The Interesting Effect of Non-uniform Sampling

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Abstract- It is well known, that the proper ratio between frequencies of testing signal and sampling signal is required, during a dynamic testing of waveform recorders or analog-to-digital converters. A lot of testing methods suggest applying a coherent sampling, which assures a uniform distribution of samples between 0 and 2π. The paper presents results obtained with Matlab numerical simulations showing influence of the sampling strategy on establishing performances of analog-to-digital converters. It was shown, that the results of tested conversions are different if a non-coherent sampling is applied. A proposal of sampling is presented in case of using a sinewave as an exciting, input signal.

I. Introduction

The characteristics of analog-to-digital converters (A/D converters) are established and assessed on the basis of the results of the conversion of a sinusoidal signal. This method of measurement has been selected because it is relatively easy to generate sinusoidal signals with the required good enough performances. Considering sampling strategy, the best situation is when all of M-samples are recorded during one period of exciting signal. In this case, there is no risk of recording samples of the same phase. Moreover, the bigger number of signal periods during recording, the more accurate frequency of exciting signal is required [1]. But selection of proper frequency of exciting signal is must meet requirements of two main limitations:

1) the signal frequency must be acceptable by the device under test,
2) the signal frequency must be included within the range of availability by signal generator.

Therefore, coherent sampling of periodic exciting signals is commonly used for recording data during the test [2][3]. The condition of coherent sampling is shown below, see Eq. 1:

$$\frac{f_X}{f_S} = \frac{L}{M} , \tag{1}$$

with: f_x – exciting signal frequency, f_s – sampling frequency, L and M - being two mutually prime numbers, and $M \neq 0$.

The equation (1) can be expressed in more intelligible manner:

$$M \cdot T_S = L \cdot T_X , \tag{2}$$

with: T_x – exciting signal period, T_s – sampling period.

It means, that during L periods T_x M samples are recorded. The main advantage of this kind of sampling is uniform location of all M samples in one equivalent sinewave period.

A simple example of coherent sampling for *fx/fs=5/22* is depicted in Fig. 1. In general, (without specific ratio $\frac{f_X}{f_S} = Q + \frac{1}{M}$, with Q being integer) to obtain proper shape of sampled signal, all samples must be reshuffled. For example the sample no. 9 is placed second after reshuffling into equivalent period (see on the right Fig. 1).

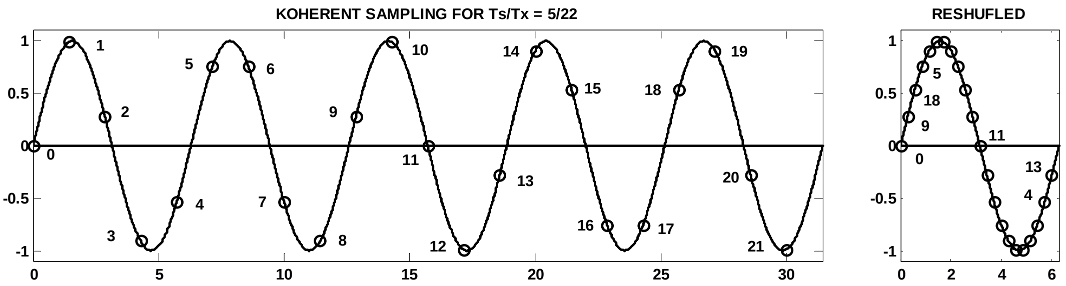


Figure 1. A coherent sampling with f_X/f_S=5/22 ratio.

The rule shown below (Eq. 3) can be applied to establish a correct sequence of recorded samples. Assume a notation:

$$p_M^L(n) = (n \cdot L)\, modulo\ M \qquad n = 0, 1, 2, 3, \ldots, M\text{-}1 \qquad (3)$$

with: $p_M^L(n)$ – denotes position of *n*-sample after reshuffled M samples in L periods of exciting signal.

$(A)\, mod\,ulo\ B$ – means the remainder of the division A/B.

For our example where N=5 and M=22, using (Eq. 3) we obtain the following sequence:

$$p_{22}^5(n) = (5 \cdot n) modulo\, 22 = \left\{ \begin{array}{lllll} 0, & 5, & 10, & 15, & 20, \\ 3, & 8, & 13, & 18, & 1, \\ 6, & 11, & 16, & 21, & 4, \\ 9, & 14, & 19, & 2, & 7, \\ 12, & 17 & \end{array} \right\} \qquad (4)$$

The numbers presented in (Eq. 4) represent the position of successively recorded samples. Note, that the numbers were intentionally arranged into five rows, to depict some regularity in columns.
The phase φ_n of *n*-sample can be established as follows:

$$\varphi_n = 2 \cdot \pi \cdot \left[\left(\frac{n \cdot L}{M} \right) mod\,ulo\, M \right] \qquad n = 0, 1, 2, 3, \ldots, M\text{-}1 \qquad (5)$$

II. Effects of non-uniform sampling

The simplified block diagram of ADC testing system with coherent sampling is shown below, see Fig. 2. To assure the condition of coherence (Eq. 1) the synchronization of generators providing the exciting signal and the sampling clock is necessary.

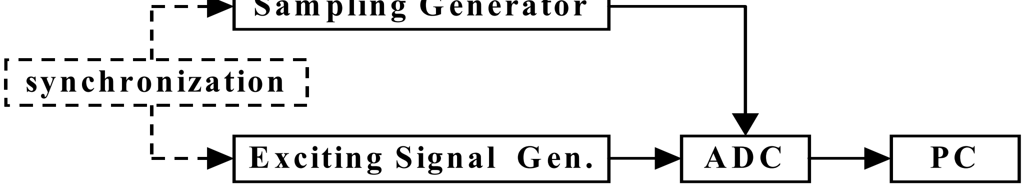
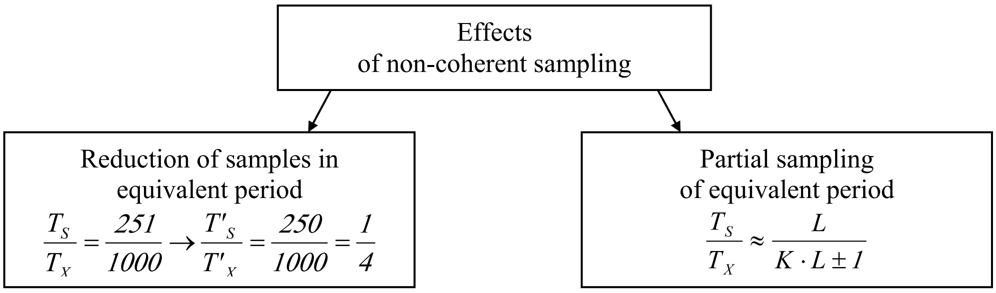


Figure 2. A simplified block diagram of ADC testing system with coherent sampling.

But in general, the condition (Eq. 1) can't be performed because of non-ideal nature of applied instrumentation during the test. Both, the sinusoidal period $T'_x \rightarrow T_x + \Delta T_x$ and the sampling interval $T'_s \rightarrow T_s + \Delta T_s$ can be biased. This deformation of biases may result in reduction of samples in equivalent period or non-uniform location of samples into equivalent interval $< 0 ; 2\pi >$.



Especially, when the condition of coherence can be expressed as:

$$\frac{T_S}{T_X} = \frac{L}{M} = \frac{L}{K \cdot L \pm 1} \ , \tag{6}$$

the samples join in some regions. Note, that numbers L and $K{\cdot}L{\pm}1$ are always mutualy prime numbers with L and K being intigers. This strategy for selection ofT_x/T_s ratio is proposed in [1].
The example of this effect simulated in Matlab is shown below (for $T_s/T_x = 10/49$, see Fig. 3a and $T'_s/T'_x = 10/49.2$, see Fig. 3b).
As marked on Fig. 3b, there are some non-sampled regions,. after reshufling all recorded samples into one single period of equivalent time. It means, that during the test of ADC, some of quantizations levels of the device are omited. Therefore, it can be interpreted as "missing bits".

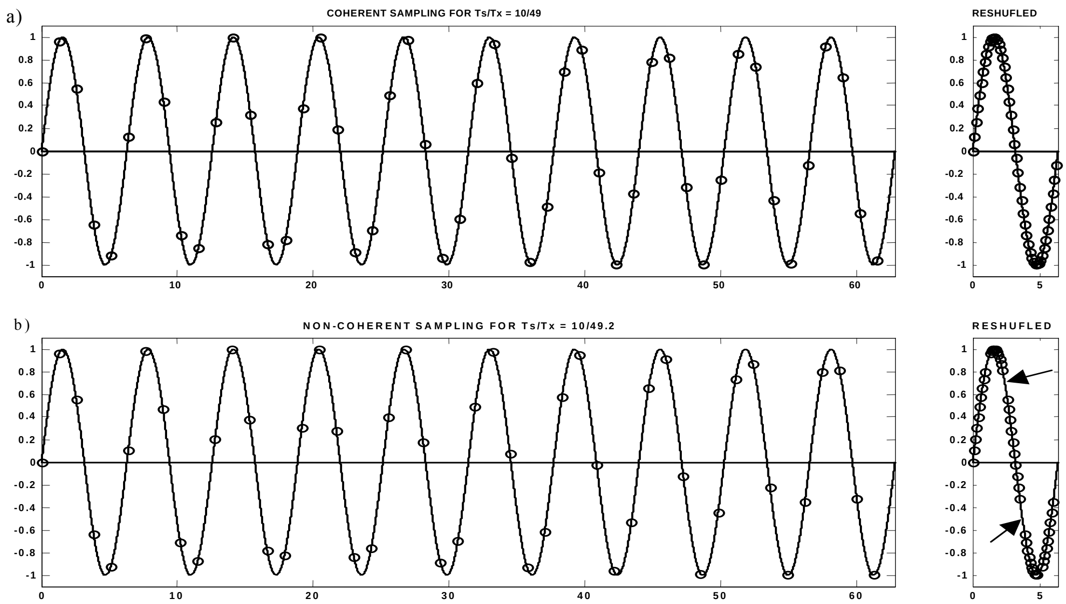


Fig. 3. An example of a) coherent sampling, b) specific non-coherent sampling of sinewave signal.

In the example, we recorded 49 samples of sinwave for assumed Ts/Tx = 10/49 ratio. But because of frequency deviation, the T_S/T_X ratio was changed to:

$$\frac{T_S}{T_X} \rightarrow \frac{T'_S}{T'_X} = \frac{10}{49{,}2} = \frac{100}{492} = \frac{25}{123} \tag{7}$$

Then, we obtain a new coherent sampling where L = 25 and M = 123 being two mutual numbers. But because of the condition (6) and only 49 recorded samples, some of non-sampled regions appeared in equivalent period. The (Eq. 8) depicts the difference of samples arrangement for both, ideal and non-ideal sampling.
The width of one non-sampled region Δ, for example, from sample no. 18 (the lowest position of first column eq. 8 right side) to sample no. 25 (the higest position of second column eq. 8 right side) can be calculated from (Eq. 9) as a part of T_X.

$$
\begin{array}{llllll}
\multicolumn{6}{l}{p_{49}^{10}(n) = (10 \cdot n) Modulo 49 =} \\
= \{ & 0, & 10, & 20, & 30, & 40, \\
& 1, & 11, & 21, & 31, & 41, \\
& 2, & 12, & 22, & 32, & 42, \\
& 3, & 13, & 23, & 33, & 43, \\
& 4, & 14, & 24, & 34, & 44, \\
& 5, & 15, & 25, & 35, & 45, \\
& 6, & 16, & 26, & 36, & 46, \\
& 7, & 17, & 27, & 37, & 47, \\
& 8, & 18, & 28, & 38, & 48, \\
& 9, & 19, & 29, & 39 & \}
\end{array}
\qquad
\begin{array}{llllll}
\multicolumn{6}{l}{p_{123}^{25}(n) = (25 \cdot n) Modulo 123 =} \\
= \{ & 0, & 25, & 50, & 75, & 100, \\
& 2, & 27, & 52, & 77, & 102, \\
& 4, & 29, & 54, & 79, & 104, \\
& 6, & 31, & 56, & 81, & 106, \\
& 8, & 33, & 58, & 83, & 108, \\
& 10, & 35, & 60, & 85, & 110 \\
& 12, & 37, & 62, & 87, & 112, \\
& 14, & 39, & 64, & 89, & 114, \\
& 16, & 41, & 66, & 91, & 116, \\
& 18, & 43, & 68, & 93 & \}
\end{array}
\tag{8}
$$

for both sets with $n = \{0, 1, 2, 3, ..., 48, 49\}$.

$$\Delta = \frac{25 - 18}{123} \cdot T_X = \frac{7}{123} \cdot T_X \tag{9}$$

III. A proposal of sampling

There is also an interesting aspect of this specific non-uniform sampling. Using the sinewave as excitating signal in histogram method, it brings some difficulties because the probability density function of sinewave isn't uniform. Hence, often an "overdriving" of the device is proposed as a solution. However, it isn't nominal situation for the tested ADC. The similar effect like "overdriving" (but with exciting signal amplitude which equals full scale of ADC), we can obtain by manipulating T_s/T_x ratio and initial phase of sampling period. The non-uniform sampling is prepared in such a way that it moves the non-sampled regions to residua of sinewave, see Fig. 4.

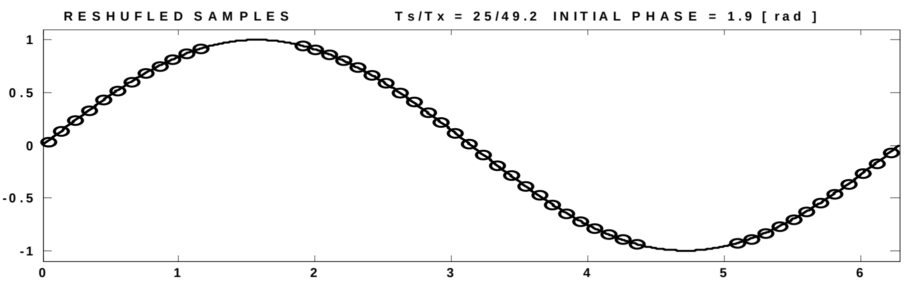


Figure 4. Reshufled samples to one equivalent period for sinewave with residua omited.

IV. Conclusions

During coherent sampling a specific kind of menace can appear. The samples of exciting signal join in some regions if the T_s/T_x ratio is biased. By combining sampling parameters, we can cause some non-sampled regions and move them to a specified location. A useful, similar to "overdriving" effect, can be obtained if residua of sinewave signal are omitted.

References

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[2] G. Chiorboli, C. Morandi, "About the number of records to be acquired for histogram testing of A/D converters using synchronous sinewave and clock generators", *Elsevier: Computer Standards & Interfaces*, Vol. 22, pp. 253-259, 2000.

[3] *IEEE Standard Terminology and Test Methods for Analog-to-Digital Converters*, IEEE Stand. 1241-2000, June 2001.