# Testing Oscillators by Waveform Comparison with the Ideal Sine-Wave

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***Abstract-*** In context of the oscillation-based-test (OBT) technique, a simple and low-cost method of testing oscillators by comparison of finite segment of waveform with the ideal sine-wave is presented. The method is based on the subtraction of two sigma-delta modulated signals with the aid of a 1-bit subtractor that employs oversampling based noise-shaping. An up/down counter extracts the average of the bit stream that is used as a fault signature. The simulation results on the example of the Van der Pol oscillator show that the proposed fault signature is sensitive to the frequency and amplitude deviations and is able to detect non-linear oscillation as well.

## I. Introduction

Testing mixed-signal circuits is nowadays a hot research topic [1]. One approach to test mixed-signal circuits that deserves significant attention is the oscillation-based-test (OBT) technique. The basic idea is to convert the circuit under test (CUT) into an oscillator during the test mode. Oscillation parameters are related to the CUT performance in the normal mode and thus faulty circuits can be detected in this way. The key issue of this technique is the on-chip or on board extraction of the main test signal features. The preferable approach is the use of measurement data encoded into a compact digital signal that is referred to as a fault signature. For the signature synthesis technique to be practical, it must possess the following properties: simplicity of the test circuitry, significant reduction (compression) of the test data, speed, and reasonable fault coverage.

In [2-4] a simple and low-cost methods for signature generation using sigma-delta modulator are proposed. The method described in [2] extracts separately amplitude, frequency and DC level. The paper [3] is focused on conversion of the both parameters, frequency and amplitude into a single digital code. The signature proposed in [4 ] is based on the analysis of the oscillator transient state.

This work is intended to design the sigma-delta signature synthesiser which provides digital signature related to the non-linear oscillation as well as the frequency and amplitude characteristics of the tested signal coming from the application of the OBT technique. Non-linear oscillation in electronic circuit is usually undesirable phenomenon and is symptom of fault.

The paper is organised as follows. Section II focuses on the concept of the sigma-delta signature synthesiser. Section III consists of simulation results obtained with the aid of Simulink toolbox of Matlab. Detection of the non-linear oscillation on an example of Van der Pol oscillator is considered in Section IV.

## II. Signature synthesiser

In the proposed signature synthesiser a finite segment of waveform generated by the oscillator under test is sampled by a first-order sigma-delta modulator (SDM) that perform quantisation and encode the analogue signals to the pulse-density modulated bit stream. This bit stream is then compared with the expected fault-free one using an oversampling bit stream subtractor. A difference is processed by an up/down digital counter over a fixed period of time.

To explain how the signature synthesiser works, we will consider the simplest sigma-delta modulator shown in Fig. 1(a). This is a first-order loop containing an integrator, a 1-bit ADC and a 1-bit DAC that can be modelled in the z-domain by the block diagram of Fig. 1(b) [5]. In this model, it is permissible to assume that the 1-bit binary quantisation is a white noise process with mean zero and mean-square value $\sigma_e^2 = \Delta^2 / 12$ , where $\Delta$ represents the step size of the quantizer. From the block diagram

$$Y(z) = z^{-1}Y(z) + U(z) - z^{-1}V(z) \tag{1}$$

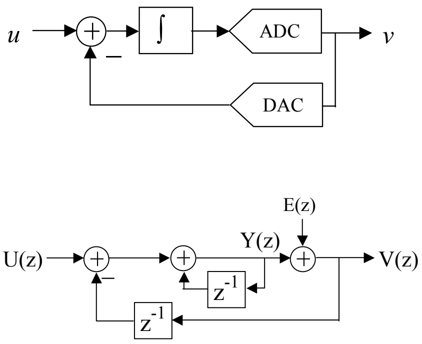
Thus

$$V(z) = Y(z) + E(z) = z^{-1}Y(z) + U(z) - z^{-1}V(z) + E(z) = U(z) + E(z) - z^{-1}(V(z) - Y(z)) = \\ U(z) + E(z) - z^{-1}E(z) = U(z) + (1 - z^{-1})E(z), \tag{2}$$

where E(z) is the quantisation noise. Equation (2) can be written in the general form

$$V(z) = STF(z)U(z) + NTF(z)E(z)\,, \tag{3}$$

where the signal transfer function (STF) is unity, and the noise transfer function is $NTF(z) = 1\text{-}z^{-1}$. For normalized frequency f/fs << 1 the squared magnitude of NTF in the frequency domain is $|NTF|^2 \approx (2\pi f)^2$. The frequency response of the NTF is clearly a highpass filter function, which suppresses the quantisation noise at and near dc and amplifies it out of useful signal band, at higher frequencies around $f_s/2$. If the integrator has a high gain in the signal band the in-band quantisation noise is strongly attenuated. This process, commonly called noise-shaping action, is the key idea of the sigma-delta modulation.




(b)

Fig. 1. (a) A first-order sigma-delta modulator and (b) its z-domain linear model.

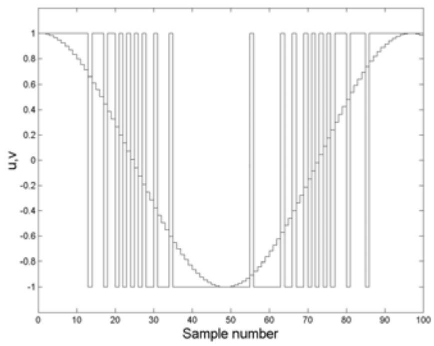



Fig. 2. Time domain plot of the input signal superimposed on the sigma-delta modulator output signal.

If we assume Δ = 2, the 1-sided power spectral density of the quantisation noise is $S_e(f) = 2\sigma_e^2 = 2/3$. Usually the energy of useful signal is concentrated in a narrow bandwidth $f_B$ at low frequencies. The in-band quantisation noise power can be approximated by

$$\sigma_q^2 = \int_0^{1/(2OSR)} [2\pi f]^2 S_e(f)df = \frac{\pi^2}{9(OSR)^3}\ , \tag{4}$$

where: $OSR = \dfrac{f_s}{2f_B}$ is the oversampling ratio, $f_s$ is the sampling frequency, $f_B$ is the signal bandwidth.

A common measure of a modulator accuracy is the signal-to-noise ratio for a sine-wave input. If the input signal is a full-scale sine wave with a peak amplitude M, then the output signal power is $\sigma_u^2 = \dfrac{M^2}{2}$. The in band signal-to-quantisation-noise ratio (SNR) can be approximated by

$$SNR = \frac{\sigma_u^2}{\sigma_q^2} = \frac{9M^2(OSR)^3}{2\pi^2}\ . \tag{5}$$

Accuracy of considered digital encoding depends on factors like modulator order, quantizer step size, oversampling ratio. The first order modulator requires rather large OSR for accurate data conversion. Sigma-delta modulators are inherently immune to a variety of imperfections, such as component mismatch.

The bit stream, used as a reference signal, is stored in existing embedded RAM or supplied from the external signal source via the appropriate test port. It is made of the sigma-delta modulated high quality sine-wave (Fig. 2). In the papers [6,7] theory is presented that finds the minimum stream length for encoding a signal into a sigma-delta modulated bit stream with a known SNR. This approach offers signals with spurious-free dynamic range as high as 80 dB for sequences as short as 100 bits [1]. The drawback to this approach is that new sequence must be downloaded for change the amplitude or frequency of the reference signal.

For performing subtraction of two sigma-delta modulated bit streams the method of addition of two bit steams, suggested in [8], was used. The arrangement is shown in Fig. 3. It consist the binary full adder with the sum-output S, carry-output $C_{out}$ and carry-input $C_{in}$. There is feedback between sum-output and carry-input through a unit delay. The roles of sum-output and carry-output are interchanged. The output Y is taken from the carry-output of the adder. The terms of Y take the values +1 or -1.

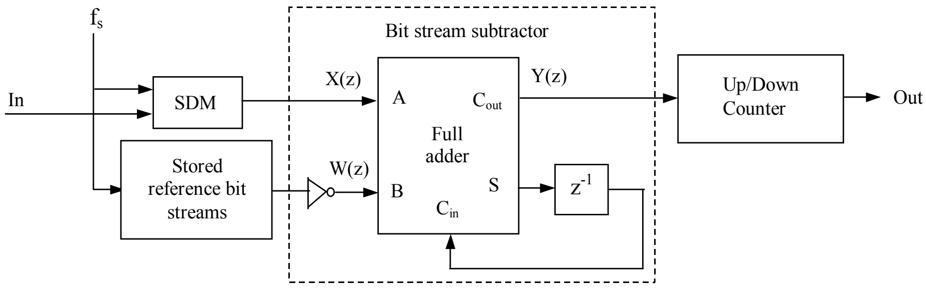



Fig. 3. Block diagram of the proposed signature synthesiser

The z-domain equation describing the bit stream adder is given by [8]

$$Y(z) = X(z) + W(z) - (1 - z^{-1})S(z) \ . \tag{6}$$

It is evident from (6) that such an arrangement performs the first-order sigma-delta noise shaping and approximates the sum of signals, provided the signal frequency is low relative to the clock rate. Low frequencies $\omega \to 0$ imply $z^{-1} \to 1$ and the output is given by $Y(z) = X(z) + W(z)$. To implement a subtraction, the reference bit stream is inverted.

The SNR of the bit stream adder may be approximated by [9]

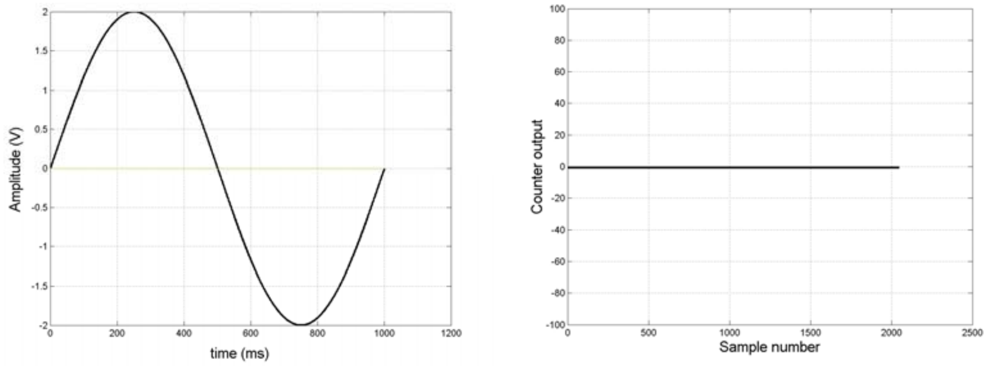
$$SNR \approx \frac{6}{\pi}(OSR)^{\frac{3}{2}} \ . \tag{7}$$

For example, if two signals with a bandwidth of 1 kHz are added at a sampling frequency $f_s$ = 1 MHz, the SNR of the sum is expected to be 86.6 dB.

Averaging the synthesiser output sequence by an up-down counter performs quantification of the result. In the case of linear oscillations with proper frequency and amplitude, the counter output sequence is equally spaced with amplitude +1 or -1. Its average value is zero. In the case where the bit stream of the tested waveform is different from the reference pattern, the pulse density of output sequence becomes higher or lower and its average value differs from zero. The plot of counter values versus time over one period duration is the useful fault signature that enables detection of different fault symptoms: frequency deviation, amplitude shift or non-linear oscillation. The signature shape and the state of the counter at the end of each evaluation period provide diagnostic information.

### III. Simulation results

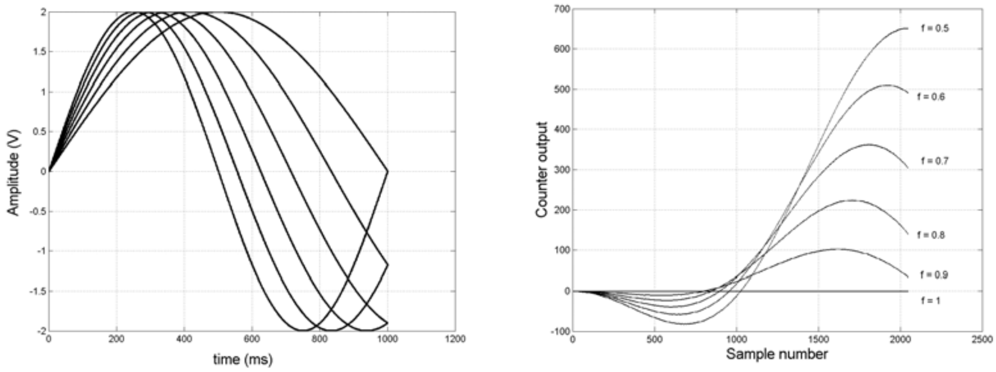
The signature synthesiser was modelled and simulated using Matlab/Simulink environment. The counting time was set to one period of the reference signal. The oversampling ratio of 1024 was used. Figures 7 - 10 present some signatures obtained from fault-free and faulty oscillator under test. The diagnostic information is contained in the shape of the signature plot. If the tested signal and the reference signal are equivalent in frequency, amplitude and phase (Fig. 7(a)), the signature contains only the noise. The signature of the fault free oscillator is displayed in Fig. 7(b).




(a) (b)

Fig. 7. (a) The fault-free oscillator sine wave equivalent in frequency, amplitude and phase with the reference signal. (b) Signature of the fault-free oscillator.

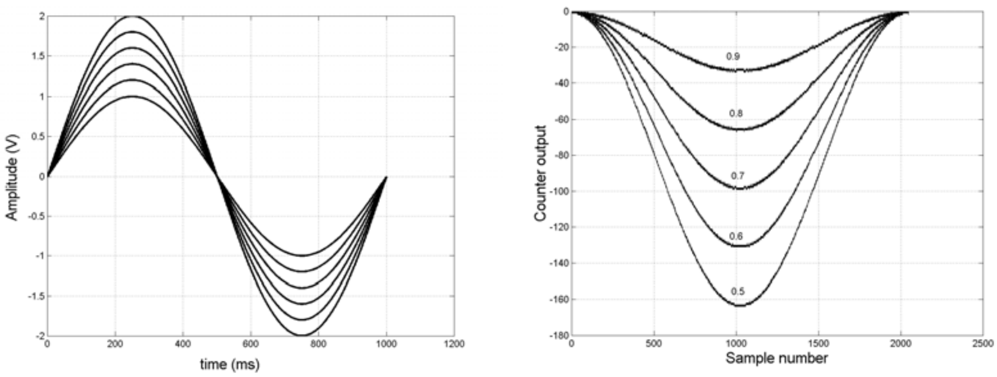
The counter state at the end of an evaluation period is directly related to the frequency deviation as is shown in Fig. 8.




(a) (b)

Fig. 8. (a) Signal frequency variability. (b) Family of the corresponding signatures.

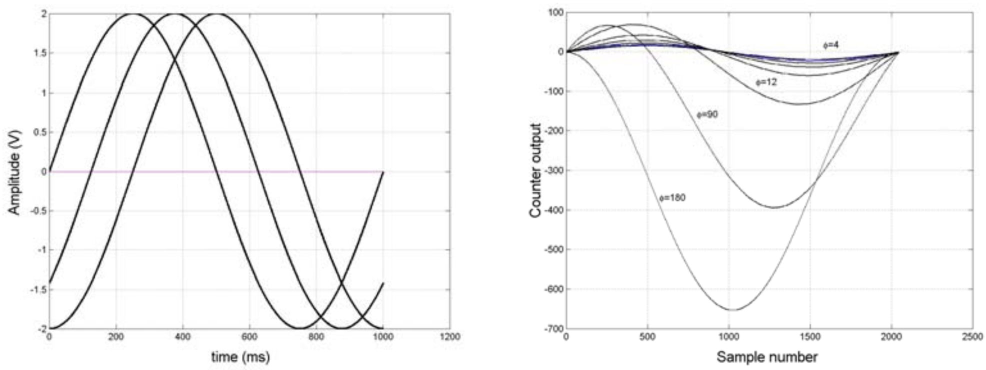
The amplitude inconsistency (Fig. 9(a)) results in the changed centre of a signature as Fig. 9(b) presents. Amplitude variability does not influence the final state of the counter, so it has no impact on detection of the frequency deviation.




(a) (b)

Fig. 9 (a) Signal amplitude variability. (b) Family of the corresponding signatures.

Generally there is a phase shift between the tested signal and the reference signal (Fig. 10(a)). This phase shift manifests itself by the signatures depicted in Fig. 10(b). To remove the phase impact on a signature, the phase of the reference signal need to be adjusted.




(a) (b)

Fig. 10. (a) Phase-shifted signals. (b) Signatures of the phase-shifted signals.

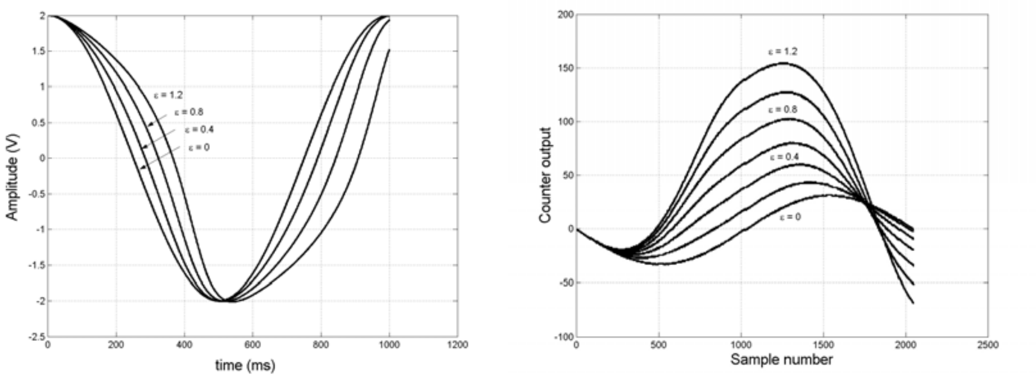
### IV. Detection of the non-linear oscillation

The model of the Van der Pol oscillator was used as the source of non-linear oscillation to examine the efficiency of the proposed methodology in detecting non-linear phenomenon. The Van der Pol state equation is

$$\frac{dx_1}{d\tau} = x_2 \qquad (8)$$

$$\frac{dx_2}{d\tau} = \varepsilon\left(1 - x_1^2\right)x_2 - x_1$$

where: $x_1$ , $x_2$ are the states, $\tau$ is dimensionless time. Parameter $\varepsilon$ gives us possibility of programming the distortion level without influencing the steady state amplitude of oscillation. Amplitude of oscillation depends on the initial conditions.




(a) (b)

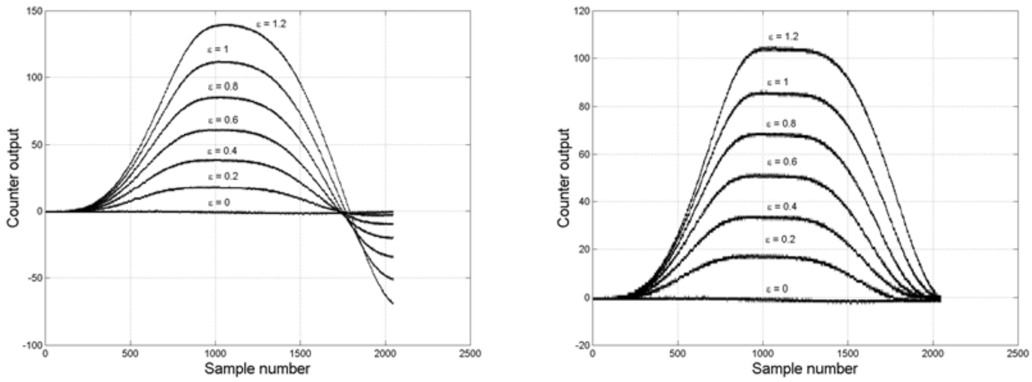
Fig. 11 (a) Non-linear oscillations coming from the Van der Pol oscillator. (b) Signatures of the distorted signals with frequency deviation and amplitude inconsistency.

The presence of harmonic terms in the oscillation wave causes a frequency deviation. The oscillation frequency of a non-linear oscillator is given by [10]

$$\omega = \omega_0\left[1 - \frac{1}{2}\sum_{n=2}^{\infty} d_n^2\left(n^2 - 1\right)\right], \qquad (9)$$

where: $\omega_0$ is the frequency of linear oscillator (nominal oscillation frequency without considering the non-linear effects), $d_n$ is the amplitude of the $n$ - th harmonic.

The signature related to the oscillation distortion only can be extracted with the aid of external adjustable reference source of signal encoded in a bit stream. Alternating adjustments of the reference frequency and amplitude leads to the unipolar signature with the zero value at the end of the counting period (Fig. 12(b)). The difference between frequencies can be removed by adjusting the frequency of the reference source until the zero final state of the counter will be reached. The consistency in both amplitude and frequency is pointed by unipolarity of the signature plot. In this situation magnitude of the signature plot in Fig. 12(b) is related only to the distortion level.




(a) (b)

Fig. 12. Extraction of the signature related to non-linear oscillation on the example of the Van der Pol oscillator. (a) Reference source amplitude adjusted to the shifted amplitude of the oscillator under test. (b) Reference source adjusted to both frequency and amplitude (it is important to notice the flat top of the plot, and the zero state of the counter at the end of the evaluation period).

## V. Conclusions

This work presents low-cost signature synthesiser applicable as the on-chip or on-board design-for-test solution for making pass/fail decision. Using compact and simple hardware, based on a sigma - delta modulator, synthesiser provides digital signature related to the frequency, amplitude and distortion characteristics of the tested signal coming from the application of the oscillation-based-test technique. It has been shown on the example of the Van der Pol oscillator that the non-linear oscillation in the CUT can be detected. The signature related to the oscillation distortion is extracted with the aid of the external reference source of the sigma-delta modulated signals, adjustable in frequency and amplitude.

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