Two-parameters Approximation Curve for NTC Resistors

Damir Ilić, Josip Butorac, Luka Ferković

Faculty of Electrical Engineering and Computing, Unska 3, HR-10000 Zagreb, Croatia
Phone: +385 1 6129 753; fax: +385 1 6129 616; e-mail: damir.ilic@fer.hr

Abstract – The table data for *R-T* dependence of NTC resistors (NTCRs), which have resistance $R_N = 10000\ \Omega$ at a temperature of 25 °C and limits of error ±0,2 °C in the range (0 to 70) °C [1-4], are analysed. The determination of two approximation curves is described: a three-parameters (abbreviated as AC1) and a two-parameters (AC2). These curves are compared with the three-parameters Steinhart-Hart equation (S&H) by using the table data of resistance for different NTCRs and calculating the following for all of them: unknown parameters by the least-squares method, approximated temperature using determined parameters, and the difference to the table data of temperature. The obtained results are presented, which show a very good agreement of curves, with their mutual differences within a few millikelvin, and the differences to the table data within ±15 mK for the range (0 to 70) °C. Therefore, presented two-parameters approximation curve AC2 can be very convenient for the use with some NTCRs for the calculation of unknown temperature from the measured resistance.

I. Introduction

NTCRs have very high sensitivity and small heat capacity, which enables the measurement of the momentary values of temperature. Their resistance can be measured directly by a DMM, since it could be at the kiloohms level, which is used in the Primary Electromagnetic Laboratory in Zagreb, Croatia, for the precise measurement of temperature [5, 6]. We are using the Fenwal uni-curve NTCRs of type UUA41J1, which have resistance $R_N = 10000\ \Omega$ at a temperature of 25 °C and limits of error ±0,2 °C in the range (0 to 70) °C, as well as of type UUA41J8 with the same data for the range (0 to 100) °C [1]; in further text for both type of these sensors the unique abbreviation FE will be used. Uncertainty can be lowered by the calibration of each sensor, which must be done if absolute temperature measurement with a lower uncertainty is needed. As the producer's table provide the resistance values in 1 °C steps, the problem of interpolation between two table data remains even after sensor's calibration. Therefore, to calculate the measured temperature from "any" measured resistance, it is necessary to use a suitable approximation curve. To compare three approximation curves AC1, AC2 and S&H, the same analysis was done for some other types of NTCRs with the same nominal value and limits of error (i.e. 10 kΩ at 25 °C, and ±0,2 °C, respectively): a BetaCURVE Thermistor Series III 10K3A1W2 (abbreviated as BTS) [2], a coated chip thermistor A1004-C3 (ATP) [3], and uni-curve NTC thermistor S 863/10k/F40 (EP) [4]. The producer's table data for the mentioned types of NTCRs are presented in Table I.

Table I. Table data of four types of NTCRs with resistance $R_N = 10000\ \Omega$ at a temperature $\vartheta = 25$ °C and limits of error ±0,2 °C in the range (0 to 70) °C

	R_T/Ω			
ϑ/°C	FE	BTS	ATP	EP
0	32650	32650,8	29490	32650
1	31030	31030,4	28157	31030
2	29500	29500,1	26891	29490
3	28050	28054,2	25689	28050
4	26690	26687,6	24547	26680
5	25390	25395,5	23462	25390
6	24170	24172,7	22431	24170
7	23010	23016,0	21450	23010
8	21920	21921,7	20518	21920
9	20880	20885,2	19631	20880
10	19900	19903,5	18787	19900
11	18970	18973,6	17983	18970
12	18090	18092,6	17219	18090
13	17250	17257,4	16490	17250
14	16460	16465,1	15797	16460
15	15710	15714,0	15136	15710
16	15000	15001,2	14506	15000
17	14320	14324,6	13906	14320
18	13680	13682,6	13334	13680
19	13070	13052,8	12788	13070
20	12490	12493,7	12268	12490
21	11940	11943,3	11771	11940
22	11420	11420,0	11297	11420
23	10920	10922,7	10845	10920
24	10450	10449,9	10413	10450
25	**10000**	**10000,0**	**10000**	**10000**
26	9573	9572,0	9606	9572
27	9167	9164,7	9229	9165
28	8777	8777,0	8869	8777
29	8407	8407,7	8525	8408

Table I continued

	R_T/Ω			
ϑ/°C	FE	BTS	ATP	EP
30	8057	8056,0	8196	8057
31	7723	7720,9	7882	7722
32	7403	7401,7	7581	7402
33	7097	7097,2	7293	7098
34	6807	6807,0	7018	6808
35	6530	6530,1	6754	6531
36	6267	6266,1	6501	6267
37	6017	6014,2	6260	6016
38	5777	5773,7	6028	5775
39	5547	5544,1	5806	5546
40	5327	5324,9	5594	5327
41	5117	5115,6	5390	5117
42	4917	4915,5	5195	4917
43	4727	4724,3	5007	4726
44	4543	4541,6	4828	4543
45	4370	4366,9	4656	4369
46	4200	4199,9	4490	4202
47	4040	4040,1	4332	4042
48	3890	3887,2	4180	3889
49	3743	3741,1	4034	3743
50	3603	3601,0	3893	3603

	R_T/Ω			
ϑ/°C	FE	BTS	ATP	EP
51	3467	3466,9	3759	3469
52	3340	3338,6	3629	3340
53	3217	3215,6	3505	3217
54	3099	3097,9	3386	3099
55	2986	2985,1	3271	2986
56	2878	2876,9	3160	2878
57	2774	2773,2	3054	2774
58	2675	2673,9	2952	2675
59	2579	2578,5	2854	2579
60	2488	2487,1	2760	2488
61	2400	2399,4	2669	2400
62	2316	2315,2	2582	2316
63	2235	2234,7	2498	2235
64	2157	2156,7	2417	2158
65	2083	2082,3	2339	2083
66	2011	2010,8	2264	2011
67	1942	1942,1	2191	1943
68	1876	1876,0	2122	1877
69	1813	1812,6	2055	1813
70	1752	1751,6	1990	1752

II. NTCR's approximation curves

In this section we will present the determination of approximation curves AC1 and AC2, as well as the calculation of unknown parameters by the least-squares method for all three curves.

A. Three-parameters approximation curve AC1

We shall analyse *R*-*T* dependence for an NTCR starting with the basic exponential equation:

$$R_T = R_{T_0} \exp\left[-\frac{B\vartheta}{T_0^2(1+\vartheta/T_0)}\right], \tag{1}$$

where from table data follows the value R_{T_0} at temperature $T_0 = 273{,}15$ K; B is coefficient (expressed in kelvins) and $\vartheta = T - T_0$ (i.e. Celsius temperature). Replacing the coefficient B by the parabolic function $B = B_0(1 + b\vartheta - c\vartheta^2)$, and rearranging (1), follows:

$$\ln(R_T/R_{T_0}) = -(B_0/T_0^2)(1+b\vartheta - c\vartheta^2)\,\vartheta\,(1+\vartheta/T_0)^{-1}. \tag{2}$$

The parameters B_0, b and c can be determined according to the least-squares method by rearranging (2):

$$k\left[\ln(R_T/R_{T_0})(1+\vartheta/T_0)\right] + b\vartheta^2 - c\vartheta^3 = -\vartheta, \tag{3}$$

where $k = T_0^2/B_0$. Introducing the substitutes $m = \ln(R_T/R_{T_0})(1+\vartheta/T_0)$, $n = \vartheta^2$, $o = -\vartheta^3$ and $f = -\vartheta$, the following system will be obtained, where for the sums the Gauss's notation is used; for instance, $[nn] = \sum_{i=1}^{r}\vartheta^4$, $[nf] = -\sum_{i=1}^{r}\vartheta^3$, etc. ($r$ is equal to the number of pairs $R_{Ti} \leftrightarrow \vartheta_i$ used in the calculation):

$$\begin{aligned} [mm]k + [mn]b + [mo]c &= [mf] \\ [mn]k + [nn]b + [no]c &= [nf] \\ [mo]k + [no]b + [oo]c &= [of]. \end{aligned} \tag{4}$$

Using the table data for R_T in the range (0 to 70) °C, the values of unknowns B_0, b and c can be calculated. If we are going a step further, (2) can be seen as the relation between the measured resistance, that can be marked as R_{m}, and the unknown temperature of interest, marked in the same way as ϑ_{m}. Thus, we can use it simply by substitution $R_T = R_{\mathrm{m}}$ and $\vartheta = \vartheta_{\mathrm{m}}$, which leads to the relation

$$b\vartheta_{\mathrm{m}}^2 + \beta\vartheta_{\mathrm{m}} + \gamma = 0 , \tag{5}$$

where

$$\beta = \left[1 + \frac{T_0}{B_0}\ln\left(R_{\mathrm{m}}/R_{T_0}\right)\right], \qquad \gamma = \left[\frac{T_0^2}{B_0}\ln\left(R_{\mathrm{m}}/R_{T_0}\right) - c\vartheta_{\mathrm{m}}^3\right]. \tag{6}$$

Finally, the unknown temperature can be calculated as follows that is, in other words, approximation curve **AC1**:

$$\vartheta_{\mathrm{m}} = -\frac{\beta + \sqrt{\beta^2 - 4b\gamma}}{2b} . \tag{7}$$

Temperature ϑ_{m} in (5) exists also in the expression for γ in (6), so to determine its value it is necessary to use an iterative procedure. Usually only one iteration is enough, the conclusion that follows from the analysis performed on mentioned NTCRs, but it is obviously a drawback of this curve.

B. Two-parameters approximation curve AC2

A two-parameters approximation curve can be obtained if the part $(1 + b\vartheta - c\vartheta^2)$ in (2) is written as $1 + x \approx (1 - x + x^2)^{-1}$; in that case follows

$$\ln\frac{R_T}{R_{T_0}} = \frac{-B_0\vartheta}{T_0(T_0 + \vartheta)(1 - b\vartheta + c\vartheta^2 + b^2\vartheta^2 - 2bc\vartheta^3 + c^2\vartheta^4)} . \tag{8}$$

After neglecting all factors except those standing with ϑ, the upper equation becomes:

$$\ln(R_T/R_{T_0}) = -C_1\vartheta(1 + C_2\vartheta)^{-1} , \tag{9}$$

where

$$C_1 = B_0/T_0^2 , \qquad C_2 = (T_0)^{-1} - b . \tag{10}$$

The parameters C_1 and C_2 can be calculated according to the least-squares method, using the table data for R_T in the range (0 to 70) °C, and writing (9) as

$$C_2\vartheta\ln(R_T/R_{T_0}) + C_1\vartheta = -\ln(R_T/R_{T_0}) . \tag{11}$$

With the substitutes $m = \vartheta\ln(R_T/R_{T_0})$, $n = \vartheta$ and $f = -\ln(R_T/R_{T_0})$ the following system will be obtained, where for the sums the Gauss's notation is used, as for the system (4):

$$\begin{aligned} [mm]C_2 + [mn]C_1 &= [mf] \\ [mn]C_2 + [nn]C_1 &= [nf] . \end{aligned} \tag{12}$$

Finally, if we make the same substitution as for the curve AC1, that is to say $R_T = R_{\mathrm{m}}$ and $\vartheta = \vartheta_{\mathrm{m}}$, from the known values of C_1 and C_2 for a particular NTCR we can rearrange (9) in that way that the measured temperature ϑ_{m} can be calculated from the measured resistance R_{m}:

$$\vartheta_{\mathrm{m}} = \frac{\ln(R_{T_0}/R_{\mathrm{m}})}{C_1 - C_2\ln(R_{T_0}/R_{\mathrm{m}})} . \tag{13}$$

This formula is the approximation curve **AC2** what we are looking for.

C. Three-parameters Steinhart-Hart equation

The well-known, widely used and empirically developed polynomial that very accurate represents R-T dependence of NTCRs is the Steinhart-Hart equation [7]:

$$\frac{1}{T} = A_0 + A_1(\ln R_T) + A_3(\ln R_T)^3 , \tag{14}$$

where T is thermodynamic temperature. To determine the parameters A_0, A_1, and A_3 according to the least-squares method and using the table data for R_T in the range (0 to 70) °C, the following system needs to be solved, where $m = 1$, $n = \ln R_T$, $o = (\ln R_T)^3$ and $f = 1/T$:

$$\begin{aligned} [mm]A_0 + [mn]A_1 + [mo]A_3 &= [mf] \\ [mn]A_0 + [nn]A_1 + [no]A_3 &= [nf] \\ [mo]A_0 + [no]A_1 + [oo]A_3 &= [of]\ . \end{aligned} \tag{15}$$

As well as for system (4), for the sums the Gauss's notation is used, with r number of pairs $R_{Ti} \leftrightarrow \vartheta_i$ used in the calculation. If we insert in (14) the calculated values for three parameters, and make substitution $R_T = R_\mathrm{m}$ like we did for two previous curves, the associated temperature $T = T_\mathrm{m}$ will be calculated. Therefore, according Celsius temperature will be

$$\vartheta_\mathrm{m} = T_\mathrm{m} - T_0\ . \tag{16}$$

III. Comparison of approximation curves AC1, AC2 and S&H

As a first step in comparison of three approximation curves, the parameters of all of them need to be determined. It was done by using the table data of resistance of different NTCRs (R_T in Table I) for the range (0 to 70) °C (number of pairs $R_{Ti} \leftrightarrow \vartheta_i$ used in the calculation is r = 71), and solving the systems (4), (12) and (15) for curves AC1, AC2 and S&H, respectively. The obtained values are pointed out in Table II.

Table II. Calculated parameters for three approximation curves and different types of NTCRs

		FE	BTS	ATP	EP
AC1	B_0 / K	3812,667	3810,631	3462,771	3812,765
	b / K^{-1}	$4{,}6722 \cdot 10^{-4}$	$4{,}9882 \cdot 10^{-4}$	$7{,}4376 \cdot 10^{-4}$	$4{,}6775 \cdot 10^{-4}$
	c / K^{-2}	$1{,}0926 \cdot 10^{-6}$	$1{,}4294 \cdot 10^{-6}$	$1{,}9666 \cdot 10^{-6}$	$1{,}1189 \cdot 10^{-6}$
AC2	C_1 / K^{-1}	$5{,}1096 \cdot 10^{-2}$	$5{,}1102 \cdot 10^{-2}$	$4{,}6443 \cdot 10^{-2}$	$5{,}1100 \cdot 10^{-2}$
	C_2 / K^{-1}	$3{,}1810 \cdot 10^{-3}$	$3{,}1810 \cdot 10^{-3}$	$2{,}9393 \cdot 10^{-3}$	$3{,}1835 \cdot 10^{-3}$
S&H	A_0 / K^{-1}	$1{,}1294 \cdot 10^{-3}$	$1{,}1288 \cdot 10^{-3}$	$1{,}0281 \cdot 10^{-3}$	$1{,}1288 \cdot 10^{-3}$
	A_1 / K^{-1}	$2{,}3405 \cdot 10^{-4}$	$2{,}3419 \cdot 10^{-4}$	$2{,}3930 \cdot 10^{-4}$	$2{,}3414 \cdot 10^{-4}$
	A_3 / K^{-1}	$8{,}8174 \cdot 10^{-8}$	$8{,}7417 \cdot 10^{-8}$	$1{,}5599 \cdot 10^{-7}$	$8{,}7893 \cdot 10^{-8}$

With the known values of parameters (Table II), we can simply compare three approximation curves by using the same table data of resistance R_T (Table I) for different NTCRs as "measured" resistance R_m in steps of 1 °C, and calculating the associated temperature ϑ_m using the formula (7), (13) and (16) for curves AC1, AC2 and S&H, respectively. Finally, we can calculate the difference between the temperature calculated in that way (ϑ_m) and the table value of temperature (ϑ) in steps of 1 °C for the whole range (0 to 70) °C, using the simple relation:

$$\Delta T = \vartheta_\mathrm{m} - \vartheta\ . \tag{17}$$

This was done for four NTCRs, and is presented in Figures 1, 2, 3 and 4.

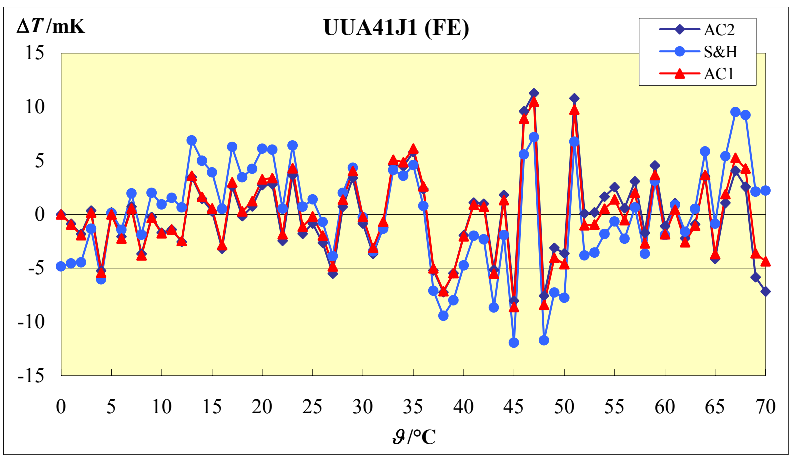

Fig. 1. Differences between the calculated and table value of temperature (in steps of 1 °C) for three approximation curves and NCTR of type Fenwal UUA41J1 (and UUA41J8)

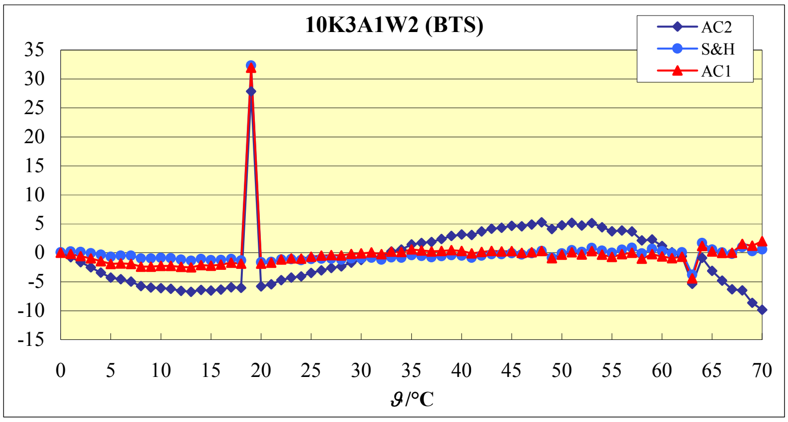

Fig. 2. Differences between the calculated and table value of temperature (in steps of 1 °C) for three approximation curves and NCTR of type BetaTHERM 10K3A1W2

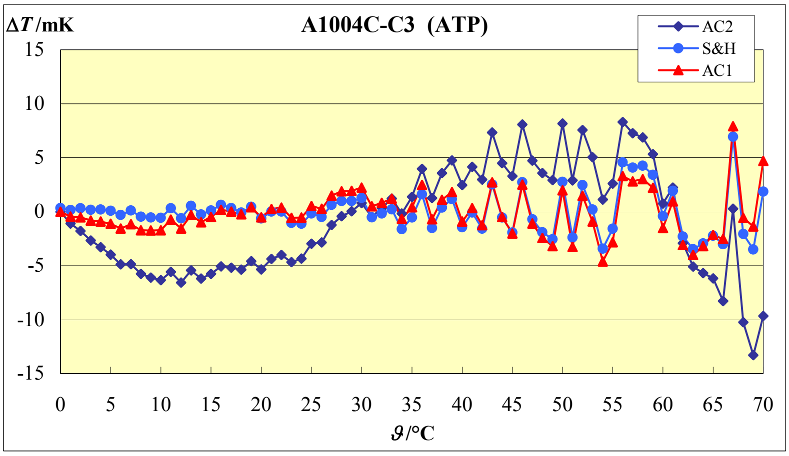

Fig. 3. Differences between the calculated and table value of temperature (in steps of 1 °C) for three approximation curves and NCTR of type Advanced Thermal Products A1004C-3

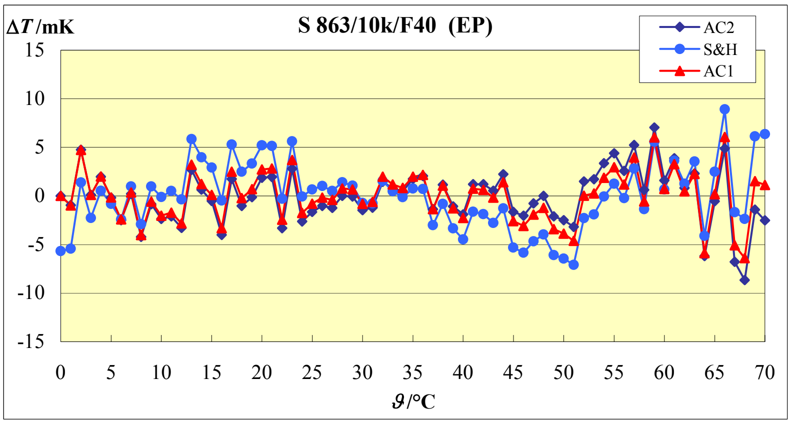

Fig. 4. Differences between the calculated and table value of temperature (in steps of 1 °C) for three approximation curves and NCTR of type Epcos (Siemens Matsushita) S 863/10k/F40

Based on the results presented in Figures 1 to 4, in Table III some numerical values of calculated ΔT for different approximation curves and types of NTCRs are expressed. For the range (0 to 70) °C, these are, respectively: maximum positive and maximum negative value, average (mean) value, and standard deviation for that series.

Table III. Comparison of ΔT calculated by (17) for three approximation curves: maximum positive and maximum negative value, average value, and standard deviation of the data

	UUA41J1 (FE)			10K3A1W2 (BTS)			A1004C-C3 (ATP)			S 863/10k/F40 (EP)		
	AC1	AC2	S&H	AC1	AC2	S&H	AC1	AC2	S&H	AC1	AC2	S&H
$-\Delta T_{max}$/mK	-8,61	-8,03	-11,91	-4,42	-9,84	-3,76	-4,62	-13,30	-3,51	-6,38	-8,65	-7,08
$+\Delta T_{max}$/mK	10,51	11,28	9,51	31,94	27,87	32,34	7,91	8,29	6,95	6,06	7,05	8,91
ΔT_{av}/mK	-0,13	-0,16	0,00	-0,15	-0,77	0,00	-0,12	-0,89	0,00	0,01	-0,08	0,00
ΔT_{sdev}/mK	3,85	3,92	4,82	3,99	5,43	3,95	2,07	4,95	1,93	2,53	2,74	3,44

IV. Conclusion

For four different types of NTCRs, which have resistance $R_N = 10000\ \Omega$ at a temperature of 25 °C and limits of error ±0,2 °C in the range (0 to 70) °C, two approximation curves were determined, a three-parameters AC1 and two-parameters AC2. For both of them, as well as for the known S&H approximation curve, the parameters were calculated by using the least-squares method and the table data for those NTCRs. This enables the calculation of the unknown temperature of interest for each value of resistance. Of course, this procedure can be done for each NTCR sensor, taking into the calculation the associated pairs of known values $R_{Ti} \leftrightarrow \vartheta_i$ (obtained, for instance, during its calibration) and in that case it is possible to calculate the associated parameters of particular sensor and chosen approximation curve.

To compare three different approximation curves, we used the same table data of resistance in steps of 1 °C as "measured" resistance from which we calculated unknown temperature, as well as the difference to the associated temperature pointed in the table. The presented results show a very good agreement of curves, with their mutual differences within a few millikelvin, and the differences to the table data within no more than ±15 mK (except only one point for BTS sensor). For the uni-curve FE and EP sensors there is no meaningful difference between three approximation curves, and therefore a two-parameters AC2 curve could be a better choice in comparison with three-parameters AC1 and S&H curves. If we analyse the results of BTS sensor, for which the table data of resistance are expressed with one decimal place (for all other types the data are integer numbers), two-parameters curve exhibit somewhat larger differences ΔT comparing to the three-parameters curves (Fig. 2). For ATP type it is interesting that the standard deviation of ΔT values is ≈2,5 times higher for AC2 curve in comparison to the three-parameters curves.

Finally, we can conclude that two-parameters approximation curve AC2 can be very convenient for the use with some NTCRs for the calculation of unknown temperature from the measured resistance.

References

[1] Fenwal Electronics: Thermistoren UNI Kurve; Typ: UUA41J1, UUA41J8; 10000 ohms bei 25 °C.

[2] BetaTHERM Sensors, http://www.betatherm.com; BetaCURVE Thermistor Series III 10K3A1W2.

[3] Advanced Thermal Products, http://www.atpsensor.com/; coated chip thermistor A1004-C3.

[4] Epcos, http://www.epcos.com; uni-curve NTC thermistor S 863/10k/F40.

[5] V. Bego, J. Butorac, D. Ilić, "PC-based regulator for oil ultrathermostat", *Proc. of the 10th IMEKO TC-4 Symposium*, vol. I, pp. 201–204, Naples, September 17–18, 1998.

[6] V. Bego, D. Ilić, N. Hlupić, "Temperature measurement by means of NTC resistors", *Proc. of the 10th IMEKO TC-4 Symposium*, vol. II, pp. 783–787, Naples, September 17–18, 1998.

[7] I.S. Steinhart, S.R. Hart, "Calibration curves for thermistors", *Deep Sea Research*, vol. 15, p. 497, 1968.