

# Hysteresis Distortions for Two-Tone Signals

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**Abstract**-As experimental evidence suggests that hysteresis type nonlinearities can be present on transmission lines, a hysteresis system is simulated numerically, and a two-tone signal is applied to its input. A FFT spectral analysis is performed on the output signal, and the dynamical behaviour of a third order beat is computed and compared to experimental data.

## I. Introduction

Telecommunications operators complain of an increased incidence of nonlinearities effects on transmissions, following the continuous increase in spectrum occupation [1]. The most harming effect of nonlinearities consists of the introduction of spurious spectral components in the signals frequency bands. Generally, polynomial, memoryless nonlinearities are considered in studies and mathematical models [2, 3]. However, evidence exist that hysteresis-like nonlinearities might also influence transmissions [4].

A lot of work has been done in research for models of basically linear systems that exhibit slight nonlinearities, such as amplifiers [5]. Memoryless, polynomial type models have been proven useful. Of many spurious sinusoidal components, third order beats are very close of the signal band, or even in band, and are the most unwanted. As known, the logarithmic amplitude (or the logarithm of the power) of a third order beat introduced by a polynomial nonlinearity depends linearly on the equal logarithmic amplitudes (or powers) of the signals in the two-tone test method, and it increases three times faster than the logarithmic amplitude of a correct output (that is, the ratio of the two slopes is 3). Slopes smaller than three are reported in some papers of nonlinear optics [6]. A special importance is given lately to second order intermodulation products, as they can affect transmission in some interesting frequency bands [7]. A majority of authors consider the nonlinearities as a result of magnetic effects [1, 4, 8], and the slope of third-order products is 3; references based on experimental data regarding second order intermodulation products are made also in [4].

Measurements on actual transmission lines have been performed and experimental data exist [2, 3]. The results show that, while third order beats are still the most important spurious components, the ratio of the slopes defined above is lower than in a third order polynomial model, of about 2.6 - 2.8 when the two tones are separated in frequency by a relative amount of 1%. This fact indicates a more involved model for the nonlinearity than a polynomial one.

In this paper, we report the simulation of the behavior of a hysteresis system having a two tone signal as input. We found that third order beats resulted at the system output, and we calculated the slope of the straight line that fits best the dependency of the logarithm of its amplitude in function of the logarithm of the input. In this way, we investigated if predictions of hysteresis models might be in accordance to experimental data.

## II. Hysteresis Model

For a certain input, an example for the behavior of a hysteresis system, characterized by the existence of a major and of minor loops is represented in Fig. 1. The system starts on the major loop at point 1 and, as the input signal decreases, it approaches point 2. However, a sudden increase in the input forces the system to proceed on a minor loop, on the path 3-4. Then, pieces of minor loops such as 4-5, 5-6, and 6-5 are followed due to the input signal variations.

We implemented a Matlab program that simulates the behavior of a hysteresis system following the very convenient and flexible algorithm proposed in [9]. The input signal is denoted by  $x$ , and its values

are supposed to be absolutely bounded by 1 (i.e. the major loop corresponds to values of  $x$  between -1 and 1). As the nonlinearities we consider are weak, actual values of  $x$  are much smaller than unity. The flexibility stems from the existence of five parameters that determine the shape of the loops; this fact is very important, as actual shapes of hysteresis loops that might be present on transmission lines are not known for the present.

In the two-tone method, the input signal consists of two sine waves of equal amplitudes. When sampled, this signal reads

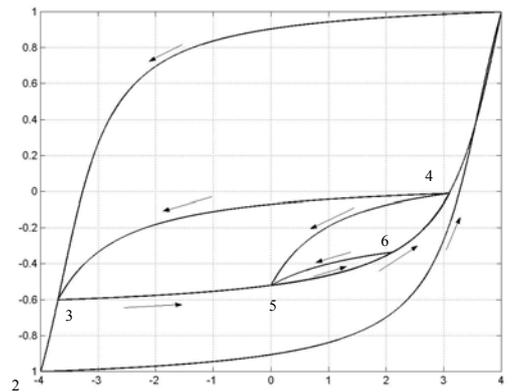


Fig. 1. Hysteresis characteristic.

$$x(n) = A \sin(2\pi F_1 n) + A \sin(2\pi F_2 n) \quad (1)$$

The discrete frequencies  $F_1$  and  $F_2$  are related to continuous time frequencies by  $F_i = f_i T_s$ , where  $T_s$  is the sampling interval. The input signal is represented in Fig. 2, and its spectrum in Fig. 3. Note that we have used very low discrete frequencies in order to avoid aliasing problems:  $F_1=0.0610$ ,  $F_2=0.0616$ .

Also note that  $\frac{F_2 - F_1}{F_2} = 1\%$ . In order to relate our simulation to experimental data, the sampling frequency is  $f_s = 14.76$  GHz.

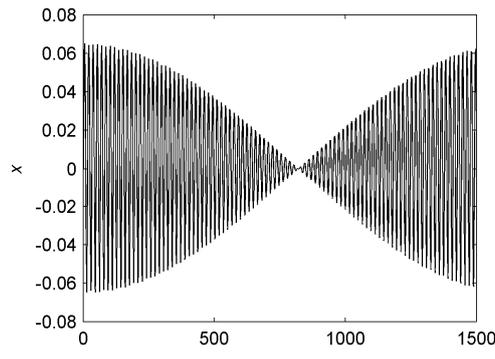


Fig. 2. Samples of the input signal.

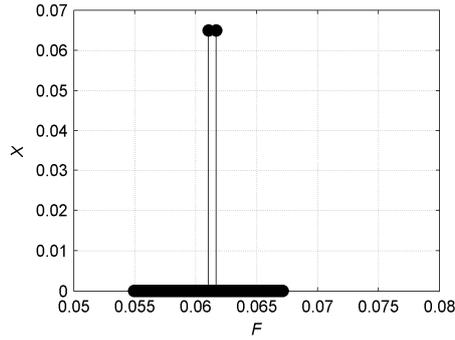


Fig. 5. Input spectrum.

### III. Simulation results

The signal  $x$  has been applied as input to a hysteresis system; a trace of the evolution of the output  $y$  is represented in Fig. 4. The time variation of  $y$  (part of the steady state, which has been also considered for spectral analysis) and its spectrum are reported in Fig. 5 and Fig. 6 respectively; a detail of the low frequency part of the output spectrum is represented in Fig. 7.

As it can be seen from Fig. 6, the baseband replicates at odd multiples of the baseband frequencies, and it is attenuated at high frequencies.

The detail of the frequency spectrum of the output  $y$  in Fig. 7 shows that the baseband is almost symmetrical, and it contains several beat components of which the third order are the most important (i.e. of frequencies  $2F_1 - F_2$  and  $2F_2 - F_1$ ).

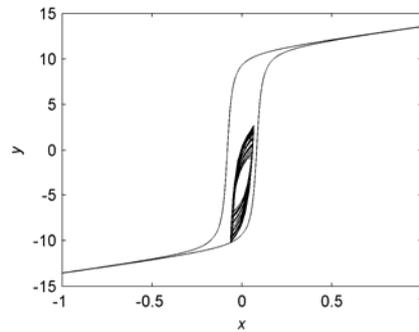


Fig. 4. Trace of the system evolution.

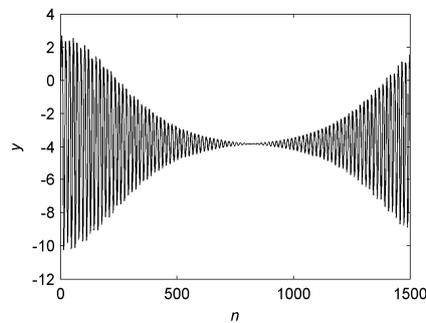


Fig. 5. Samples of the output signal.

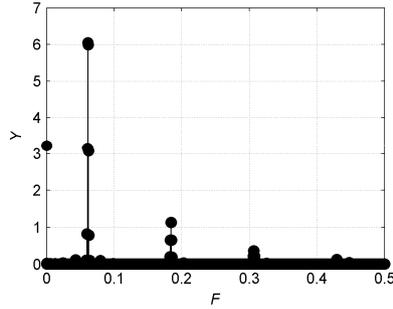


Fig. 6. Output spectrum.

As the results presented above are qualitatively in accordance to experimental data, we tested the slope of the straight line that approximates best the plot of the logarithm of the third order beat amplitude in function of the logarithm of one of the amplitudes of the input signal. Amplitudes of the input signal have been taken equidistant over a decade, ending at 0.065 in linear units. We found that there exist hysteresis loops such that the slope is in the interval 2.6 - 2.8 (see Fig. 8 for an example with slope 2.8).

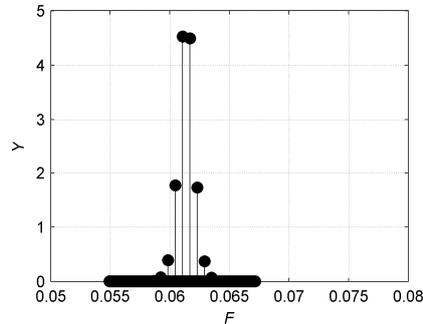


Fig. 7. Detail of the output spectrum.

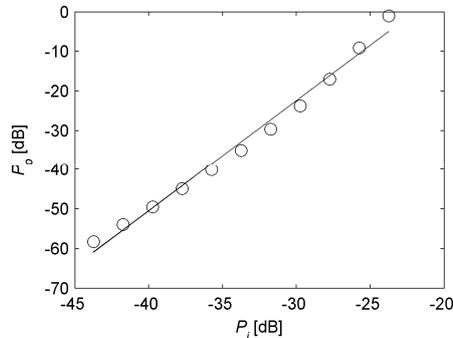


Fig. 8. Third order component power in function of the input power (arbitrary units).

#### IV Conclusions

We investigated whether a hysteresis model could explain the nonlinear behavior of transmission lines. As we disposed of experimental data gathered by the two-tone method, we implemented a computer program to simulate the response of a hysteresis loop to a two-tone signal. We found two important similarities between experimental and simulated behaviors: 1. the input signal baseband is replicated at the system output at odd multiples of the baseband frequencies and 2. there exist parameters of the hysteresis system for which the slope of the variation of the third order signal power in function of the input power in logarithmic coordinates is smaller than in the case of a third degree polynomial nonlinearity.

Experimental evidence shows that a premagnetisation of the material introduce second order nonlinearities. This and dependency on frequency will be investigated in the future.

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