}_n)\cdot[\mathbf{Y}-\mathbf{G}(\mathbf{X}_n)] \quad n=0,1,2,\ldots \quad with\ J_{lm}=\frac{\partial G_l}{\partial X_m} \tag{5}$$

where $\underline{\mathbf{J}}$ is the (2*M*+1)x(2*M*+1) *Jacobian* matrix associated with the vector function **G**.

Such a general technique offers very good convergence properties even in the presence of highly non-linear systems and it has been exploited in this work in order to carry out the results presented in the next section. Unfortunately, the evaluation of the Jacobian (usually performed by means of FFT-based routines) and its algebraic inversion are high-cost requirements from a computational standpoint, which are needed at *each* iteration of the algorithm: this can compromise the capability of the numerical resources of achieving the convergence, especially when a large number of harmonics are involved within the problem. Many solutions can be found in the literature in order to increase the efficiency of the algorithm described, which are usually based on approximations of different nature on the Jacobian matrix, with the purpose of speeding up the numerical manipulations it is involved in. For example, the Jacobian terms which are relatively far from the main diagonal can be neglected (i.e., the influence of input harmonics on output ones, which are placed into very different regions of the spectrum is forced to be reduced, and the actual non-linearity of the system is smoothed), making possible the subdivision of the problem in several smaller ones. Another solution adopted is that of considering the same Jacobian evaluation for several subsequent iterations (Newton-Samanskii technique), with evident benefits in terms of overall computational time.

The Newton algorithm in its most general form (5), which guarantees the best convergence speed (in terms of number of iterations) and reliability (approximated-Jacobian variations often do not lead to the convergence), can be successfully adopted with the purpose of validating the compensation technique based on the DTCM characterization and modelling of the digital channel, when a modern workstation with a suitable software is exploited "off-line" with respect to the measurement. Instead, its implementation in the framework of actual sampling instrumentation and its use for real-time correction on the output of digital acquisition channels can be not feasible, or too much expensive in terms of the additional hardware and software needed.

For these reasons, an alternative solution to the problem of inverting the system (4) is briefly proposed in the following subsection, which exploits the *mild* (from an analytical standpoint, and with respect to other families of systems usually approached with HB techniques, but clearly not in terms of measurement accuracy) overall non-linearity usually associated with the behaviour of digital channels. It is based on a quasi-linear approximation of the system in order to define a modified problem, to be solved with an iterative *direct* algorithm, which reliably (i.e., the convergence is guaranteed) carries out the estimate of vector $\mathbf{X}^*$ with the same accuracy of the Newton method but at a computationally "light" cost.

B. Fixed-point iteration algorithm

The problem defined by Eq. (4) can be analytically manipulated and transformed into the following explicit form without any loss of generality

$$\mathbf{X} = \widetilde{\mathbf{G}}^{-1}\left\{\widetilde{\mathbf{G}}(\mathbf{X}) - \mathbf{G}(\mathbf{X}) + \mathbf{Y}\right\} \doteq \mathbf{F}(\mathbf{X}) \qquad (6)$$

by introducing a generic vector function $\widetilde{\mathbf{G}}$ of dimension (2M+1), with the only constraint of being invertible. By assuming the channel as described by a mild non-linearity (in the sense discussed above), a useful choice for $\widetilde{\mathbf{G}}$ is a *linear* vector function that satisfies the relationship $\widetilde{\mathbf{G}}(\mathbf{X}) \cong \mathbf{G}(\mathbf{X})$ in all the subspace $\mathbf{\Omega} \subset \Re^{(2M+1)}$ of points $\mathbf{X}$ of interest for the application according to a given level of approximation, which can be considered good due to the assumptions made on the system under investigation. Under these hypotheses, the solution of equation $\mathbf{X} = \mathbf{F}(\mathbf{X})$, which is called *fixed-point* of $\mathbf{F}$ and coincides by definition with $\mathbf{X}^*$, can be univocally achieved through the recursive recipe

$$\mathbf{X}_{n+1} = \mathbf{F}(\mathbf{X}_n) \qquad n = 0,1,2,\ldots \quad \mathbf{X}_0 \in \mathbf{\Omega} \qquad (7)$$

The convergence to $\mathbf{X}^*$ of the iterative algorithm (7) is guaranteed since it is possible to demonstrate that, under the assumptions made of mild non-linearity of the acquisition channel (i.e., $\mathbf{G}$ mildly non-linear within $\mathbf{\Omega}$), the function $\mathbf{F}$ is a *contraction* [10,11] from the (2M+1)-dimensional subspace $\mathbf{\Omega}$ into $\mathbf{\Omega}$**,** i.e., a constant $L < 1$ exists such that

$$\left\|\mathbf{F}(\mathbf{A}) - \mathbf{F}(\mathbf{B})\right\| \le L\left\|\mathbf{A} - \mathbf{B}\right\| \qquad \mathbf{A},\mathbf{B} \in \mathbf{\Omega} \qquad (8)$$

where $\|\cdot\|$ indicates the *norm* or *length* operator. The set of all the possible values of $\mathbf{X}_n$ involved into the algorithm (7), which depends on the starting guess $\mathbf{X_0}$ made, is a sub-set of $\mathbf{\Omega}$.

The recursive algorithm described by (7) can be easily implemented without the need for expensive computational resources, since it only involves the evaluation of vector equation $\mathbf{G}$ at each iteration step**,** without any time-consuming inversion of differential matrices. The choice for the linear approximation $\widetilde{\mathbf{G}}(\mathbf{X}) \cong \mathbf{G}(\mathbf{X})$ can be accomplished by considering the simple transformation $\widetilde{\mathbf{G}}(\mathbf{X}) = G_0\mathbf{X}$, G_0 being the linear gain of the ADC evaluated in quasi-static operation. The fixed-point iteration procedure assumes thus the form:

$$\mathbf{X}_{n+1} = \frac{1}{G_0}\{G_0\mathbf{X}_n - \mathbf{G}(\mathbf{X}_n) + \mathbf{Y}\} \qquad n = 0,1,2,\ldots \tag{9}$$

The solution $\mathbf{X}^*$ to the DTCM-derived non-ideality compensation problem (4) can be achieved by exploiting simple routines implementing the algorithm (9) operated by the standard controlling hardware available in most of sampling instrumentation. Non-expensive DSP-based boards can be instead embedded within custom acquisition systems.

IV. Experimental results and tests

The DTCM model has been fully implemented, among other numerical environments, in the framework of Agilent-ADS commercial package for circuit analysis [2]. This implementation is particularly suitable for the validation of the compensation technique proposed, since it makes available, at the user interface, many reliable tools for the HB analysis of non-linear dynamic systems by means of different algorithms, including the Newton-Raphson iterative procedure in the exact-Jacobian version. In the following, the DTCM characterization of a commercial PCI-bus 12-bit digital acquisition board operated at 250 kSa/s will be considered. In Fig. 3 (*left*) the spectral components evaluated at the output of this device when a 4.5-V amplitude, 21-kHz sinusoidal signal $s_I(t)$ is applied at its input are shown. Since the input excitation is well-below the input analogue bandwidth (100 kHz) the estimation of components Y_k can be accomplished through a FFT transformation of the output samples acquired according to a conventional real-time periodic sampling. In the same graph, the input spectrum is shown after the application of the compensation procedure described. In the same figure (*right*), the input frequency (90 kHz) has been shifted up to the upper boundary of the analogue bandwidth. Under these conditions, a time-equivalent sampling has been necessary in order to obtain exhaustive information about the spectrum of output $y(t)$: as expected, the overall distortion due to the non-linear effects has increased. Nevertheless, the inversion of the DTCM non-linear dynamic laws, starting from the rich information acquired on harmonics Y_k , has allowed for a even better compensation of such effects. The poor capabilities of the technique of correcting high-order, low-level distortion harmonics is due to two different reasons. First, no precautions have been taken on the effect of quantization noise: thus a residual uncertainty still remains when passing from output samples to harmonics Y_k , which are defined at the *input* of the ideal quantizer in Fig. 1. Secondly, the spurious harmonics, which survive to the non-linear problem (4) inversion, are amplified by the application of $1/H(\omega)$ when they are located well-above the upper boundary of the device input bandwidth. In Fig. 4 (*left*), the results of the acquisition channel simulation are shown, which has been performed through the DTCM model with a 50-kHz, 4-V peak-to-peak square waveform with 0.2-μs rise/fall times applied at the input. These excitation conditions, which are well-outside the bandwidth specifications of the board, have led to a strongly corrupted output (equivalent-time sampling), both in terms of non-linear distortion and harmonic attenuation. Since the DTCM model of the system has been characterized by means of input test signals at frequencies much more higher than the upper limit of the analogue bandwidth, it is valid also under such operation: the application of the compensation technique proposed has been thus possible and it has allowed to reconstruct the square waveform by smoothing the ripples and correcting the rise/fall transitions. Right-side of Fig. 4 shows the spectra involved into the test: the compensation has carried out an accurate correction of output harmonics both by raising the odd-orders up to the right levels (compensation of linear attenuation) and reducing the even ones (introduced by non-linear static and dynamic effects), at least up to the 1-MHz values.

V. Conclusions

The Discrete-Time Convolution Model [1-4] capabilities of describing the overall non-idealities (i.e., static as well as dynamic non-linearities, along with purely-linear dynamic effects) can be exploited for the adequate compensation of these effects at the output of digital data acquisition channels for sampling sub-systems and instrumentation. Harmonic Balance techniques have been proposed for the inversion of the DTCM non-linear dynamic laws, which exhaustively describe the behaviour of the channel, and the reconstruction of the original spectrum of the input signal from the samples at the output of the system. Computationally robust algorithms (Newton-Raphson with exact Jacobian) implemented in the framework of refined numerical environments have been exploited in order to validate through experimental tests the approach proposed. Moreover, an efficient numerical technique have been described for the implementation of the DTCM-based compensation procedure in real-time instrumentation, which guarantees an accurate convergence to the solution at a low computational cost.

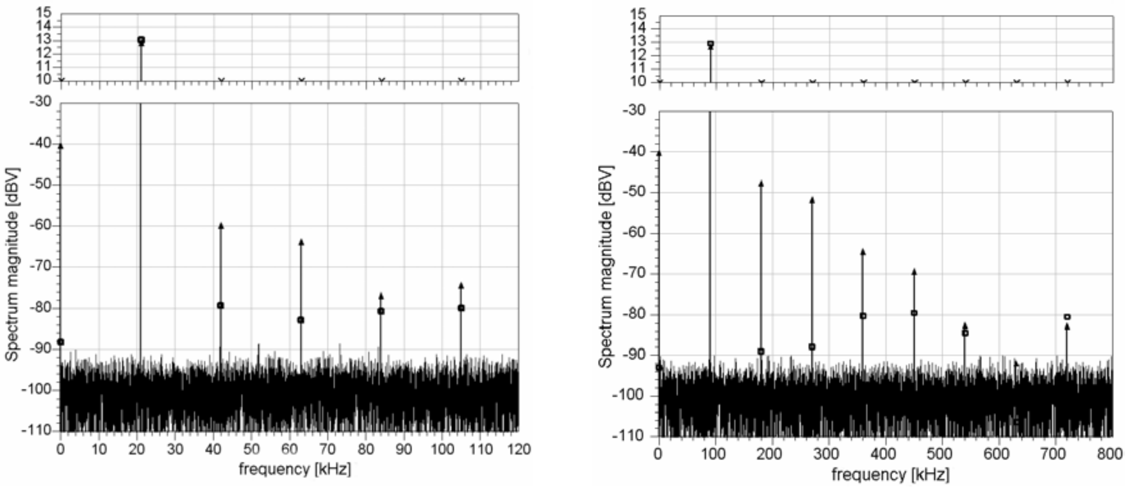

Figure 3. Response of the digital channel (triangles) to a 21-kHz (*left*) and a 90-kHz (*right*) (equivalent-time sampling) 4.5-V amplitude pure sinusoid, and estimate of the input spectrum after the DTCM-based compensation (squares).

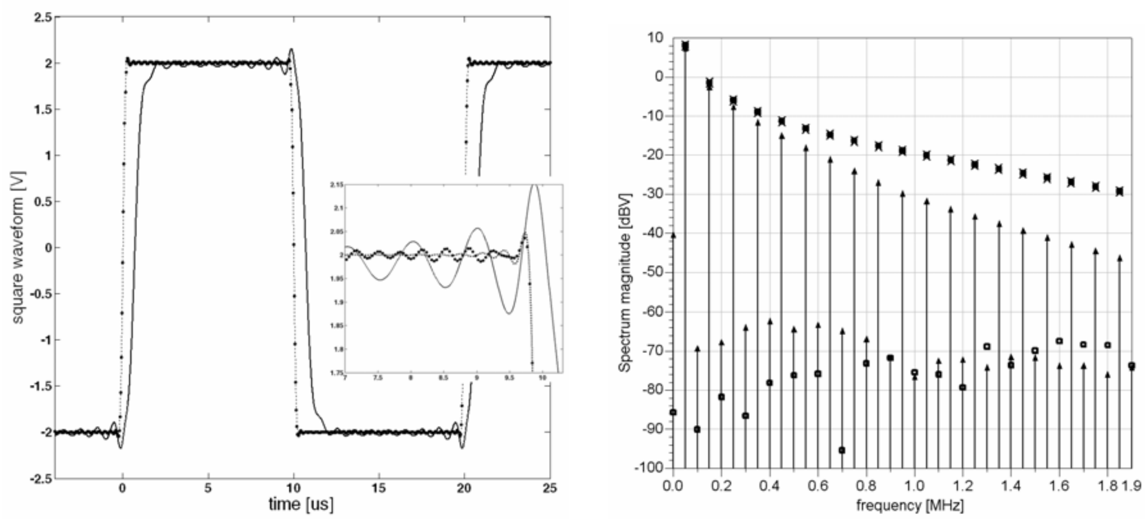

Figure 4. (*left*)- Time-domain DTCM response (solid line) of the channel (equivalent-time sampling) to a 50-kHz 4-V peak-to-peak square waveform (dotted), compared to the input reconstruction after DTCM-based compensation (circles). Magnification of the three waves within the ripple region. (*right*)- Spectra of input waveform (crosses), channel response (triangles) and reconstructed input (squares) after compensation based on the inversion of DTCM equations.

References

[1] D. Mirri, G. Pasini, P.A. Traverso, F. Filicori, G. Iuculano, "A finite-memory discrete-time convolution approach for the nonlinear dynamic modelling of S/H–ADC devices," *Computer Standards & Interfaces,* vol. 25, pp. 33-44, Jan. 2003.

[2] P.A. Traverso, D. Mirri, G. Pasini, F. Filicori, "A Nonlinear Dynamic S/H-ADC Device Model Based on a Modified Volterra Series: Identification Procedure and Commercial CAD Tool Implementation," *IEEE Trans. on Instrumentation and Measurement,* vol. 52, No. 4, Aug. 2003, pp. 1129-1135.

[3] P.A. Traverso, G. Pasini, D. Mirri, F. Filicori, G. Iuculano," Experimental validation of a general-purpose non-linear dynamic model for S/H-ADC devices," in: *Proc. 8th Workshop on ADC Modelling and Testing*, Perugia, Italy, Sep. 2003, pp. 235-238.

[4] D. Mirri, G. Iuculano, P.A. Traverso, G. Pasini, F. Filicori, "Non-linear dynamic system modelling based on modified Volterra series approaches," *Measurement*, vol. 33, No. 1, pp. 9-21, Jan. 2003.

[5] F.Filicori, V.A. Monaco, C. Naldi, "Simulation and design of microwave class-C amplifiers through harmonic analysis," *IEEE Trans. on MTT*, vol. MTT-27, Dec. 1979, pp. 1043-1051.

[6] V. Rizzoli, C. Cecchetti, A. Lipparini, F. Mastri, "General-purpose harmonic balance analysis of non-linear microwave circuits under multitone excitations," *IEEE Trans. on MTT*, vol. MTT-36, Dec. 1988, pp.1650-1660.

[7] I. Bilinskis and M. Mikelsons, *Randomized Signal Processing*. London, U.K., Prentice-Hall International, 1992.

[8] D. Mirri, G. Iuculano, F. Filicori, G. Pasini, P.A. Traverso, "Accuracy Evaluation of a New Broad-band Vector Digital Spectrum Analyzer," in: *Proc. IEEE Instr. Meas. Tech. Conf. (IMTC'03)*, Vail, CO, USA, May 2003, pp. 467-472.

[9] G. Pasini, P.A. Traverso, D. Mirri, G. Iuculano, F. Filicori, "Hardware Implementation of a Broad-Band Vector Spectrum Analyzer Based on Randomized Sampling," *IEEE Trans. on Instrumentation and Measurement*, Aug. 2005.

[10] J. Ortega, W.C. Rheinboldt, *Iterative Solution of Nonlinear Equations in Several Variables.* New York, Academic Press, 1970.

[11] L.O. Chua, P.M. Lin, *Computer-Aided Analysis of Electronic Circuits: Algorithms and Computational Techniques*. New Jersey, Prentice-Hall, Inc, 1975.