# Parameters of Band Pass ΣΔ ADC and the Comparison with the Standard Ones

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***Abstract-*** The bandpass sigma delta analog-digital converters (BP ΣΔ-ADC) represent a subgroup from the more general group of the unconventional ADCs. The BP ΣΔ-ADCs are mostly embedded into digital communication systems (DCS) in various applications. Their main task is to provide a frequency down conversion along with the conversion of chosen parameter of analog signal into digit. These converters are analog front end for DCS like software radios, UMTS, GSM and GPS systems. Parameters for characterisation of BP ΣΔ-ADC are until now taken intuitively and they are different from the standard conventional ADCs. The paper reviews existing parameters for conventional ADC with the aim to midificate them for BP ΣΔ-ADC.

## I. Introduction

The typical DCS architecture with minimum of analog components is shown in Fig.1. The analog front end represented by Linear Input Amplifier (LNA) and Analog-to Digital Converter (ADC) has to process full signal bandwidth for which the DCS has been designed. In order to cover all services to be supported by the DCS a limited frequency band has to be selected out of the full band by means of intermediate frequency filtering and analog to digital conversion. The digital signal processing block from the output of ADC performs sample rate conversion and channelisation. Channelisation comprises all tasks necessary to select the channel of interest. This includes conversion to baseband, channel filtering and possible despreading. In the future new different standards will be developed and applied and therefore no unification of them can be expected. The future DCS terminals will be designed for different services in different networks. Moreover, the DCS manufacturers can reduce the cost of their products by unifying the hardware platform. Looking at existing standards we can recognise three basic principles of sharing the limited resource frequency bandwidth. Those principles are code, time and frequency division multiplexing access (CDMA, TDMA and FDMA).
Selecting the bandwidth Δf to be digitized the only one basic signal characteristics is of highest importance and that could be named bandwidth-dynamic range trade-off. The dynamic range is proportional to the signal to noise ratio (SNR). Given a fixed digitization bandwidth Δf, the dynamical range of a mobile communications signals diminishes as the channel bandwidth increases. A fixed digitization bandwidth means that independent of the current standard of operation the sample rate and antialiasing filter are fixed. The mentioned trade-off suggests that a very high dynamic range is the only necessary in relatively narrow bands where the channels of interest lie.

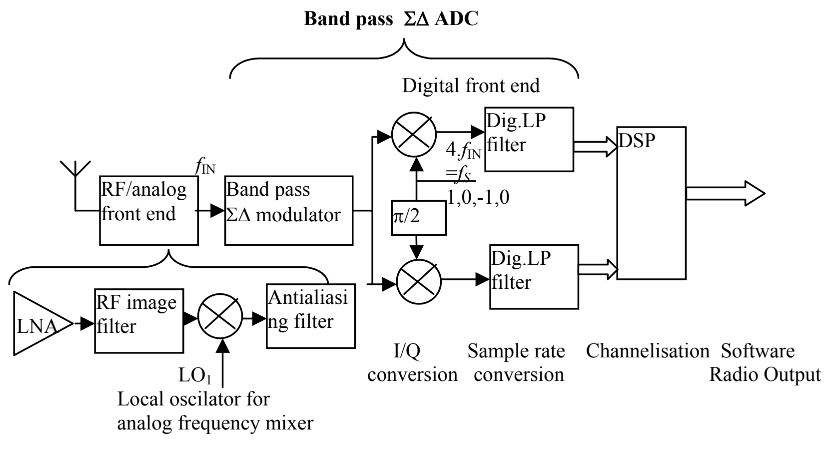


Fig.1 Architecture of the digital communication system with BP ΣΔ analog to digital converter

The high dynamic range in connection with relatively wide bandwidth makes ADC the key component of every software radio. Analog to digital conversion is to be performed either at the radio or interferer frequency. In order to simplify farther analysis let consider $2^N$ level-QAM modulation. It is based on the principle that harmonic carrier signal posses $2^{N}{}_1$ phase positions and $2^{N}{}_2$ different amplitudes where $N=N_1+N_2$. If the $N_1=4$ and $N_2=0$ the 4-QAM modulation is identical with the Quadruple Phase Shift Keying (QPSK). If the $N_1=8$ and $N_2=0$ the 8-QAM modulation is identical with 8-th level Phase Shift Keying (8-PSK). The case of $N_1=8$ and $N_2=8$ we are speaking about 64-QAM modulation. Here the phasor gets 64 position in the phase plain mutually shifted about $\pi/2$ with 8 different amplitudes. Besides flash ADC the band pass sigma delta ADC are the AD converters utilised in the software radio [1].

In contrast to the theoretically uniform noise transfer characteristic of the flash ADC, the band pass sigma delta ADC (BP ΣΔ-ADC) are noise shaping converters. BP ΣΔ-ADC are split into ΣΔ modulator and digital demodulator. Digital demodulator is realised by the multiplying block performing conversion into I/Q components and digital low pass filter that together carry out the digital down conversion (Fig.2). The relation between the input frequency $f_{IN}$ and the sampling frequency $f_S$ is $f_S=4.U.f_{IN}$. The coefficient U is being defined as conversion ratio. The block diagram of the second order BP ΣΔ-ADC is shown in Fig.2.

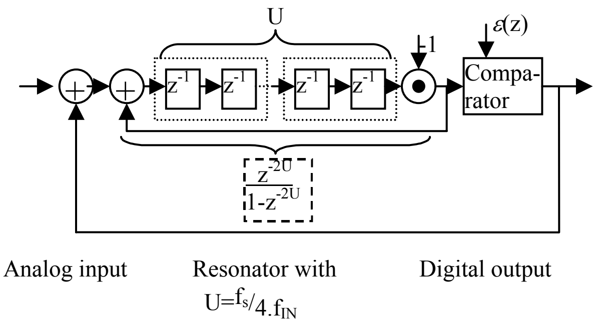


Fig.2. Structure of the BP ΣΔ-ADC

## II. Main parameters of BP ΣΔ-ADC

I/Q conversion by BP ΣΔ-ADC for U=1 can be performed by multiplying the output binary flux from BP ΣΔ modulator with the sequences [0,1,0,-1] and [1,0,-1,0]. The first sequence generates the digital samples proportional to the real part and the second sequence gives signal proportional to the imaginary part of the input frequency $f_{IN}$. The resonator transfer characteristic $H_{res}(z)$ is $H_{res}(z)=\frac{-z^{-2}}{1-z^{-2}}$. The output signal $B_{OUT}(z)$ in the z-domain is determined by the input signal $B_{IN}(z)$ multiplied by the signal transfer function $STF(z)$; the additional quantisation noise $\varepsilon(z)$ is shaped by the noise transfer function $NTF(z)$.

$$B_{OUT}(z)=B_{IN}(z).STF(z)+\varepsilon(z)NTF(z)=B_{IN}(z).z^{-2}+\varepsilon(z)\left(1-z^{-2}\right) \tag{1}$$

The main feature of BP ΣΔ-ADC is the direct conversion of a parameter from the input periodic signal with the peak value $A$ to its digital representation $k$. Already this first statement is vagueness and needs more precise definition. Which is the input signal parameter whose digital value is being provided at the output of BP ΣΔ-ADC?

For the harmonic signal with the amplitude $A$ the data stream at the output of I/Q multiplier represents the phasor of the input signal which carries the modulated information. In general, the period of the phasor changes at the input signal is $4L/f_S$,, where $L$ is the output digital low pass filter length. Taking in the account the periodicity of the input signal $b_{IN}(i)=b_{IN}(i+4.j)$ the final digital value at the filter output for I and Q component is given by

$$\begin{aligned} k_{OUT,I}(4L.j)&=\text{round}\left[\frac{\frac{1}{4L}\sum_{i=0}^{4L-1}b_{IN}\left(4L(j-1)-2+i\right).\sin\left(\frac{\pi}{2}i\right)}{A}\right]=\frac{\frac{1}{4L}\sum_{i=0}^{4L-1}b_{IN}\left(4L(j-1)-2+i\right).\sin\left(\frac{\pi}{2}i\right)}{A}+\varepsilon_{NS}(4L.j)\\ k_{OUT,Q}(4L.j)&=\text{round}\left[\frac{\frac{1}{4L}\sum_{i=0}^{4L-1}b_{IN}\left(4L(j-1)-2+i\right).\cos\left(\frac{\pi}{2}i\right)}{A}\right]=\frac{\frac{1}{4L}\sum_{i=0}^{4L-1}b_{IN}\left(4L(j-1)-2+i\right).\cos\left(\frac{\pi}{2}i\right)}{A}+\varepsilon_{NS}(4L.j)\end{aligned} \tag{2}$$

Here the operation **round[]** represents rounding of the analog value to its integer part. The quantization noise is shaped by the resonator in the BP ΣΔ modulator according to the formula in $z$-domain:

$$\varepsilon_{NS}(z) = \frac{\varepsilon(z)\left(1 - z^{-2}\right)}{A} \tag{3}$$

The noise spectrum is symmetrically spread around the central frequency $f_{IN}$ in the frequency range <0;2$f_{IN}$ >. Transforming to the frequency domain the quantisation power is

$$P_{NS}^2 = \frac{2}{3.f_S} \int_{f_{IN}-\frac{\Delta f}{2}}^{f_{IN}+\frac{\Delta f}{2}} \left|1 - e^{-j4\pi T_S}\right|^2 df = \frac{4}{3f_S}\left[\Delta f - \frac{f_S}{2\pi}\sin\left(\frac{2\pi}{f_s}\Delta f\right)\right] \tag{4}$$

where $f_S=4f_{IN}$. Approximating the second term by first three components of the Taylor´s series

$$P_{NS}^2 = \frac{4}{3f_S}\left[\Delta f - \Delta f + \frac{f_S}{2\pi}\frac{1}{3!}\left(\frac{2\pi\Delta f}{f_S}\right)^3\right] = \frac{\pi^2}{9}\left(\frac{2\Delta f}{f_S}\right)^3 \tag{5}$$

The output signal I or Q is maximal when the phase shift between input signal and multiplying harmonic component sin(iπ/2) or cos(iπ/2) is zero. I is maximal for minimal Q and vice versa. In general, the output digital value for coherency between input signal and multiplying harmonic component is the proportional to the sum of values of the input signal in the first and inverted value in the third quarter of the one signal period.

$$k(4Lj) = \text{round}\left[\frac{\frac{1}{4L}\left\{\sum_{i=0}^{L-1} b_{IN}\left(4L(j-1)+1+i.4\right) - \sum_{i=0}^{L-1} b_{IN}\left(4L(j-1)+3+i.4\right)\right\}}{A}\right]$$
$$k(4Lj) = \text{round}\left[g(b_{IN}(1), b_{IN}(2), ..b_{IN}(L/2))\right] \tag{6}$$

Formula (6) represents mathematical description of the BP ΣΔ-ADC with the sampling frequency $f_S=4.f_{IN.}$ The minimal down sampling ratio is 4 and the digital values could be taken in the intervals $4/f_S$. Doubling the conversion ratio U=2 requires the multiplication by the sequences [0,1,1,1,0,-1,-1,-1] and [1,1, 0, -1,-1,-1,0,1] both for real and imaginary part of the input frequency $f_{IN}$. For increasing value of $U$ the transfer characteristic converge to the digital representation of the mean value multiplied by the phase shift.

The transfer characteristic (6) is the relation between the input modulating parameter $p(iT_s)$ in the time window $T_w=4LT_s$ and the corresponding mean digital value $k(j4LT_s)$ at the output. Each transition level $T(k)$ in the transfer characteristic using the analogy with the parameters of the standard ADC should be defined as the value of the input parameter which causes the changes between two neighbouring digital values $k$,($k$+1) with the probability 50%:50%.

Taking in the account the operation mode, the BP ΣΔ-ADC is closed to the digital measuring instruments where the measured signal parameter is the converted to the digital output value during the measuring time $T_w=4LT_s$. For this reason more convenient parameters describing properties of BP ΣΔ-ADC are those of measuring instruments. On the contrary, the conventional ADC converts the input signal from analog to the digital domain without changes of their form and frequency spectra.

This fact suppresses the sense of some ADC parameters as defined in the IEEE standards. For example, the integral and differential nonlinearity of the conventional ADC describe signal distortion. The deviation from the linear relation in the conversion characteristic in the BP ΣΔ-ADC does not create high frequency components. It is just deviation from the transfer characteristic determined by the maximal deviation $\Delta_{MAX}$. In contrary to the conventional ADC, the quantization noise of the BP ΣΔ ADC is more close to the uncertainty as used for the measuring instruments.

Taking in the account the BP ΣΔ-ADC as an instrument measuring the deviation of the digital result to the ideal input value of the modulating parameter $p(iT_s)$ consists of the random and the measuring error. The final error introduced into converted signal consists of various sources:

1. Random noise superimposed to the input signal. Because of synchronous processing of the input signal the incoherent random noise will be suppressed in the output digital low pass filter.
2. Quantisation noise of the comparator in the structure of the BP ΣΔ-ADC. This noise is shaped by the frequency dependent feedback.
3. The deviation of real transfer characteristic as the relation between digital output and input modulating parameter $p(iT_s)$ to the ideal one. This error is caused by the various effects in the realisation of the converter. In general, the error function could be expressed by Volterra power series [6], which for static case is simplified by polynomial function.

Using the analogy to the measuring instruments the more effective metrological parameter should be the combined uncertainty of the conversion procedure [2],[3]. The first error source could be expressed by the Gaussian white noise with the standard deviation σ. The second error source is represented by the quantisation noise which power is defined in (5).
The third error source consists of following components:

1. Gain error $\Delta G$ as the difference of the transfer characteristic slope to the ideal slope equal to $1/A$. The gain error is caused by the changes in the amplification of the input buffering amplifier.
2. Offset error $O$ caused by the buffering amplifier and feedback circuit.
3. Deviation between real and linearised transfer characteristic (after compensation of the offset and gain error) $\Delta_{MAX}$, which is caused by the various factors like hysteresis, saturation of circuit components etc.

The combined uncertainty of the expression (6) is according to [2] caused by the components with systematic impact (i) noise superimposed on the measured samples and components with casual impact, (ii) gain error, (iii) offset error (vi) error of the transfer characteristic for the modulated parameter $p(j4LT_s)$ and (v) the quantisation noise. A needed hypothesis is the zero mean value of all error sources and their mutual independence.

$$US(k)=\sqrt{\sum_{1}^{L/2}\left(\sigma_n\frac{\partial g}{\partial b_{IN}}\right)^2+\left(\sum_{1}^{L/2}\sigma_G b_{IN}\frac{\partial g}{\partial b_{IN}}\right)^2+\left(\sum_{1}^{L/2}\sigma_O\frac{\partial g}{\partial b_{IN}}\right)^2+\sigma_{MAX}^2+\sigma_Q^2} \tag{7}$$

Taking in account that the input values $b_{IN}$ and their partial derivation, $\partial g/\partial b_{IN}$ are alternating by the sign for even and odd samples, the combined uncertainty for the BP ΣΔ-ADC could be simplified

$$US(k)=\sqrt{\sigma_G^2\left(\sum_{l=1}^{L/2}\frac{|b_{IN}(l)|}{4LA}\right)^2+\left(\sigma_n^2+\sigma_O^2\right)\sum_{1}^{L/2}\left(\frac{1}{4LA}\right)^2+\sigma_{MAX}^2+\frac{\pi^2}{9}\left(\frac{2\Delta f}{f_S}\right)^3} \tag{8}$$

The quantisation noise depends on the frequency band of the low pass digital filter characterised by the window function in the frequency domain. The qauntisation noise will be expressed by the formula:

$$\sigma_Q^2=\frac{\pi^2}{9}\left(\frac{M}{L}\right)^3 \tag{9}$$

where $M$ is determined by the type of the window according to Table 1.

Table 1 Coefficients of the digital low pass filter

| *Window* | *Coefficient M* | *Maximal stopband ripple* [dB] |
|---|---|---|
| Rectangular | 0.9 | -21 |
| Hanning | 3.1 | -44 |
| Hamming | 3.3 | -53 |
| Blackman | 5.5 | -74 |

The combined uncertainty is determined by the gain error multiplied by the averaged input value, the thermal noise added to the analog input samples, the dispersion related to the maximal error of the transfer characteristic and the quantisation noise. If the distributions of gain error, maximal error and offset error are considered as uniform with maximal values $\pm b_G$, $\pm p_{MAX}$ $\pm b_O$, the corresponding values of the dispersion are $\sigma_G=b_G/\sqrt{3}$; $\sigma_{MAX}=p_{MAX}/\sqrt{3}$; and $\sigma_O=b_O/\sqrt{3}$. The superimposed noise has Gaussian distribution where the $\sigma_n=u_{TRMS}$ is equivalent to the effective value of the noise at the output of LNA amplifier.
The effective number of bits (ENOB) could be used for the determining the effective resolution of BP ΣΔ-ADC. The effective number of bits at the output of digital LP filter will be increased comparing the value of bits at the output of the ΣΔ modulator according to formula

$$ENOB=1-\log_2\left(\frac{US(k)}{1/\sqrt{3}}\right) \tag{10}$$

The higher harmonics caused by the nonlinearity of the buffering amplifier are out of studied frequency range. For this reason the parameter signal-to-noise and distortion ratio (SINAD) and total harmonic distortion (THD) could be discussed.
The only one parasitic frequency component which is converted together with the input frequency $f_{IN}$ is the frequency $3f_S/4=3f_{IN}$. This third harmonics of the processed input signal could be generated from the input

amplifier (LNA). The BP ΣΔ-ADC testing has to be performed using a harmonic generator with notch filter and neglected value of higher harmonics.
The frequency dependence of the chosen parameter $p(t)$ of the input signal $b_{IN}$ is quite difficult to implement and its predicative force is week. The digital communication systems utilise pulse changes of the modulating parameter $p(t)$. That is why the very important parameter of BP ΣΔ-ADC seems to be its dynamic behaviours represented by the settling time$\tau_0=4LT_s$. It could be used as defined in the standards for LP ADC [4],[5]. Another possibility is to define threshold parameter – the modulation speed for which the output digital values are able to settled in two mutually distinguished values.

## III.Conclusion

The band pass ΣΔ-ADC as a representative of the unconventional ADC could be understood as a digital instrument measuring chosen signal parameter. From the point of view, its parameters were discussed and compared with the parameters of the standard ADC.

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